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# COMPUTERLAB, EXERCISE 1.2.3-2, SOLUTION

## Abstract

Wave numbers, phase space motion, at fixed energy.

## Contents

# 1 1.2.3-2.a - Multiple turns around the ring, elliptical motion

It is recommended to set IL=0 under `DIPOLE`, in order to save on CPU time, and avoid a big zgoubi.plt file.

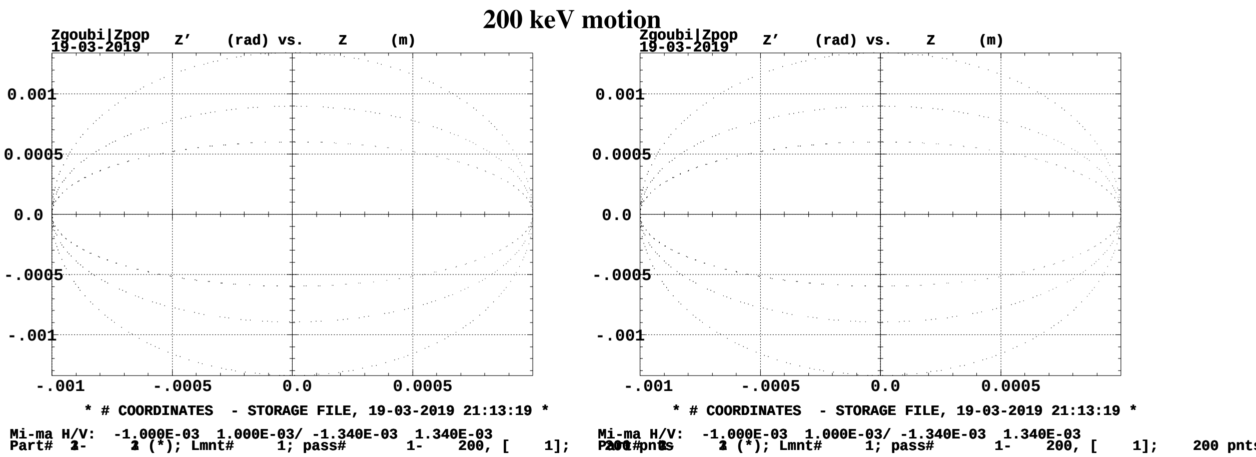


Figure 1: Left: horizontal phase space motion. Right: vertical phase space motion.

Note $\beta = \rho/\sqrt{1+\mathrm{k}}$. Multiply Eqs. 1.14 by $1/\sqrt{\beta}$ and $\sqrt{\beta}$ respectively, this gives

$$\begin{cases} \mathrm{x(s)}/\sqrt{\beta} = & \mathrm{x}_0/\sqrt{\beta}\cos(\mathrm{s}-\mathrm{s}_0)/\beta + \mathrm{x}'_0\sqrt{\beta}\sin(\mathrm{s}-\mathrm{s}_0)/\beta \\ \mathrm{x}'(\mathrm{s})\sqrt{\beta} = & -\mathrm{x}_0/\sqrt{\beta}\sin(\mathrm{s}-\mathrm{s}_0)/\beta + \mathrm{x}'_0\sqrt{\beta}\cos(\mathrm{s}-\mathrm{s}_0)/\sqrt{\beta} \end{cases}$$

Thus

$$\left(\frac{\mathrm{x}}{\sqrt{\beta}}\right)^2 + \left(\mathrm{x}'\sqrt{\beta}\right)^2 = \left(\frac{\mathrm{x}_0}{\sqrt{\beta}}\right)^2 + \left(\mathrm{x}'_0\sqrt{\beta}\right)^2$$

$\epsilon = \frac{\mathrm{x}^2}{\beta} + \beta \mathrm{x}'^2$ is an invariant of the motion. The motion in (x,x') phase space is on an upright ellipse with half-axes $\sqrt{\beta\epsilon}$ horizontal and $\sqrt{\epsilon/\beta}$ vertical.

## Input file to track and save many turns

Three particles are tracked, at 200 keV, 1 MeV and 5 MeV, for 200 turns.

```
Cyclotron, classical.
 'OBJET'
64.62444403717985                              ! BORO[kG cm] for 200keV proton
2
3  1
1.29248888E+01  0. 0. 0.  0.00  1.000E+00 'm'        ! 200keV. R=Brho/B=BORO/5[kG]
3.01078986E+01  0. 0. 0.  0.00  2.23654451E+00 'm'   ! 1 MeV
7.57546708E+01  0. 0. 0.  0.00  5.00638997E+00 'M'   ! 5 MeV. R=Brho/B=BORO*5.00639/5[kG]
1 1 1
 'PARTICUL'
PROTON
 'FAISCEAU'

'INCLUDE'
1
60degSector_k-0.03.dat[60DegSector_#S:60DegSector_#E]
'INCLUDE'
1
60degSector_k-0.03.dat[60DegSector_#S:60DegSector_#E]
'INCLUDE'
1
60degSector_k-0.03.dat[60DegSector_#S:60DegSector_#E]
'INCLUDE'
1
60degSector_k-0.03.dat[60DegSector_#S:60DegSector_#E]
'INCLUDE'
1
60degSector_k-0.03.dat[60DegSector_#S:60DegSector_#E]
'INCLUDE'
1
60degSector_k-0.03.dat[60DegSector_#S:60DegSector_#E]

'FAISTORE'                ! Store coordinates here,
zgoubi.fai                ! in this file,
1                         ! every turn.

'REBELOTE'
 199 0.1 99               ! Repeat 199 times (that will make 200 turns in total).

 'END'
```

## Plot phase space motion and ellipse, using gnuplot

```
set key maxcol 1
set key outside
set key t c font  "sans,14"
```

# 2 1.2.3-2.b - Wave numbers from Fourier analysis

Describe the harmonic motion in the $(\mathrm{y}, \frac{1}{\nu}\frac{\mathrm{dz}}{\mathrm{d}\theta})$.

In that space, the particle motion $(\mathrm{y}, \frac{1}{\nu}\frac{\mathrm{dz}}{\mathrm{d}\theta})$ is on a circle of radius $\hat{y}$.

The projections are

$$\mathrm{y} = \hat{\mathrm{y}}\cos(\nu\theta + \phi)$$

$$\frac{1}{\nu}\frac{\mathrm{dy}}{\mathrm{d}\theta} = -\hat{\mathrm{y}}\sin(\nu\theta + \phi)$$

This establishes that the motion is clockwise on the circle. The pahse advance per turn is $\Delta\phi = 2\pi\nu < 2\pi$.

• Indetermination on the wave number $\nu$:

- at turn 0 (start) the observer measures an excursion $\mathrm{z}_0 = \hat{\mathrm{z}}\cos\phi_0$ (z = x or y)
- at turn 1 the observer measures $\mathrm{z}_1 = \hat{\mathrm{z}}\cos(2\pi\nu + \phi_0)$ (or $\mathrm{z}_0 = \hat{\mathrm{z}}\cos(2\pi(1-\nu) + \phi_0)$, he can't tell, he knows, though, that $\nu < 1$)
- at turn n the observer measures $\mathrm{z}_\mathrm{n} = \hat{\mathrm{z}}\cos(2\pi\nu\mathrm{n} + \phi_0$ (or $\mathrm{z}_0 = \hat{\mathrm{z}}\cos(2\pi(1-\nu)\mathrm{n} + \phi_0)$, he can't tell)

So, because he does not know whether y increases or decreases, the observer cannot determine, from a frequency $\nu$ or $1-\nu$.