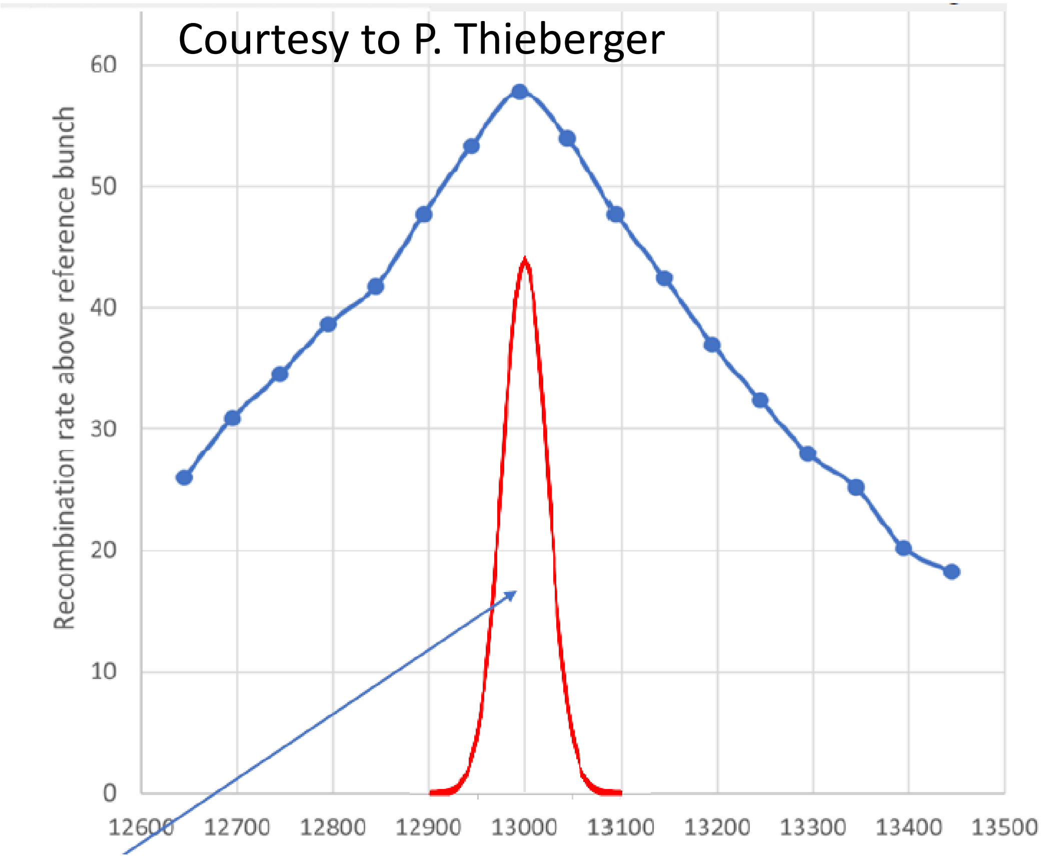
Dependance of recombination rate on energy deviation of electrons

G. Wang

Introduction

- As presented in the previous meeting by Peter, the recombination curve (recombination rate as a function of energy difference between electrons and ions) measured during CeC experiment in run 21 seems to be much wider than what obtained from scaling with LeREC results.
- Energy spread in the electrons does not seem to be enough to explain the observed width of the recombination curve.
- This study is to investigate how transverse angular spread as well as orbit angle due to misalignment affect the recombination rate and at what level, they can be responsible for the observed widening of the recombination curve.



$$CeC\ RMS\ width = 6.15 * 3.9\ keV = 24\ keV$$

Equations to calculate recombination rate

Recombination cross section for an electron to be captured by an ion:

$$\sigma(v) = A\frac{2h\nu_0}{m_e v^2}\left[\ln\left(\sqrt{\frac{2h\nu_0}{m_e v^2}}\right)+\gamma_1+\gamma_2\left(\frac{m_e v^2}{2h\nu_0}\right)^{1/3}\right]$$

$A = 2.11\times10^{-22}\,cm^2$

$\gamma_1 = 0.1402$

v: relative velocity between an electron and an ion

$\gamma_2 = 0.525$

To calculate recombination rate of an ion bunch moving together with an electron bunch, the following formula is used

$h\nu_0 = Z^2\alpha^2 m_e^2 / 2 = Z^2 \times 13.6eV$

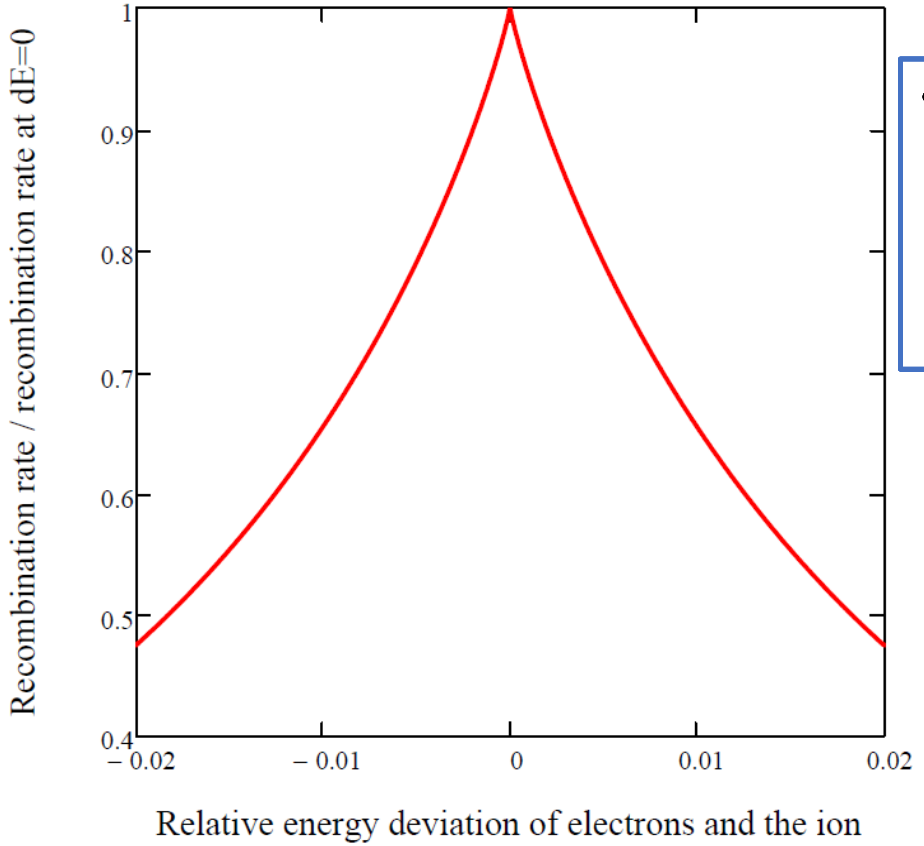
$$\alpha_r = \frac{\int_{-\infty}^{\infty} d^3v_i d^3v_e f_e(v_e)\, f_I(v_i)\left|\vec{v}_e-\vec{v}_i\right|\sigma\left(\left|\vec{v}_e-\vec{v}_i\right|\right)}{\int_{-\infty}^{\infty} d^3v_i d^3v_e f_e(v_e)\, f_I(v_i)}$$

$Z = 79$

Longitudinally cold electrons and ions

$$f_e\left(v_e\right)=\frac{1}{2\pi\beta_{e,\perp}^2}\exp\left(-\frac{v_{e,x}^2+v_{e,y}^2}{2\beta_{e,\perp}^2}\right)\delta\left(v_{e,z}-v_{z0}\right)$$

$$f_I\left(v_i\right)=\frac{1}{2\pi\beta_{i,\perp}^2}\exp\left(-\frac{v_{i,x}^2+v_{i,y}^2}{2\beta_{i,\perp}^2}\right)\delta\left(v_{i,z}\right)$$

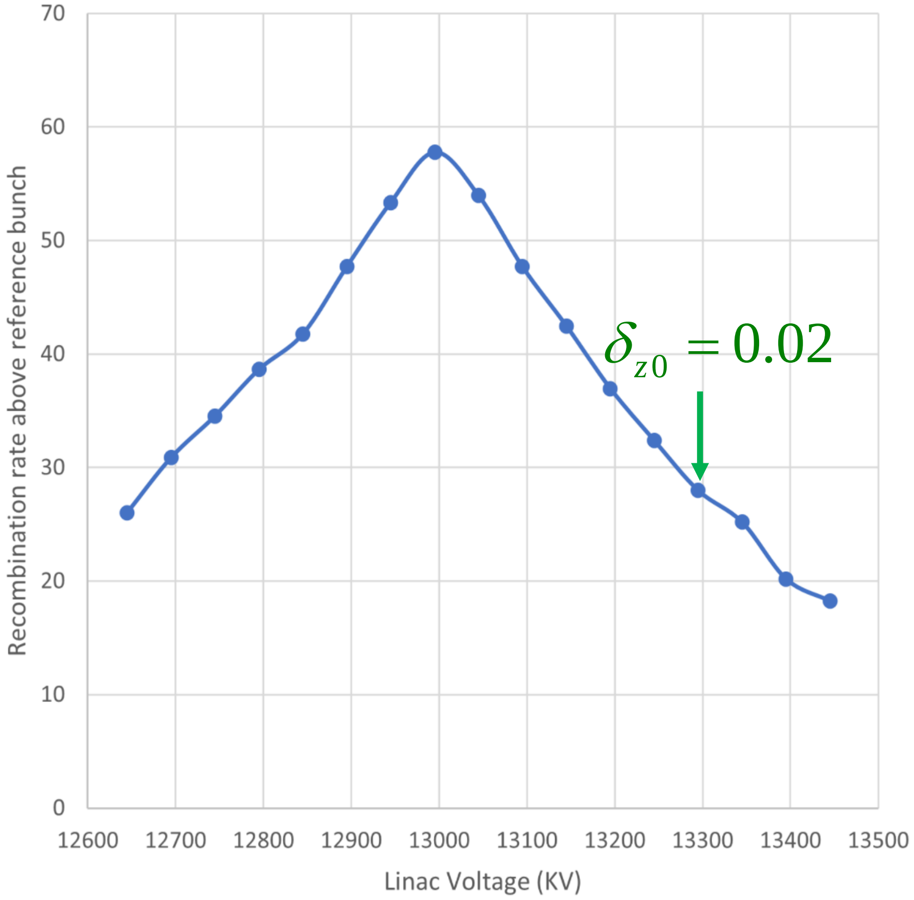


For electron beam with 0.9 mrad of angular spread

$$\beta_{e,x}=\beta_{e,y}=8\times10^6\, m/s$$

$$\beta_{i,x}=\beta_{i,y}=1.2\times10^6\, m/s$$

- **Without orbit angle,** 0.9 mrad angular spread is needed to explain the observed recombination curve.

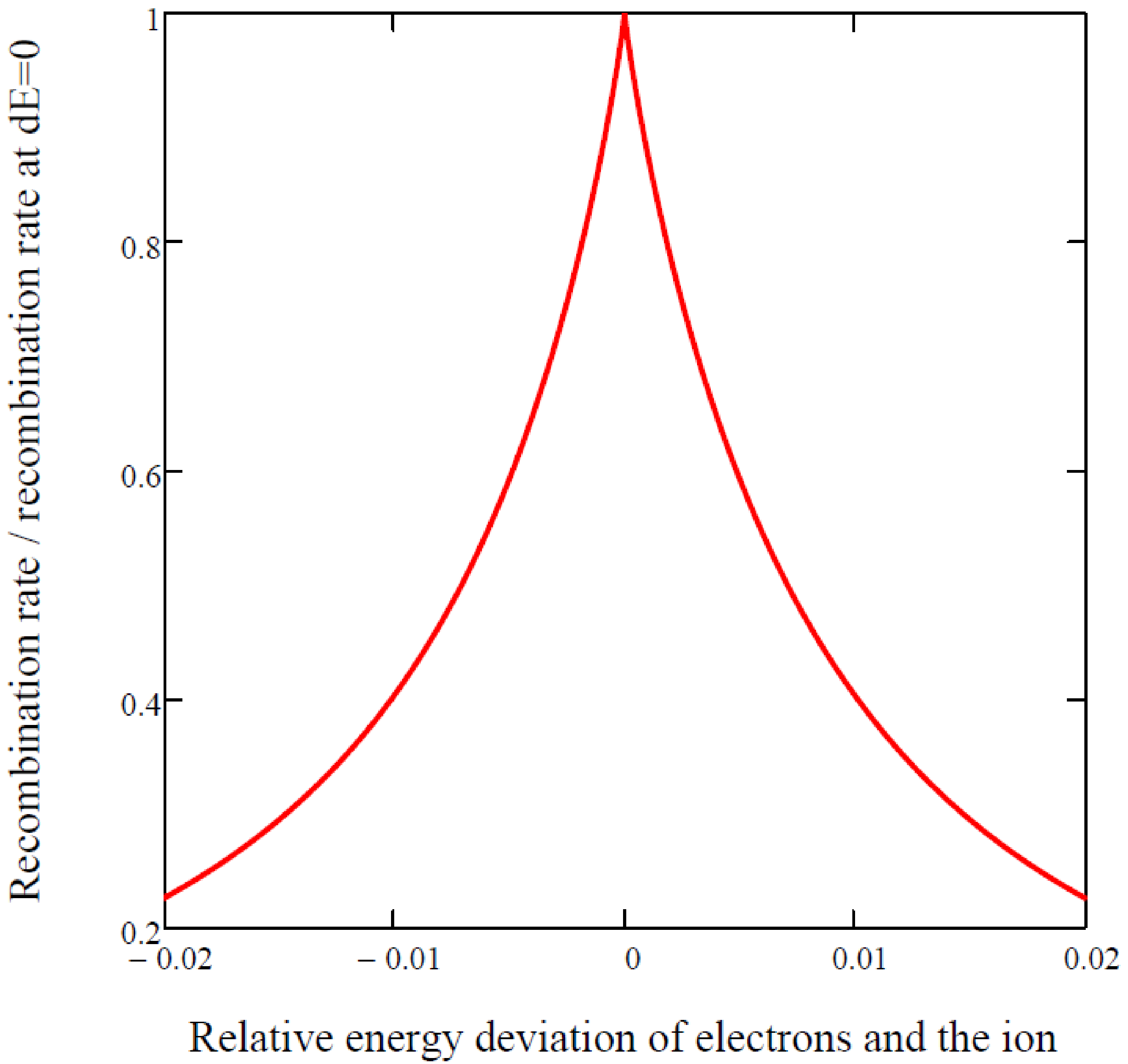


$$\alpha_r=Ac\frac{h\nu_0}{kT_{ei}}\sqrt{\frac{2h\nu_0}{m_ec^2}}\exp\left(\frac{m_ec^2}{2kT_{ei}}\delta_{z0}^2\right)\times\int_{\frac{m_ec^2\delta_{z0}^2}{2h\nu_0}}^{\infty}\frac{1}{\sqrt{y}}\left[-\frac{1}{2}\ln y+\gamma_1+\gamma_2y^{1/3}\right]\exp\left(-\frac{h\nu_0}{kT_{ei}}y\right)dy$$

$$\delta_{z0}=\frac{v_{z0}}{c}=\frac{\gamma_e-\gamma_i}{\gamma_i}$$

With designed angular spread in modulator and kicker, 0.3 mrad

- With the designed angular spread in modulator and kicker, 0.2~0.3 mrad, the recombination curve calculated from the theory is a factor of 2 too narrow compared with what measured in the CeC experiment.



Longitudinally cold electrons and ions with orbit angle

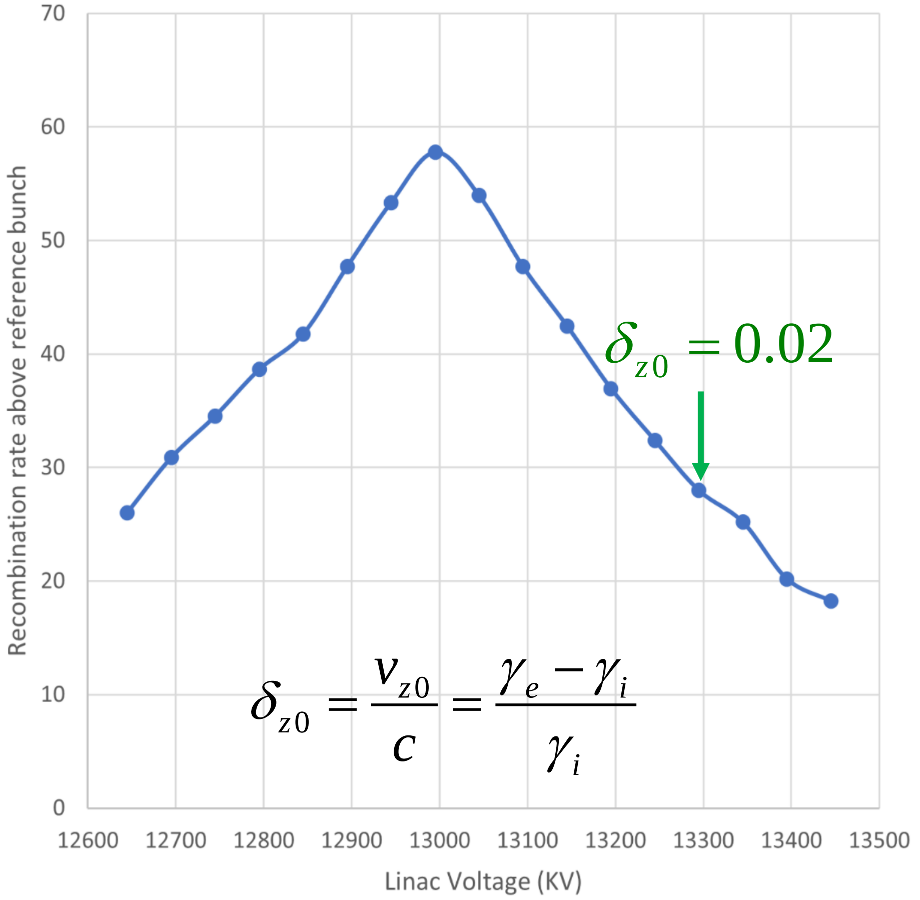
$$f_e(v_e) = \frac{1}{2\pi\beta_{e,\perp}^2}\exp\left(-\frac{(v_{e,x}-v_{x0})^2+v_{e,y}^2}{2\beta_{e,\perp}^2}\right)\delta(v_{e,z}-v_{z0})$$

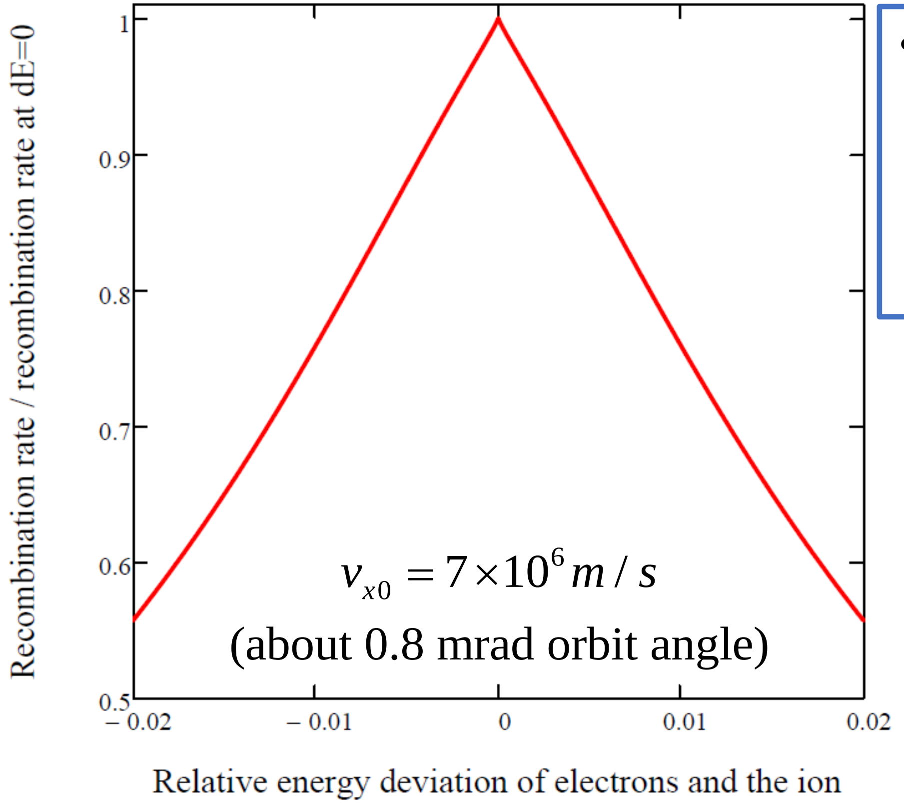
$$f_I(v_i) = \frac{1}{2\pi\beta_{i,\perp}^2}\exp\left(-\frac{v_{i,x}^2+v_{i,y}^2}{2\beta_{i,\perp}^2}\right)\delta(v_{i,z})$$

For electron beam with 0.3 mrad of angular spread

$$\beta_{e,x} = \beta_{e,y} = 2.6\times10^6 m/s$$

$$\beta_{i,x} = \beta_{i,y} = 1.2\times10^6 m/s$$





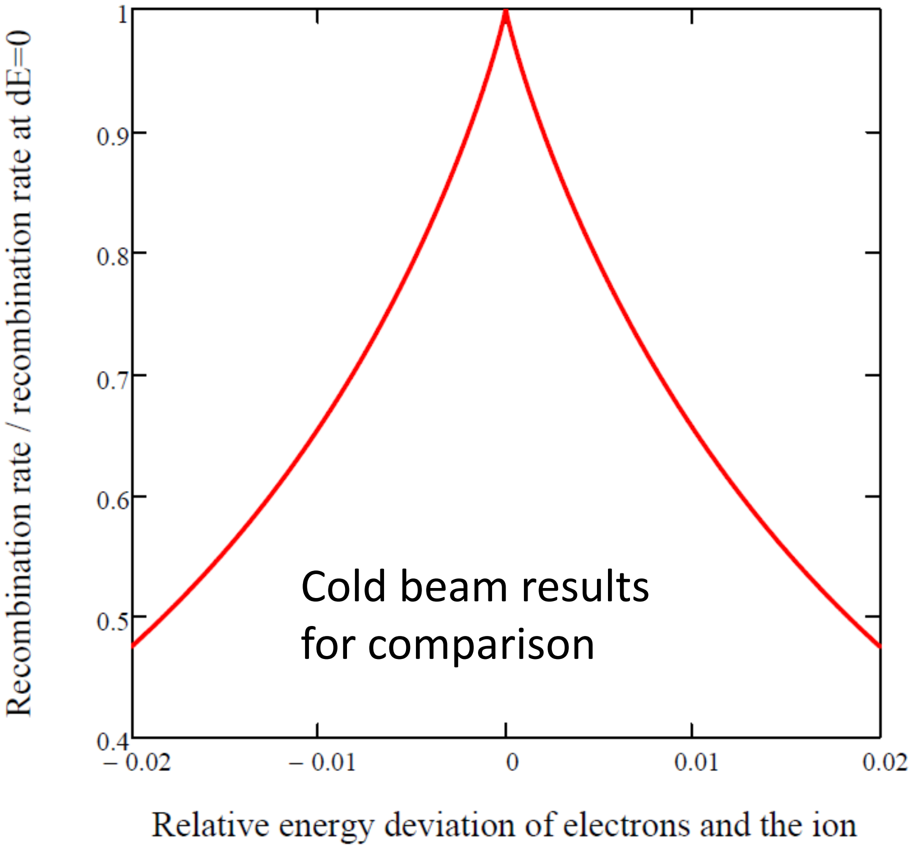
- For 0.3 mrad of angular spread, **an orbit angle of 0.8 mrad is adequate** to explain the observed width of the recombination curve.

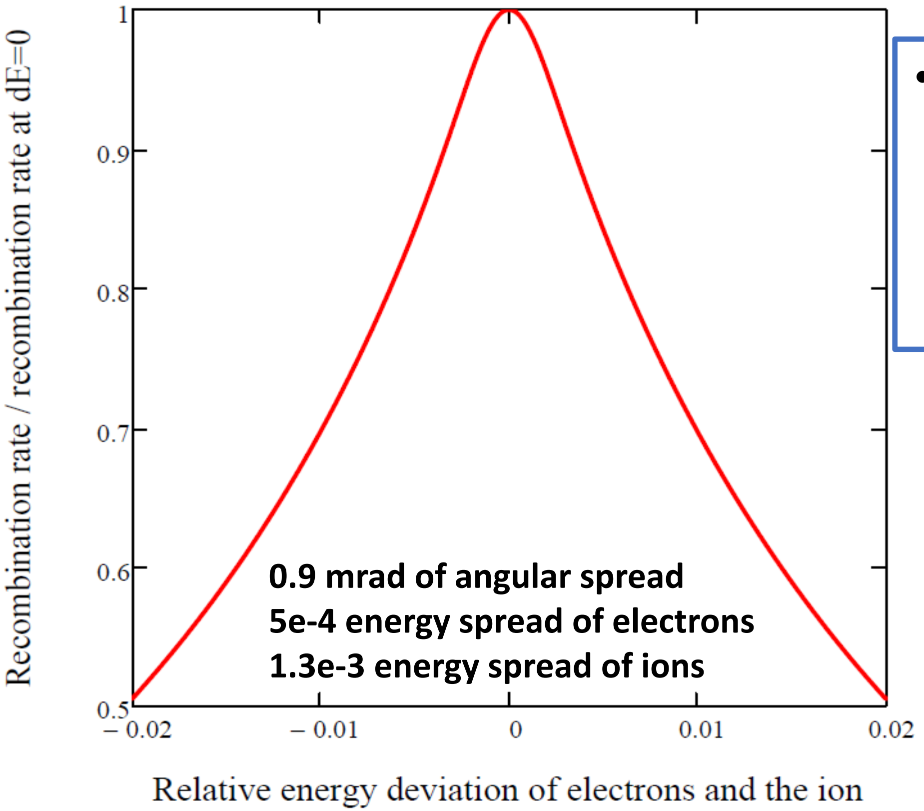
$$\alpha_r = Ac\frac{h\nu_0}{kT_{ei}}\sqrt{\frac{2h\nu_0}{m_ec^2}}\exp\left(\frac{v_{z0}^2 - v_{x0}^2}{2kT_{ei}/m_e}\right)$$
$$\times\int_{\frac{m_ev_{z0}^2}{2h\nu_0}}^{\infty}\frac{1}{\sqrt{y}}\left[\ln\left(\sqrt{\frac{1}{y}}\right)+\gamma_1+\gamma_2y^{1/3}\right]I_0\left(\frac{v_{x0}\sqrt{2m_eh\nu_0}}{kT_{ei}}\sqrt{y-\frac{m_ev_{z0}^2}{2h\nu_0}}\right)\exp\left[-\frac{h\nu_0y}{kT_{ei}}\right]dy$$

Longitudinally warm electrons and ions

$$f_e(v_e)=\frac{1}{(2\pi)^{3/2}\beta_{e,\perp}^2\beta_{e,z}}\exp\left(-\frac{v_{e,x}^2+v_{e,y}^2}{2\beta_{e,\perp}^2}\right)\exp\left(-\frac{(v_{e,z}-v_{z0})^2}{2\beta_{e,z}^2}\right)$$

$$f_I(v_i)=\frac{1}{(2\pi)^{3/2}\beta_{i,\perp}^2\beta_{i,z}}\exp\left(-\frac{v_{i,x}^2+v_{i,y}^2}{2\beta_{i,\perp}^2}\right)\exp\left(-\frac{{v_{i,z}}^2}{2\beta_{i,z}^2}\right)$$



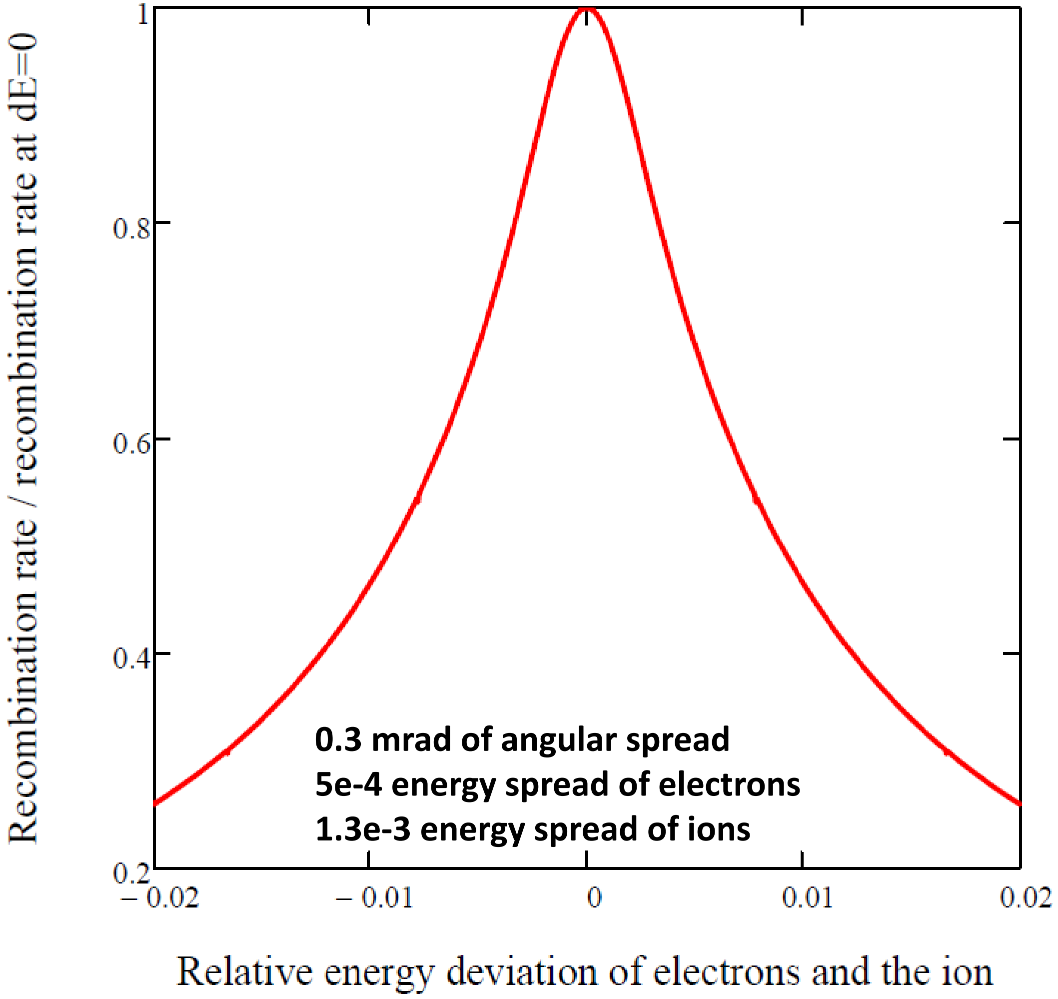


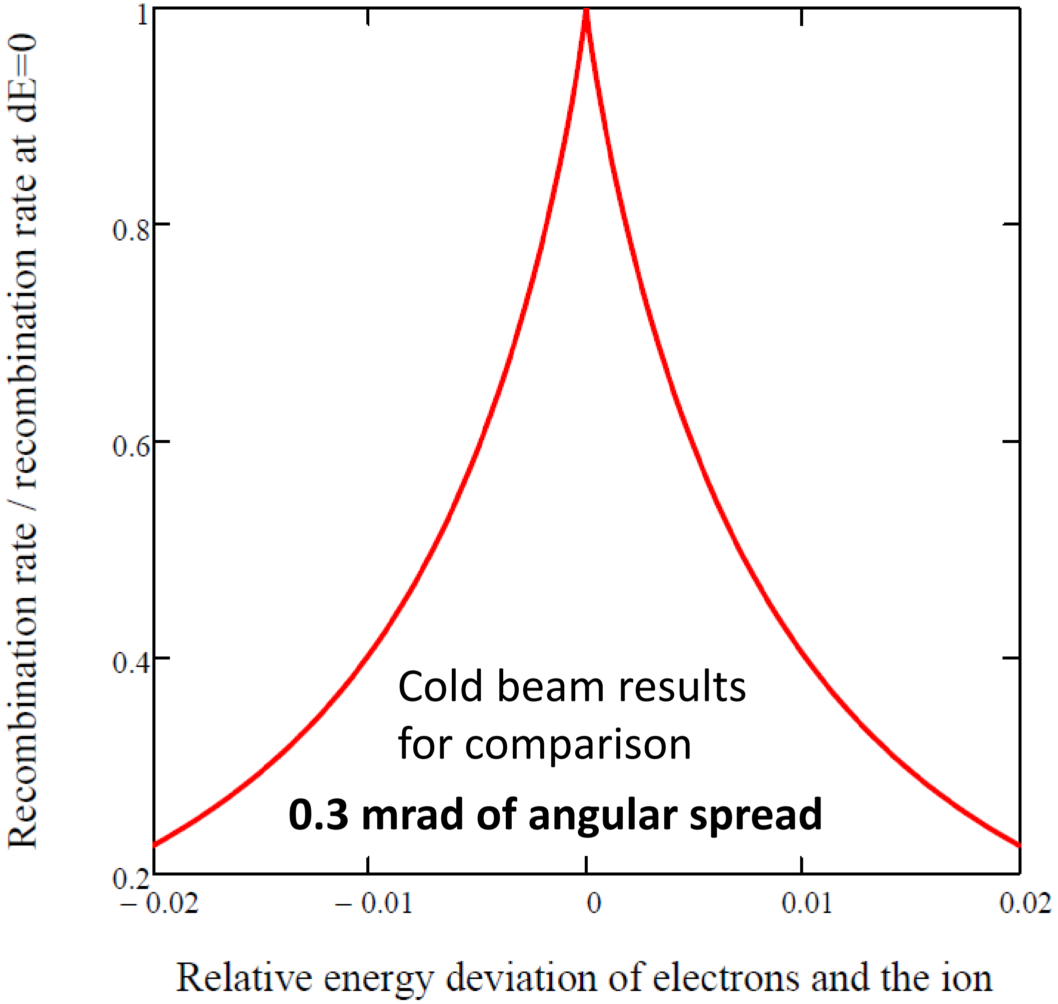
- Including longitudinal energy spread for parameters of CeC experiment does not change results significantly from the cold beam results

$$\alpha_r=\frac{Ach\nu_0}{kT_{ei}\eta}\sqrt{\frac{2h\nu_0}{m_ec^2}}\exp\left(\frac{m_e{v_{z0}}^2}{2kT_{ei}\eta^2}\right)\int_0^\infty\left(-\ln y+\gamma_1+\gamma_2y^{2/3}\right)\exp\left(-\frac{h\nu_0}{kT_{ei}}y^2\right)$$
$$\times\left\{Erf\left[\eta\sqrt{\frac{h\nu_0}{2kT_z}}\left(y+\sqrt{\frac{m_e}{2h\nu_0}}\frac{v_{z0}}{\eta^2}\right)\right]-Erf\left[\eta\sqrt{\frac{h\nu_0}{2kT_z}}\left(\frac{v_{z0}}{\eta^2}\sqrt{\frac{m_e}{2h\nu_0}}-y\right)\right]\right\}dy$$

Longitudinally **warm** electrons and ions

- Including longitudinal energy spread for parameters of CeC experiment does not change results significantly from the cold beam results





Summary

- An analytical expression in form of 1-D integral has been derived for calculating the recombination rate in the presence of transverse angular spread and orbit angle due to misalignment in the limit of zero longitudinal momentum spread (longitudinally code beams).
- For electrons and ions with Gaussian longitudinal velocity distribution, An analytical expression in form of 1-D integral has been derived in the absence of orbit angle.
- Calculation based on these analytical results show
  - It requires 0.9 mrad angular spread of the electron beam to explain the observed width of the recombination curve if the electron beam and ion beam are perfectly aligned in the cooling section, i.e. without any orbit angle between them.
  - For the designed angular spread of the electrons, 0.3 mrad, it requires an orbit angle of 0.8 mrad to explain the measured width of the recombination curve.
  - The energy spread does not significantly change the results for beam parameters relevant to the CeC experiment.