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COMPUTERLAB, EXERCISE 1.2.3-1, SOLUTION

Abstract

Projected particle trajectories.

Contents

1 1.2.3-1.a - Horizontal and vertical trajectories around the ring

We use zgoubi input data material of the previous 1.2.2-2 exercise, which uses `DIPOLE`.

First, make sure that there is tight consistency between Y0 of 200 keV proton and RM in DIPOLE. The reason: RM id the reference radius in the defintion of the index, k. It has been chosen to take RM=orbital radius for 2 200 keV proton, this is an arbitray choice, but an easy way of checking the setting up of DIPOLE data. *i.e.*, on the one hand:

```
  Cyclotron, classical.
 'OBJET'                                                                                                 1
64.62444403717985                              ! BORO[kG cm] for 200keV proton
2
3  1
12.924889 0.   0.    0.    0.   1. 'm'         ! Y0 for 200keV
30.107899 0.   0.    0.    0.   2.23654451125 'm'    ! 1 MeV
75.754671 0.   0.    0.    0.   5.0063899693  'M'    ! 5 MeV. R=Brho/B=BORO*5.00639/5[kG]
1 1 1
```

and on the other hand:

```
 'DIPOLE'
0
60.  12.924889                           ! AT, RM
30.  5.  -0.03  0. 0.                    ! ACENT, B_RM, N, N', N''
0.  0.                                   ! EFB 1  hard-edge
```

The expected value for RM is BORO/5[kG]=12.924889. `FIT[2]` can be used to check that RM=Y0.

Fig. 1 shows the radial and axial motions over a few turns, at 200 keV and 1 MeV.

An estimate of the wave numbers can be obtained from the number of turns per oscillaiton, from zgoubi.plt content, as follows:

- inspection of x(s/C) for the 200 keV particle, plotting from zgoubi.plt around s/C$\in$ $[1.012 : 1.018]$ in the region of the maximum, shows that of x(s) reaches a maximum at $\mathrm{s/C} = 1.015^+$ (reducing the integration step size is a way to increase the accuracy on the location of the maximum). That makes (Eq. 1.7 in the course) $\nu_\mathrm{R} = \omega_\mathrm{R}//\omega_\mathrm{rev} =\approx 1.015^{-1} = 0.9852^-$ comparable to the expected value $\sqrt{1+\mathrm{k}} = \sqrt{1-0.03} = 0.9849$.

- inspection of y(s/C) for the 200 keV particle, plotting from zgoubi.plt around s/C$\in$ $[5.75 : 5.79]$ in the region of the maximum, shows that of y(s/C) reaches a maximum at $\mathrm{s/C} \approx 5.7725$. That makes $\nu_\mathrm{R} = \omega_\mathrm{R}//\omega_\mathrm{rev} =\approx 5.7725^{-1} = 0.1732$, which is the expected $\sqrt{-\mathrm{k}} = \sqrt{-0.03} = 0.1732$.

NOte in addition, $\nu_\mathrm{R}^2 + \nu_\mathrm{y}^2 =$, as expected.

Fig. 2 shows that the wave numbers at 1 MeV are different from the 200 keV value. Could that be expected? Explain.

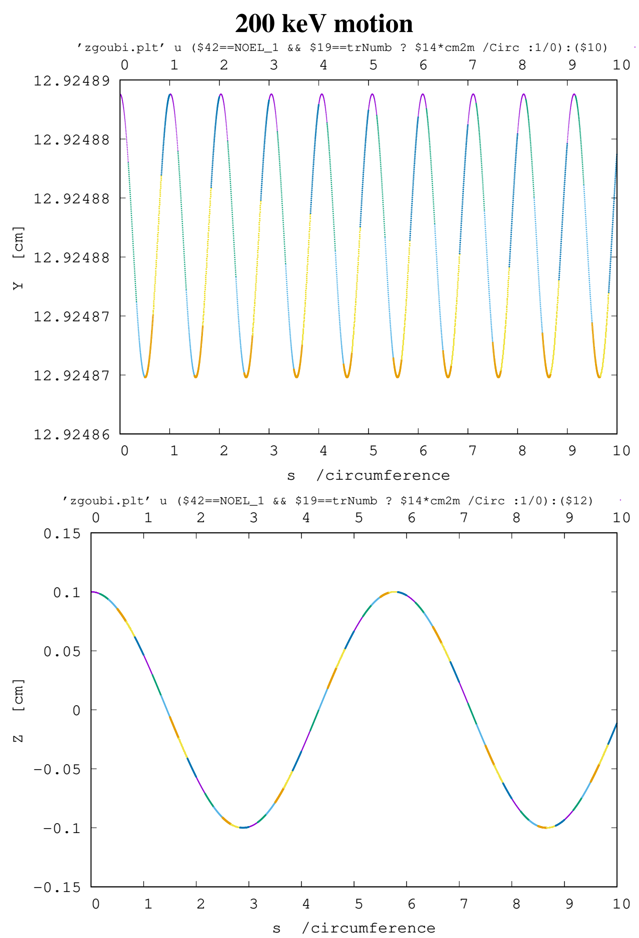

Figure 1: Radial and axial motion around the 200 keV closed orbit (which is a circle of radius $\mathrm{R}_{1\,\mathrm{MeV}} = 12.92$ cm, lying in the median plane). It can be observed that the radial (resp. axial) motion has an oscillation period of a little more than 1 turn (less than 6 turns), consistent with $\omega_\mathrm{R}//\omega_\mathrm{rev} = \sqrt{1+\mathrm{k}} = \sqrt{1-0.03} \approx 1.0153^{-1}$ ($\omega_\mathrm{y}//\omega_\mathrm{rev} = \sqrt{-\mathrm{k}} = \sqrt{0.03} \approx 5.7735^{-1}$).

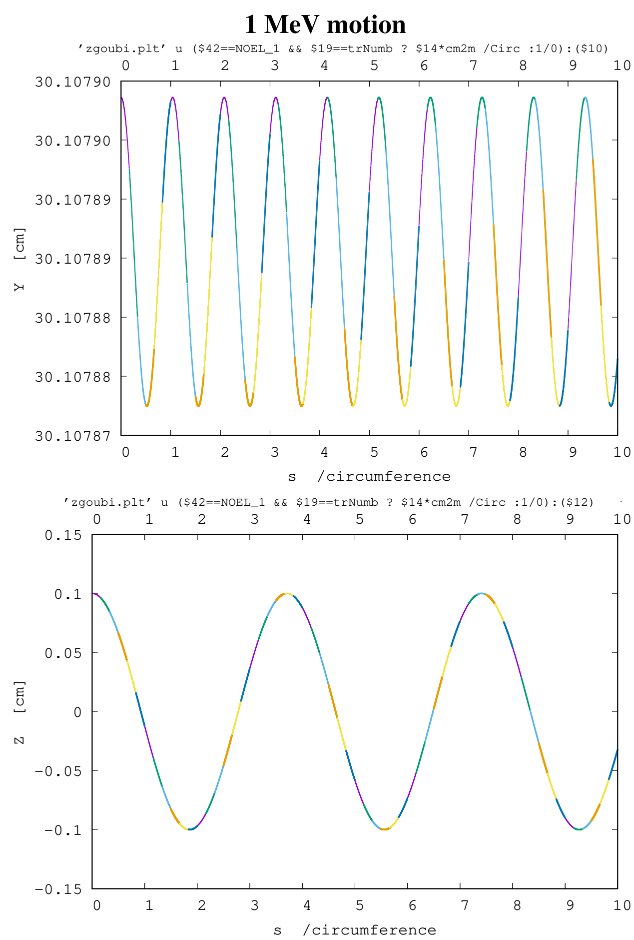

Figure 2: Radial and axial motion around the 1 MeV closed orbit (which is a circle of radius $\mathrm{R}_{1\,\mathrm{MeV}} = 12.92$ cm, lying in the median plane). It can be observed that the number of oscillations in the radial and axial motions differ from the 200 keV case. This means that the value of the index k in the R=30 cm region differs from the value in the R=12.9 cm region.

Input file to compute a few turns

```
Cyclotron, classical.
 'OBJET'
64.62444403717985                              ! BORO[kG cm] for 200keV proton
2
3  1
1.29248888E+01  0. 0. 0.  0.00  1.000E+00 'm'       ! 200keV. R=Brho/B=BORO/5[kG]
3.01078986E+01  0. 0. 0.  0.00  2.23654451E+00 'm'   ! 1 MeV
7.57546708E+01  0. 0. 0.  0.00  5.00638997E+00 'M'   ! 5 MeV. R=Brho/B=BORO*5.00639/5[kG]
1 1 1
 'PARTICUL'
PROTON
 'FAISCEAU'

'INCLUDE'
1
60degSector_k-0.03.dat[60DegSector_#S:60DegSector_#E]
'INCLUDE'
1
60degSector_k-0.03.dat[60DegSector_#S:60DegSector_#E]
'INCLUDE'
1
60degSector_k-0.03.dat[60DegSector_#S:60DegSector_#E]
'INCLUDE'
1
60degSector_k-0.03.dat[60DegSector_#S:60DegSector_#E]
'INCLUDE'
1
60degSector_k-0.03.dat[60DegSector_#S:60DegSector_#E]
'INCLUDE'
1
60degSector_k-0.03.dat[60DegSector_#S:60DegSector_#E]

'REBELOTE'
 2 0.1 99

 'END'
```

60degSector_k-0.03.dat file:

```
  Cyclotron, classica
 'OBJET'
64.62444403717985                              ! BORO[kG cm] for 200keV proton
2
3  1
1.29248888E+01  0. 0. 0.  0.00  1.000E+00 'm'       ! 200keV. R=Brho/B=BORO/5[kG]
3.01078986E+01  0. 0. 0.  0.00  2.23654451E+00 'm'   ! 1 MeV
7.57546708E+01  0. 0. 0.  0.00  5.00638997E+00 'M'   ! 5 MeV. R=Brho/B=BORO*5.00639/5[kG]
1 1 1
 'PARTICUL'
PROTON
 'FAISCEAU'

 'DIPOLE'   60DegSector_#S
2
60.  12.924889                                 ! AT, RM
30.00    5.00   -0.03    0.00    0.00          ! ACNT,  HNORM, indices
0.  0.                                         ! EFB 1  hard-edge
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
30. 0.  1.E6  -1.E6  1.E6  1.E6
0.   0.                                        ! EFB 2
4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
-30. 0.  1.E6  -1.E6  1.E6  1.E6
0. 0.                                          ! EFB 3
0  0.      0.      0.      0.      0. 0.  0.
0. 0.  1.E6  -1.E6  1.E6  1.E6 0.
2   10.             ! Interpolation 2nd order ;  grid size = step size/10.
.2                  ! step size (cm)
2     0.00     0.00     0.00     0.00          ! KPOS, RE, TE, RS, TS
 'FAISCEAU'   60DegSector_#E

'END'
```

Plot orbital motion, using gnuplot

```
set key maxcol 1
set key outside
set key t c font  "sans,14"

#set logscale y

set xtics mirror font  "sans,18"
set x2tics mirror font  "sans,18"
set ytics mirror font  "sans,18"

set xlabel 's  /circumference(1MeV)' font  "sans,18"
set ylabel 'Y  [cm]' font  "sans,18"

cm2m = 0.01
MeV2eV = 1e6
am = 938.27203
c = 2.99792458e8
pi = 4. * atan(1.)

NOEL_1 = 4    #   number of 1st TOSCA in zgoubi,plt (col. 42)
NOEL_2 = 6    #   number of 2nd TOSCA in zgoubi,plt (col. 42)
NOEL_3 = 8    #   number of 3rd TOSCA in zgoubi,plt (col. 42)
NOEL_4 = 10   #   number of 4th TOSCA in zgoubi,plt (col. 42)
NOEL_5 = 12   #   number of 5th TOSCA in zgoubi,plt (col. 42)
NOEL_6 = 14   #   number of 6th TOSCA in zgoubi,plt (col. 42)

dev = 2.*pi/6.
R1MeV = 3.01078986E+01  *cm2m
Circ = 2. *pi *R1MeV

#set size ratio -1

trNumb = 2

plot  \
   'zgoubi.plt' u ($42==NOEL_1 && $19==trNumb ? $14*cm2m /Circ :1/0):($10) w p ps .2  ,\
   'zgoubi.plt' u ($42==NOEL_2 && $19==trNumb ? $14*cm2m /Circ :1/0):($10) w p ps .2 notit ,\
   'zgoubi.plt' u ($42==NOEL_3 && $19==trNumb ? $14*cm2m /Circ :1/0):($10) w p ps .2 notit ,\
   'zgoubi.plt' u ($42==NOEL_4 && $19==trNumb ? $14*cm2m /Circ :1/0):($10) w p ps .2 notit ,\
   'zgoubi.plt' u ($42==NOEL_5 && $19==trNumb ? $14*cm2m /Circ :1/0):($10) w p ps .2 notit ,\
   'zgoubi.plt' u ($42==NOEL_6 && $19==trNumb ? $14*cm2m /Circ :1/0):($10) w p ps .2 notit

     set terminal postscript eps blacktext color  enh  "Times-Sans" 18
       set output "gnuplot_zgoubi.plt_ex1231a_orbitsY.eps"
       replot
       set terminal X11
       unset output

pause 2   # don't change this: needed for proper running of sector180deg
exit
```

2 1.2.3-1.b - Compare with theory

There is various possible ways:

- program that in gnuplot, as gnuplot can have the function that it plots take its values at abscissa which are read from a file (zgoubi.plt for the circumstance),

- write a program to (i) read all $\mathrm{s_{i,i=1,N\ steps}}$ path distances from zgoubi.plt, (ii) compute theoretical $x_{theoretical}(s_{i,i=1,N\ steps})$ ($y_{theoretical}(s_{i,i=1,N\ steps})$) values, (iii) log the outcomes into some "theoretical.plt" file. Use gnuplot "paste" (same effect as linux's) to plot $x_{zgoubi.plt} - x_{theoretical}$ as a function of $\mathrm{s_{i,i=1,N\ steps}}$.

- use an interpolation method (for instance in python) to compute x (y) as a function of s, from the values logged in zgoubi.plt, and plot together with theoretical x(s), y(s).

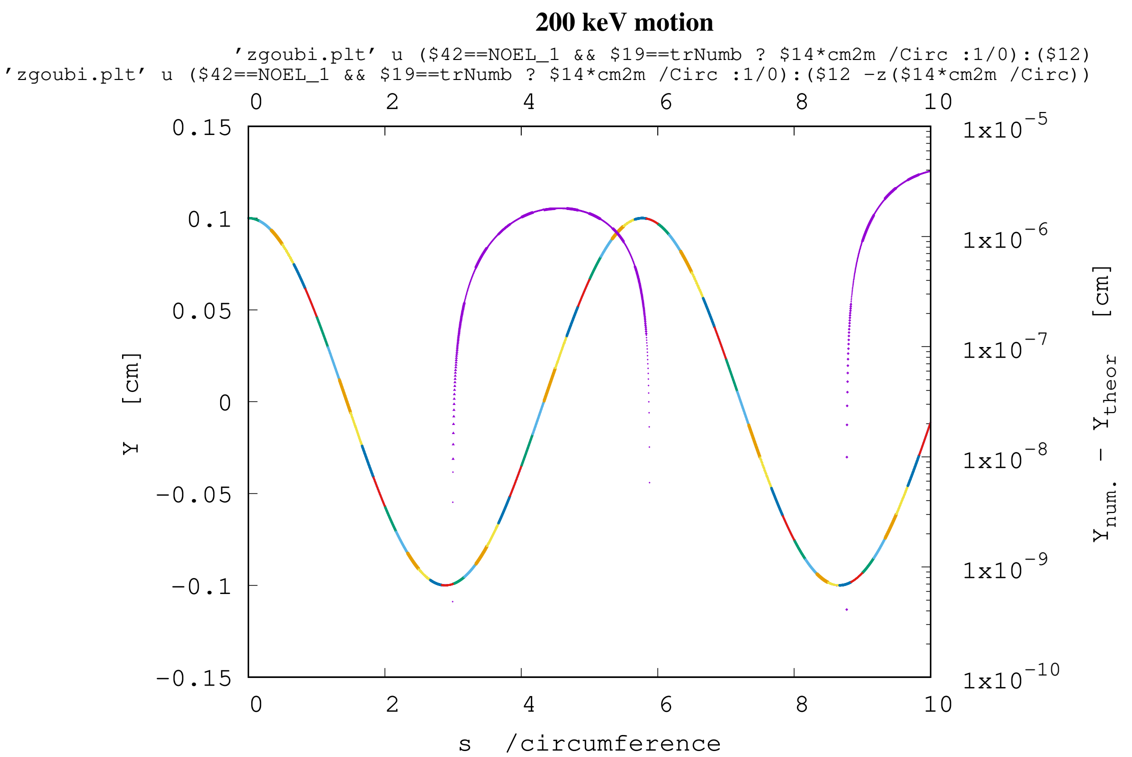

Figure 3: Vertical trajectory as a function of the path length (left vertical axis) and (right axis) its difference with $y(s) = \mathrm{y_0} \cos \frac{\sqrt{-\mathrm{k}}}{\mathrm{R_0}}(\mathrm{s} - \mathrm{s_0}) + \mathrm{y'_0} \frac{\mathrm{R_0}}{\sqrt{-\mathrm{k}}} \sin \frac{\sqrt{-\mathrm{k}}}{\mathrm{R_0}}(\mathrm{s} - \mathrm{s_0})$. Here, we have taken the initial conditions $\mathrm{y_0} = 0.1\,\mathrm{cm}$ and $\mathrm{y'_0} = 0$. Plotted using gnuplot file in p. 6.

gnuplot file to plot the numerical difference between tracking and theory:

```
set key maxcol 1
set key outside
set key t c font  "sans,14"

set logscale y2

set xtics mirror font  "sans,18"
set x2tics  font  "sans,18"
set ytics nomirror font  "sans,18"
set y2tics nomirror font  "sans,18"

set xlabel 's  /circumference' font  "sans,18"
set ylabel 'Y  [cm]' font  "sans,18"
set y2label 'Y_{num.} - Y_{theor}  [cm]' font  "sans,18"

cm2m = 0.01
MeV2eV = 1e6
am = 938.27203
c = 2.99792458e8
pi = 4. * atan(1.)

NOEL_1 = 4    #   number of 1st TOSCA in zgoubi,plt (col. 42)
NOEL_2 = 6    #   number of 2nd TOSCA in zgoubi,plt (col. 42)
NOEL_3 = 8    #   number of 3rd TOSCA in zgoubi,plt (col. 42)
NOEL_4 = 10   #   number of 4th TOSCA in zgoubi,plt (col. 42)
NOEL_5 = 12   #   number of 5th TOSCA in zgoubi,plt (col. 42)
NOEL_6 = 14   #   number of 6th TOSCA in zgoubi,plt (col. 42)

dev = 2.*pi/6.

#set size ratio -1

R1MeV = 3.01078986E+01  *cm2m
Circ = 2. *pi *R1MeV
trNumb = 2

R200keV = 1.29248888E+01  *cm2m
Circ = 2. *pi *R200keV
trNumb = 1

k = -0.03
Z0 = 0.1
z(x) = Z0*cos(sqrt(-k) * x*Circ/R200keV)

#plot [0:] z(x)
#pause 44
#exit

plot [0:10] \
   'zgoubi.plt' u ($14*cm2m /Circ):(z($14*cm2m /Circ)) w p ps .2  notit ,\
   'zgoubi.plt' u ($42==NOEL_1 && $19==trNumb ? $14*cm2m /Circ :1/0):($12) w p ps .2  ,\
   'zgoubi.plt' u ($42==NOEL_2 && $19==trNumb ? $14*cm2m /Circ :1/0):($12) w p ps .2 notit ,\
   'zgoubi.plt' u ($42==NOEL_3 && $19==trNumb ? $14*cm2m /Circ :1/0):($12) w p ps .2 notit ,\
   'zgoubi.plt' u ($42==NOEL_4 && $19==trNumb ? $14*cm2m /Circ :1/0):($12) w p ps .2 notit ,\
   'zgoubi.plt' u ($42==NOEL_5 && $19==trNumb ? $14*cm2m /Circ :1/0):($12) w p ps .2 notit ,\
   'zgoubi.plt' u ($42==NOEL_6 && $19==trNumb ? $14*cm2m /Circ :1/0):($12) w p ps .2 notit ,\
   'zgoubi.plt' u ($42==NOEL_1 && $19==trNumb ? $14*cm2m /Circ :1/0):($12 -z($14*cm2m /Circ)) axes x1y2 w p ps .2 lc 1  ,\
   'zgoubi.plt' u ($42==NOEL_2 && $19==trNumb ? $14*cm2m /Circ :1/0):($12 -z($14*cm2m /Circ)) axes x1y2 w p ps .2 lc 1 notit ,\
   'zgoubi.plt' u ($42==NOEL_3 && $19==trNumb ? $14*cm2m /Circ :1/0):($12 -z($14*cm2m /Circ)) axes x1y2 w p ps .2 lc 1 notit ,\
   'zgoubi.plt' u ($42==NOEL_4 && $19==trNumb ? $14*cm2m /Circ :1/0):($12 -z($14*cm2m /Circ)) axes x1y2 w p ps .2 lc 1 notit ,\
   'zgoubi.plt' u ($42==NOEL_5 && $19==trNumb ? $14*cm2m /Circ :1/0):($12 -z($14*cm2m /Circ)) axes x1y2 w p ps .2 lc 1 notit ,\
   'zgoubi.plt' u ($42==NOEL_6 && $19==trNumb ? $14*cm2m /Circ :1/0):($12 -z($14*cm2m /Circ)) axes x1y2 w p ps .2 lc 1 notit

     set terminal postscript eps blacktext color  enh  "Times-Sans" 18
       set output "gnuplot_zgoubi.plt_ex1231a_diffTheor.eps"
       replot
       set terminal X11
       unset output

pause 2   # don't change this: needed for proper running of sector180deg
exit
```