F. Méot
BNL C-AD
SBU-SUNY PHY689

COMPUTERLAB, EXERCISE 1.2.4-2, SOLUTION

Abstract

Exercise 1.2.4-2 The cyclotron equation.

Contents

1 1.2.4-2.a

The cyclotron equation writes

$$\cos\phi = \cos\phi_{\mathrm{i}} - \frac{\pi}{\mathrm{e}\hat{\mathrm{V}}}\left[\frac{\omega_{\mathrm{rf}}}{2\mathrm{E}_0\omega_{\mathrm{rev}}}(\mathrm{E}^2 - \mathrm{E}_{\mathrm{i}}^2) - (\mathrm{E} - \mathrm{E}_{\mathrm{i}})\right] \tag{1}$$

where the indec i denotes injection parameters, ϕ is the phase of the RF when the particle arrives at the accelerating gap, $\mathrm{E} = \mathrm{W} + \mathrm{E}_0$ is the total energy with $\mathrm{E}_0 = \mathrm{m}_0\mathrm{c}^2$ the rest energy.

The value of W at maximum $\cos\phi$ is drawn from $\mathrm{d}(\cos\phi)/\mathrm{dW} = 0$ and writes

$$\mathrm{W}_{\mathrm{m}} = (\frac{\omega_{\mathrm{rev0}}}{\omega_{\mathrm{rf}}} - 1)\mathrm{E}_0 \tag{2}$$

$$\omega_{\mathrm{rf}} = \omega_{\mathrm{rev0}}/(1 + \mathrm{W}_{\mathrm{m}}/\mathrm{E}_0) \tag{3}$$

Taking $\mathrm{W} \approx 10\,\mathrm{MeV} \approx 10^{-2}E_0$ one gets $\frac{\omega_{\mathrm{rev0}}}{\omega_{\mathrm{rf}}} \approx 1 + 10^{-2}$, $\omega_{\mathrm{rf}} \approx 0.99\omega_{\mathrm{rev0}}$

Figure 1: Accelerated trajectory. Namely, a succession of half-circles, with increasing radius.

Optical sequence in zgoubi, accelerated orbit

```
Uniform field sector
 'OBJET'                                                                                                    1
64.62444403717985                 ! Reference: 200keV proton.
2                                 ! Generate a
1 1                               ! single particle.
4.087013 0. 0. 0. 0. 0.3162126 'o' ! p[MeV/c]=  6.126278, Brho[kG.cm]= 20.435064, kin-E[keV]=  20
1
 'PARTICUL'            ! This is required only because we want to get the time-of-flight                    2
PROTON

 'FAISTORE'                                                                                                 3
zgoubi.fai   #End
1
 'TOSCA'                                                                                                    4
0  2
1.  1. 1. 1.
HEADER_8
315 151 1 22.1 1.                 !  IZ=1 -> 2D ; MOD=22 -> polar map ; .MOD2=.1 -> one map file.
geneSectorMap_180deg.out
0 0 0 0
2
1.
2
0. 0. 0. 0.
 'FAISCEAU'                                                                                                 5

 'CAVITE'                                                                                                   6
3                                 ! dW = qVsin(phi_s), independent of time
0. 0.
100e3  1.57079632679

 'TOSCA'                                                                                                    7
0  2
1.  1. 1. 1.
HEADER_8
315 151 1 22.1   1.               ! Just for the fun : field maps can be (linearly) superimposed.
geneSectorMap_180deg.out
0 0 0 0
2
1.
2
0. 0. 0. 0.

 'FAISCEAU'  #End                                                                                           8

 'CAVITE'                                                                                                   9
3                                 ! dW = qVsin(phi_s), independent of time
0. 0.
100e3  1.57079632679

 'FAISCEAU'                                                                                                10

 'REBELOTE'            ! K = 99 : coordinates at end of previous pass are used as initial                  11
60  1.1 99             ! coordinates for the next pass ; idem for spin components.
!                      ! Note that, (i) Y0 remains constant (due to the "microtron configuration"),
!                      ! (ii) is updated by passage through CAVITE
 'FAISCEAU'                                                                                                12

 'SYSTEM'                                                                                                  13
6
cp gnuplot_zgoubi.plt.cmd gnuplot_zgoubi.plt_temp.cmd
sed -i 's@pause 2@pause 0@g' gnuplot_zgoubi.plt_temp.cmd
gnuplot < gnuplot_zgoubi.plt_temp.cmd
mv -f gnuplot_zgoubi.plt_XYLab.eps gnuplot_zgoubi.plt_XYLab_acceleration.eps
okular gnuplot_zgoubi.plt_XYLab_acceleration.eps &
rm -f gnuplot_zgoubi.plt_temp.cmd

 'END'                                                                                                     14
```

Plot trajectories, using gnuplot

```
set title "Plotted from file zgoubi.plt  \n From zgoubi's polar frame to lab frame \n u ($10 *cm2m *cos($22)):($10 *cm2m *sin($22)) " font "

set key maxcol 1
set key t r

#set logscale y

set xtics mirror font  "sans, 14"
set ytics mirror font  "sans, 14"

set xlabel 'Y * cos(X)  [m]' font  "sans, 14"
set ylabel 'Y * sin(X)  [m]' font  "sans, 14"

cm2m = 0.01
MeV2eV = 1e6
am = 938.27203
c = 2.99792458e8
pi = 4. * atan(1.)

NOEL_1 = 4   #   number of 1st TOSCA in zgoubi,plt (col. 42)
NOEL_2 = 7   #   number of 2nd TOSCA in zgoubi,plt (col. 42)

set size ratio -1

plot  \
   'zgoubi.plt' u ($42==NOEL_1 ? $10 *cm2m *cos($22) :1/0):($10 *cm2m *sin($22)) w l lc rgb 'red' tit 'x\_lab, y\_lab' ,\
   'zgoubi.plt' u ($42==NOEL_2 ? $10 *cm2m *cos($22+pi) :1/0):($10 *cm2m *sin($22+pi)) w l lc rgb 'blue' notit

     set terminal postscript eps blacktext color  enh  "Times-Sans" 12
       set output "gnuplot_zgoubi.plt_XYLab_acceleration.eps"
       replot
       set terminal X11
       unset output


pause 2   # don't change this: needed for proper running of sector180deg
exit
```

2 1.1-2.b - Momentum and energy, compare with theory

Theoretical curves need be added.

Figure 2: Momentum and total energy versus kinetic energy, compare with theory.

3 1.1-2.c - Normalized velocity, compare with theory

Theoretical curve need be added.

Figure 3: Normalized velocity $\beta = \mathrm{v}/\mathrm{c}$, compare with theory.

Plot trajectories, using gnuplot

```
set title "Plotted from file zgoubi.fai  \n Isochronism: T(R) is not exactly constant" font "sans, 16 \n ~ "

set key maxcol 1
set key spac 3
set key t c  font "Roman, 18"

#set logscale y

set xtics mirror font  "sans, 16"
set ytics nomirror font  "sans, 16"
set y2tics mirror font  "sans, 16"

# set size ratio 1.

set xlabel "E_k [MeV]" font  "sans, 18"
set ylabel "p [MeV/c]" font  "sans, 18"
set y2label "E [MeV/c]" font  "sans, 18"

cm2m = 0.01
MeV2eV = 1e6
am = 938.27203
c = 2.99792458e8

set xrange [0:]        # m
set yrange [0:]        # m

plot  \
   "zgoubi.fai" u ($24):(sqrt($25**2 -am*am)) axes x1y1 w p pt 4 lc rgb "red" tit "p(E_k)"  ,\
   "zgoubi.fai" u ($24):($25) axes x1y2 w p pt 5 lc rgb "blue" tit "E(E_k)"

#     set terminal postscript eps blacktext color  enh size 8cm,8cm "Times-Sans" 16
     set terminal postscript eps blacktext color  enh  "Times-Sans" 16
     set output "gnuplot_zgoubi.fai_E.vs.Ek.eps"
     replot
     set terminal X11
     unset output


pause 1

set xtics mirror font  "sans, 16"
set ytics mirror font  "sans, 16"
unset y2tics

# set size ratio 1.

set xlabel "E_k [MeV]" font  "sans, 18"
set ylabel "v/c" font  "sans, 18"
unset y2label

plot  \
   "zgoubi.fai" u ($24):(sqrt($25**2 -am*am)/$25) axes x1y1 w p pt 4 lc rgb "red" tit "{/Symbol b}(E_k)"

#     set terminal postscript eps blacktext color  enh size 8cm,8cm "Times-Sans" 16
     set terminal postscript eps blacktext color  enh  "Times-Sans" 16
     set output "gnuplot_zgoubi.fai_bta.vs.Ek.eps"
     replot
     set terminal X11
     unset output

pause 1
exit
```