

# Today schedule:

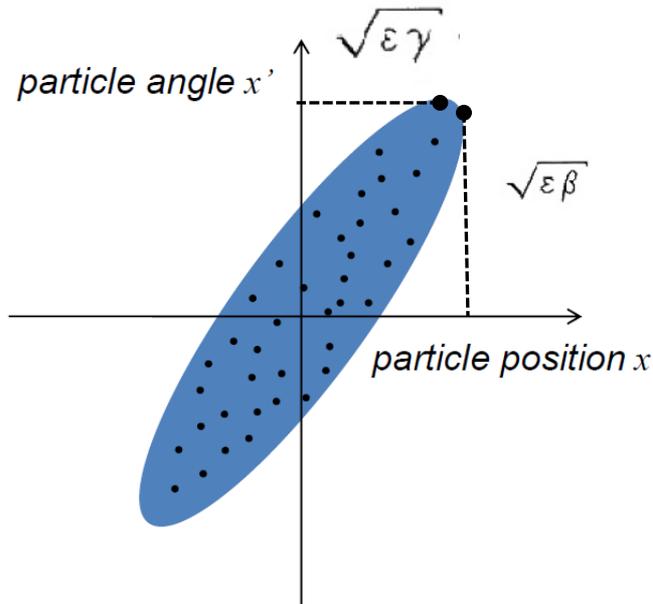
1. Emittance measurements short lecture.(30-40 minutes)
2. Questions from last class (10 min.)
3. 10 minutes break.
4. Demonstration of emittance measurements: go to ATF control room (~30 min)
  1. 4 profile monitors
  2. Quadrupole scan
5. Data collection for homework analyzes at ATF control room (~60 min)

# PHY542. Emittance Measurements

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# Emittance, what is it ?



$\epsilon$  = Area in  $x, x'$  plane occupied by beam particles divided by  $\pi$

Beam ellipse and its orientation is described by 4 parameters

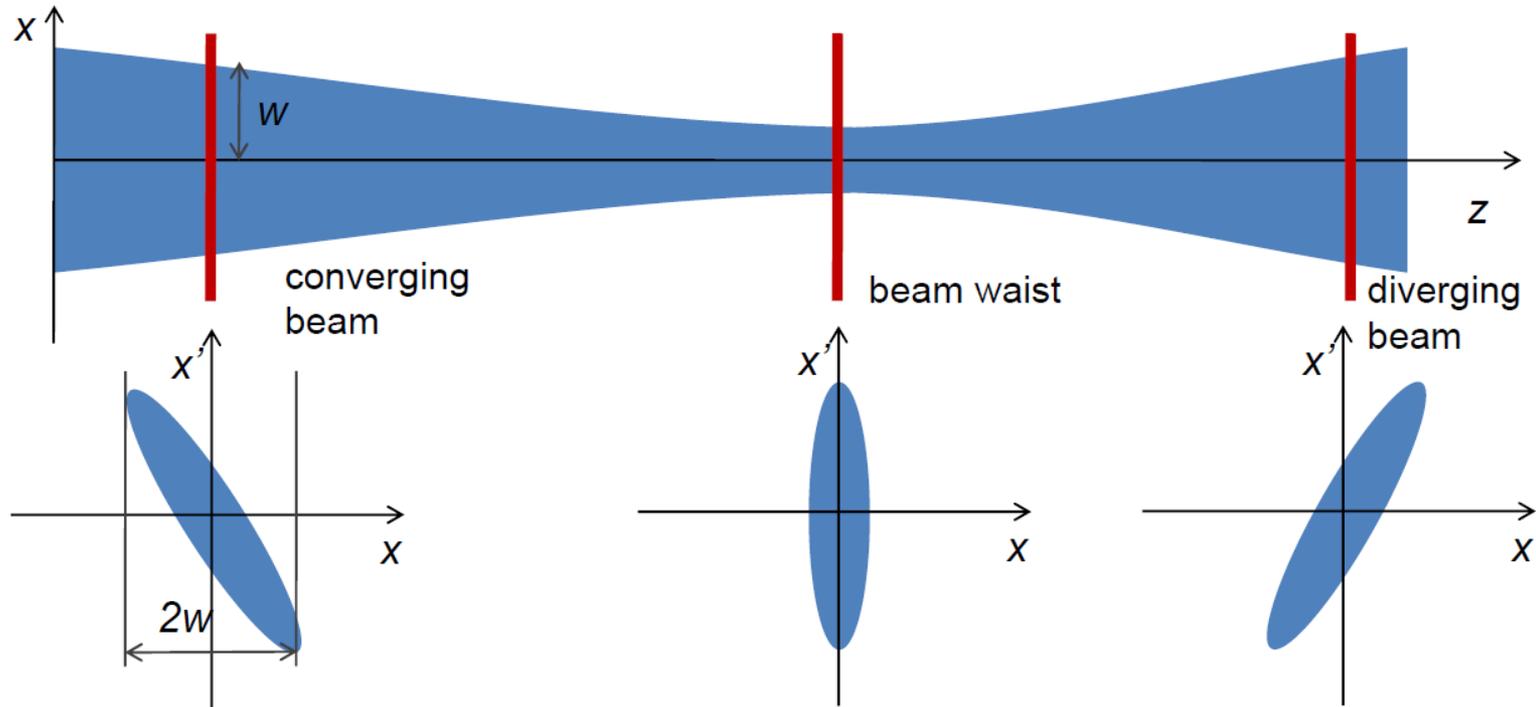
$$\epsilon = \gamma x^2 + 2 \alpha x x' + \beta x'^2$$

- $\sqrt{\beta\epsilon}$  Is the beam half width
- $\sqrt{\gamma\epsilon}$  Is the beam half divergence
- $\alpha$  Describes how strong  $x$  and  $x'$  are correlated
  - $\alpha < 0$  beam diverging
  - $\alpha > 0$  beam converging
  - $\alpha = 0$  beam size is maximum or minimum (waist)

The three orientation parameters are connected by the relation

$$\gamma = \frac{1 + \alpha^2}{\beta}$$

# *Beam envelope along a beamline.*

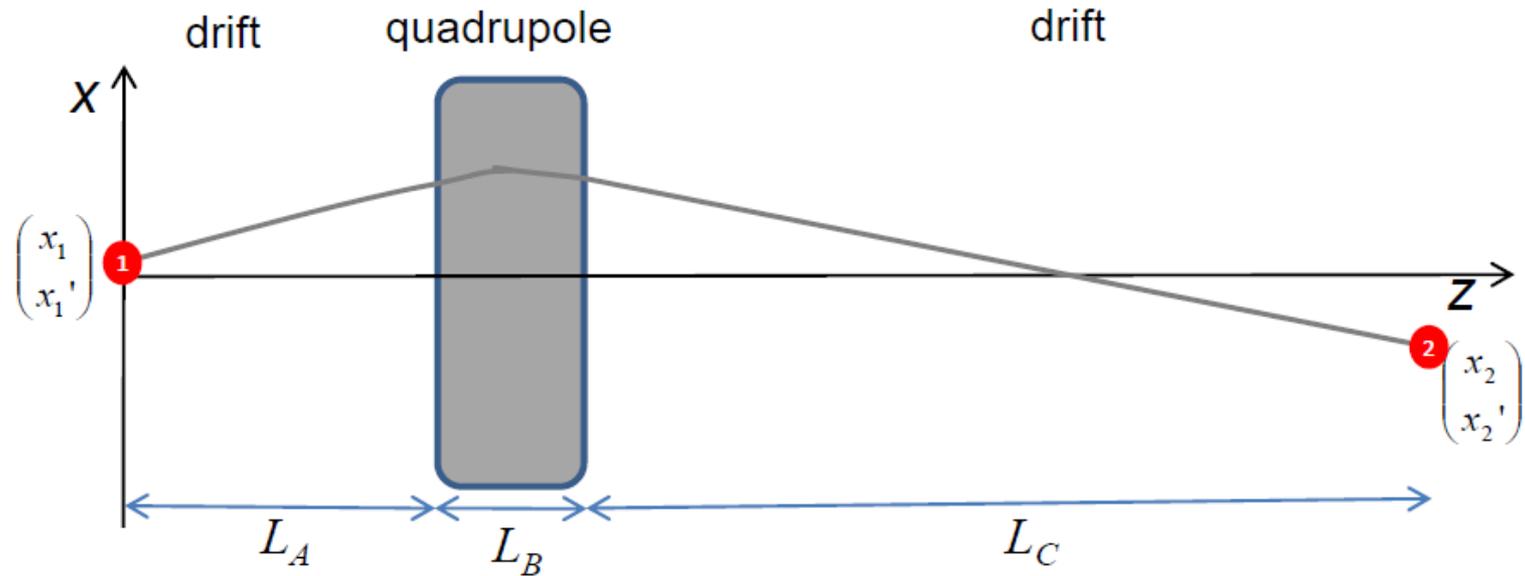


Along a beamline the orientation and aspect ratio of the beam ellipse in  $x, x'$  changes, but area (emittance) remains constant.

Beam width along  $Z$  is described

$$w(z) = \sqrt{\beta(z) \varepsilon}$$

# Transport of single particle described with matrix

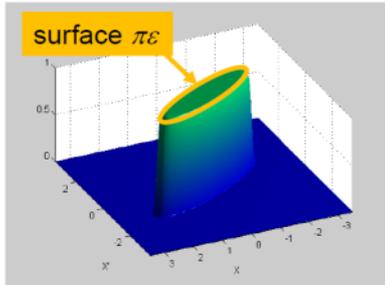


$$M_{\text{Drift}} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \quad M_{\text{Quadrupole}} = \begin{pmatrix} \cos(\sqrt{k}L) & 1/\sqrt{k} \sin(\sqrt{k}L) \\ -\sqrt{k} \sin(\sqrt{k}L) & \cos(\sqrt{k}L) \end{pmatrix}$$

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = M \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} \quad \text{There} \quad M = \begin{pmatrix} 1 & L_C \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\sqrt{k}L_B) & 1/\sqrt{k} \sin(\sqrt{k}L_B) \\ -\sqrt{k} \sin(\sqrt{k}L_B) & \cos(\sqrt{k}L_B) \end{pmatrix} \cdot \begin{pmatrix} 1 & L_A \\ 0 & 1 \end{pmatrix}$$

# R.m.s emittance

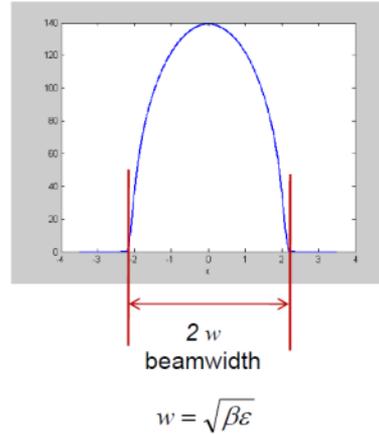
particle density in  $x, x'$  space



projection on  $x$  axis

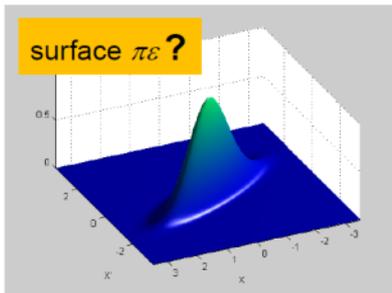
$$f(x) = \int_{-\infty}^{\infty} \rho(x, x') dx'$$

transverse beam profile



In reality beam density in  $x, x'$  space is rarely a area with sharp boundary

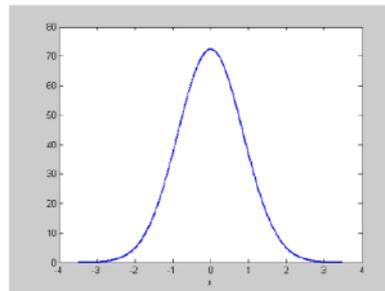
particle density in  $x, x'$  space



projection on  $x$  axis

$$f(x) = \int_{-\infty}^{\infty} \rho(x, x') dx'$$

transverse beam profile



beamwidth ?

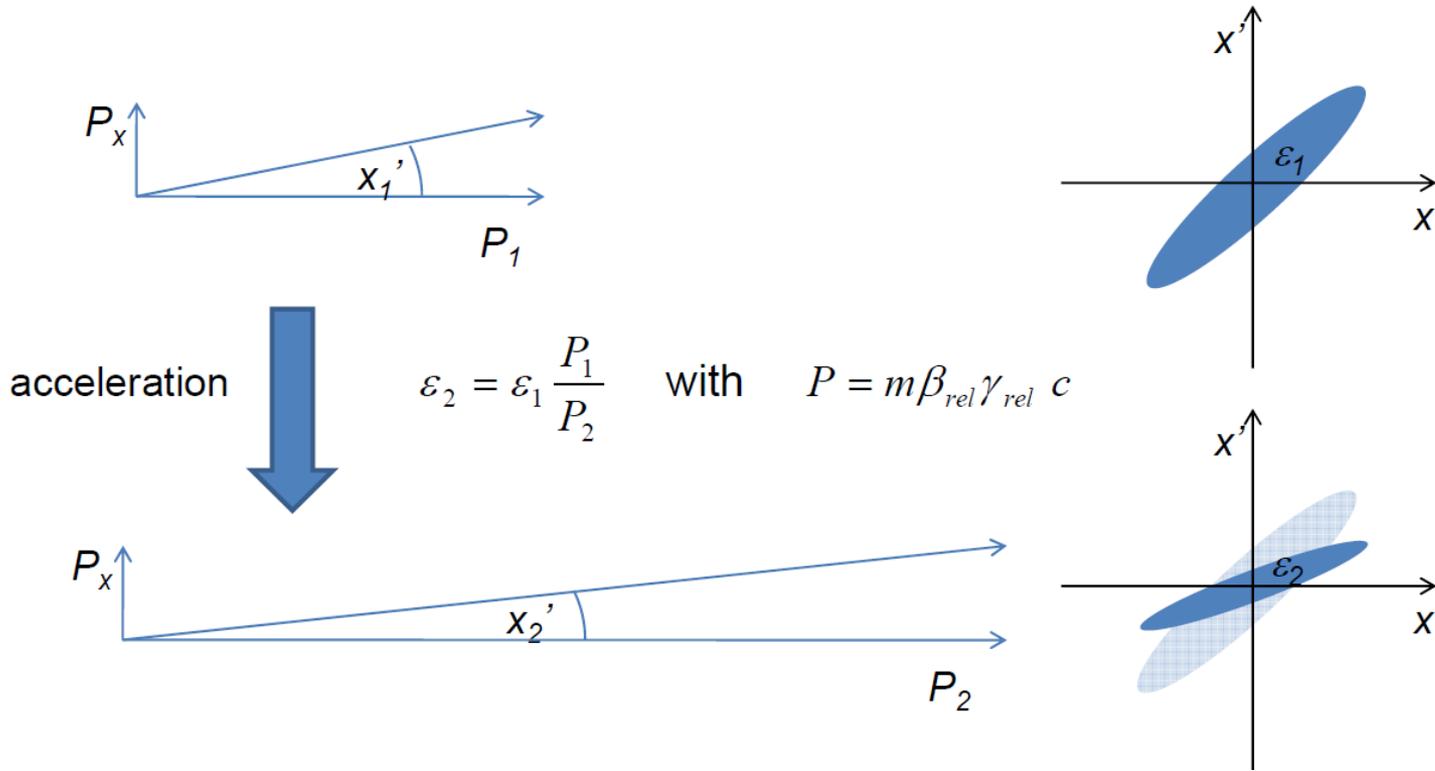
$$w_{RMS} = \sqrt{\frac{\int_{-\infty}^{+\infty} (x - x_{COG})^2 f(x) dx}{\int_{-\infty}^{+\infty} f(x) dx}}$$

For Gauss beam distribution

$$w_{rms} = \sigma_X$$

$$\epsilon_{rms}, 1 \text{ sigma r.m.s. emittance} \Leftrightarrow w_{RMS} = \sqrt{\beta\epsilon}$$

*Geometrical emittance is only constant in beamlines without acceleration*

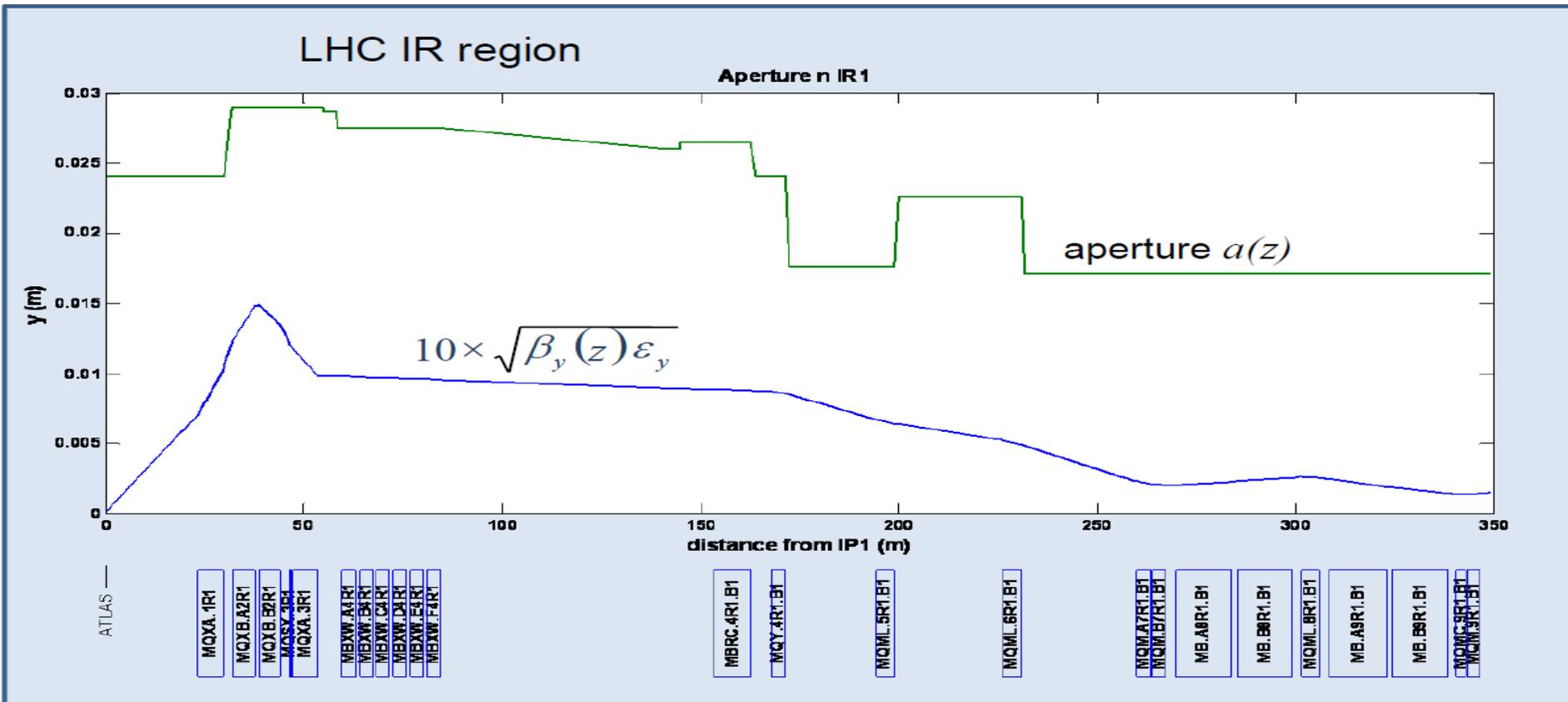


Normalized emittance preserved with acceleration

$$\varepsilon_N = \beta_{rel}\gamma_{rel} \varepsilon$$

# Emittance, why measuring it ?

The emittance tells if a beam **fits** in the vacuum chamber or not  $w(z) = \sqrt{\beta(z)} \varepsilon < a(z)$



**Emittance is one of key parameters for overall performance of an accelerator:**

- Luminosity of colliders for particle physics
- Brightness of synchrotron radiation sources
- Wavelength range of free electron lasers
- Resolution of fixed target experiments

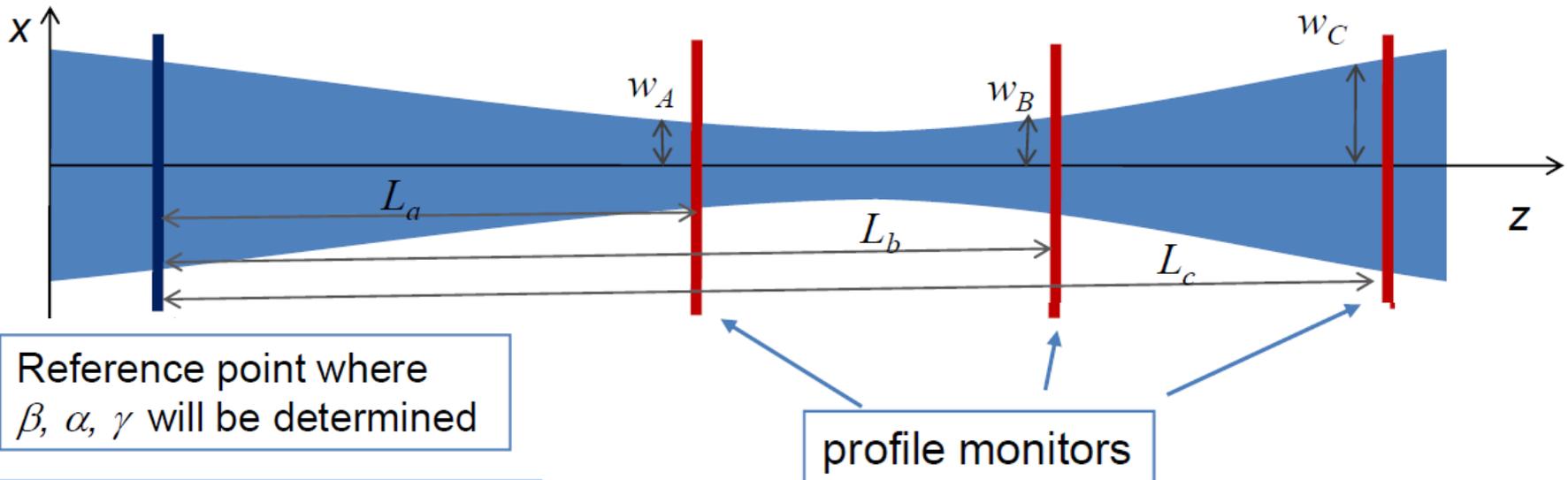
# *Emittance, how to measure it ?*

- Methods based on transverse beam profile measurements
  - Quadrupole scan
  - Different location beam profile measurements
- Slit and pepper pot(multi slit) methods

# Emittance measurement in transfer line or linac

Twiss parameters  $\alpha, \beta, \gamma$  are a priori not known, they have to be determined together with emittance  $\epsilon$

Method A



$$w_A^2 = \beta \epsilon - 2 L_A \alpha \epsilon + L_A^2 \gamma \epsilon$$

$$w_B^2 = \beta \epsilon - 2 L_B \alpha \epsilon + L_B^2 \gamma \epsilon$$

$$w_C^2 = \beta \epsilon - 2 L_C \alpha \epsilon + L_C^2 \gamma \epsilon$$

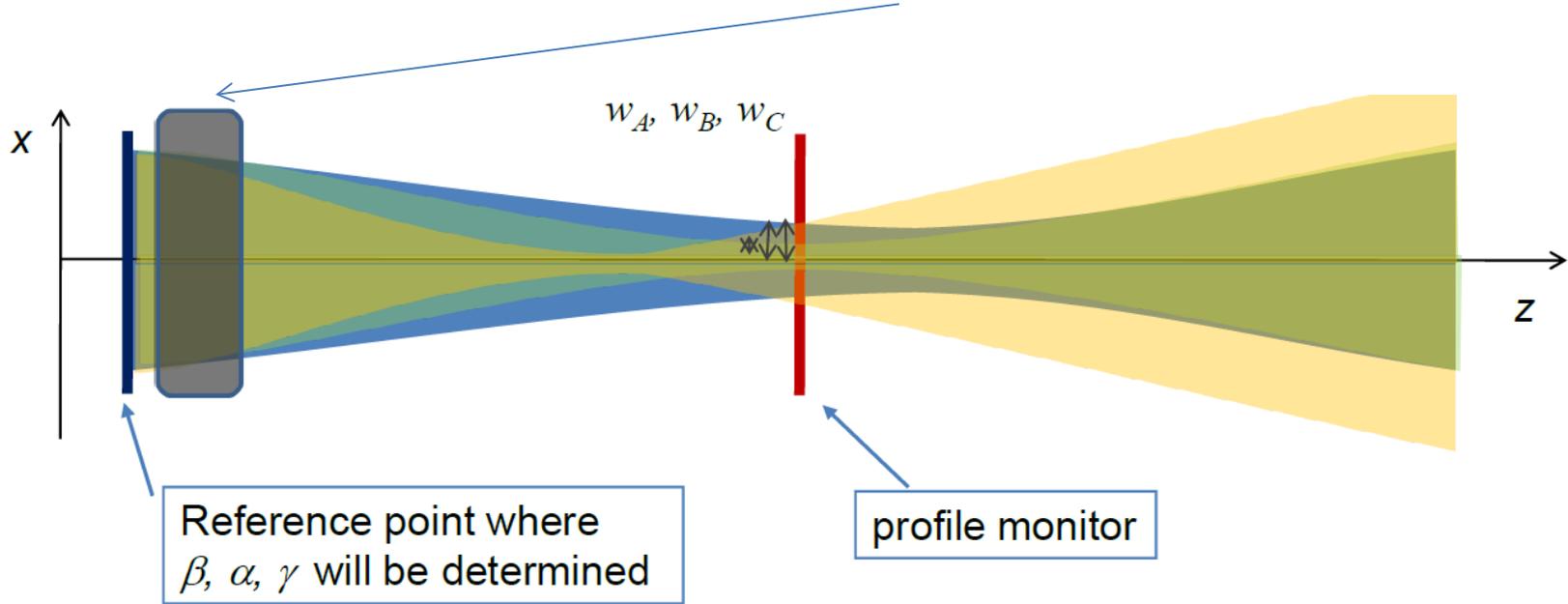
3 linear equations, 3 independent variable  
Solved by inverting matrix.

$$\beta \epsilon \cdot \gamma \epsilon - (\alpha \epsilon)^2 = \epsilon^2 (\beta \cdot \gamma - \alpha^2) = \epsilon^2 \Rightarrow \sqrt{\beta \epsilon \cdot \gamma \epsilon - (\alpha \epsilon)^2} = \epsilon, \quad \beta = \frac{\beta \epsilon}{\epsilon}, \quad \alpha = \frac{\alpha \epsilon}{\epsilon}$$

# Emittance measurement in transfer line or linac, (count.)

- Method B

Adjustable magnetic lens with settings  $A, B, C$   
(quadrupole magnet, solenoid, system of quadrupole magnets...)



$$w^2 = c^2 \beta \varepsilon - 2cs \alpha \varepsilon + s^2 \gamma \varepsilon, \quad \begin{pmatrix} c & s \\ c' & s' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} m_{11}(I_{mag}) & m_{12}(I_{mag}) \\ m_{21}(I_{mag}) & m_{22}(I_{mag}) \end{pmatrix}$$

$$w_A^2 = c_A^2 \beta \varepsilon - 2c_A s_A \alpha \varepsilon + s_A^2 \gamma \varepsilon$$

$$w_B^2 = c_B^2 \beta \varepsilon - 2c_B s_B \alpha \varepsilon + s_B^2 \gamma \varepsilon$$

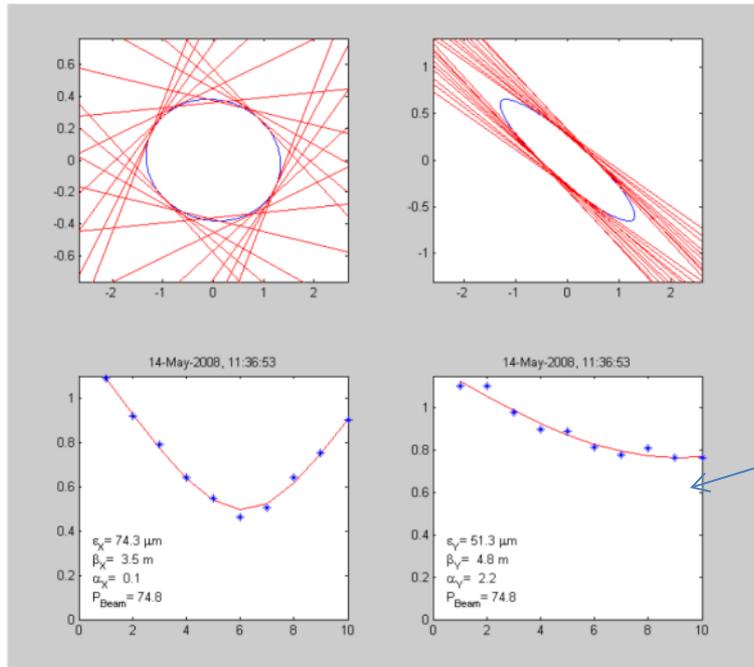
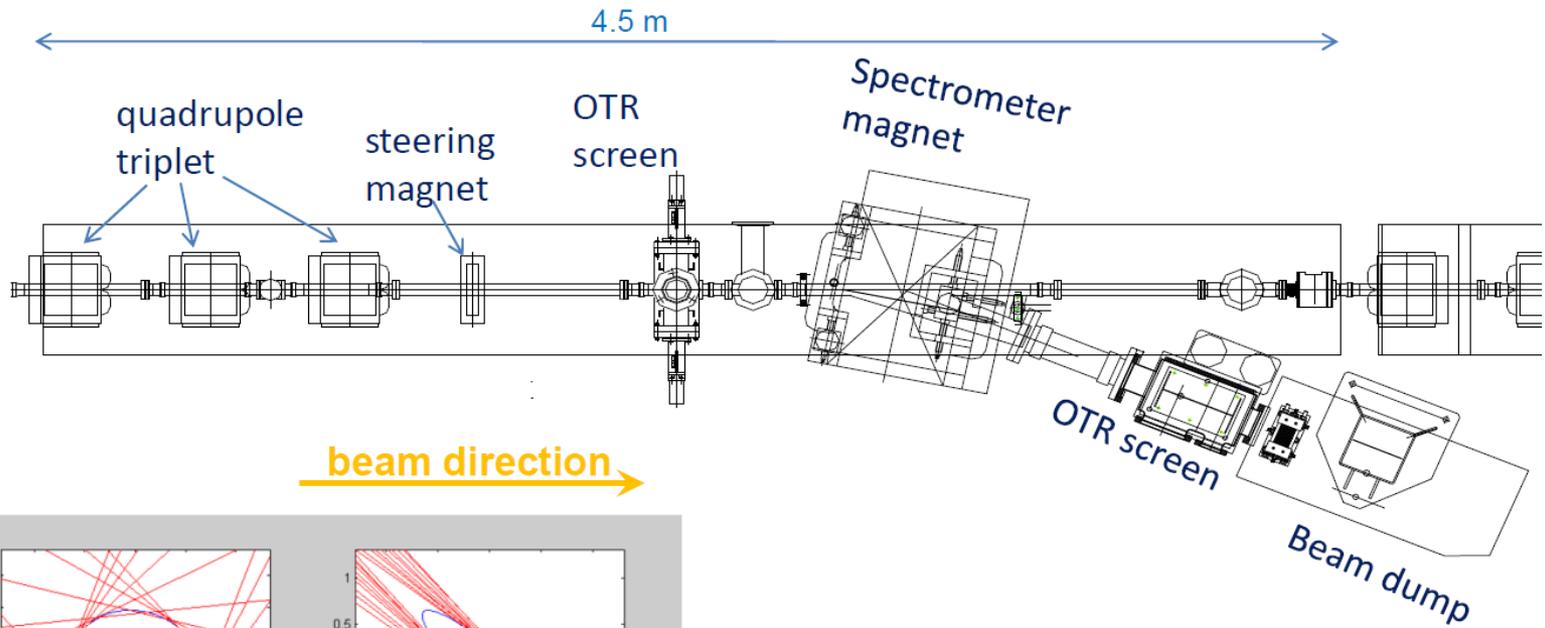
$$w_C^2 = c_C^2 \beta \varepsilon - 2c_C s_C \alpha \varepsilon + s_C^2 \gamma \varepsilon$$

3 linear equations, 3 independent variable

Solved by inverting matrix.

$$\beta \varepsilon \cdot \gamma \varepsilon - (\alpha \varepsilon)^2 = \varepsilon^2 (\beta \cdot \gamma - \alpha^2) = \varepsilon^2 \Rightarrow \sqrt{\beta \varepsilon \cdot \gamma \varepsilon - (\alpha \varepsilon)^2} = \varepsilon, \quad \beta = \frac{\beta \varepsilon}{\varepsilon}, \quad \alpha = \frac{\alpha \varepsilon}{\varepsilon}$$

# Emittance measurement example:

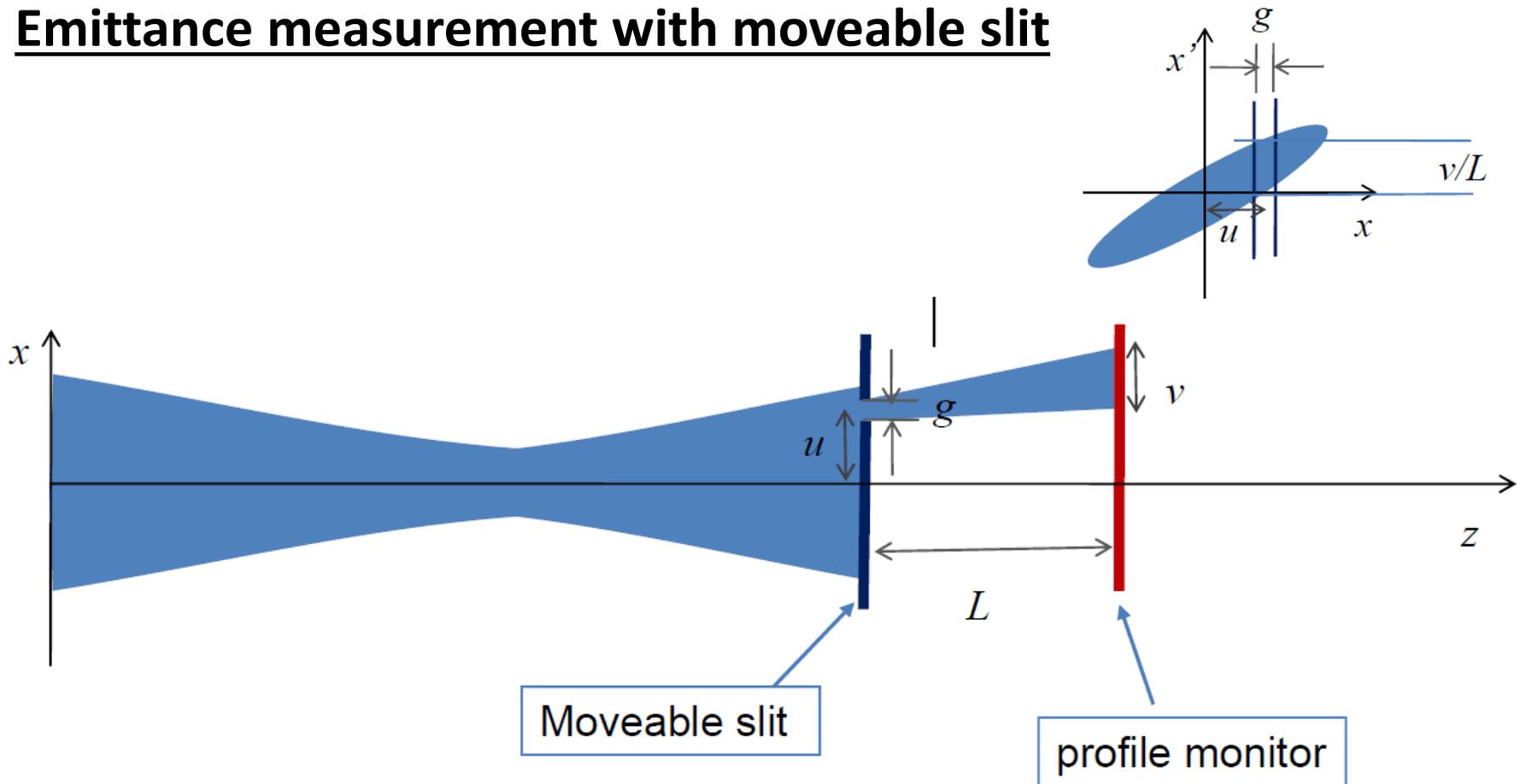


Parabola fit for quadrupole scan

# Summary beam profile technics

- To determine  $\varepsilon$ ,  $\beta$ ,  $\alpha$  at a reference point in a beamline one needs at least three  $w$  measurements with different transfer matrices between the reference point and the  $w$  measurements location.
- Different transfer matrices can be achieved with different profile monitor locations, different focusing magnet settings or combinations of both.
- Once  $\beta$ ,  $\alpha$  at one reference point is determined the values of  $\beta$ ,  $\alpha$  at every point in the beamline can be calculated.

# Emittance measurement with moveable slit



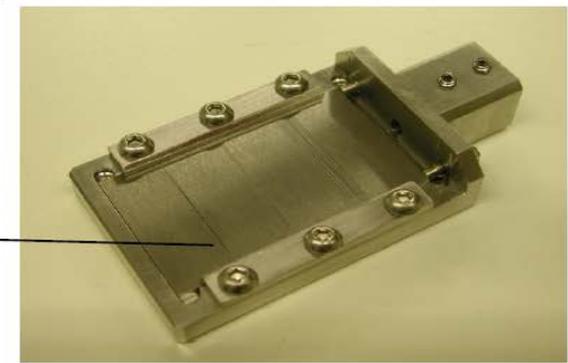
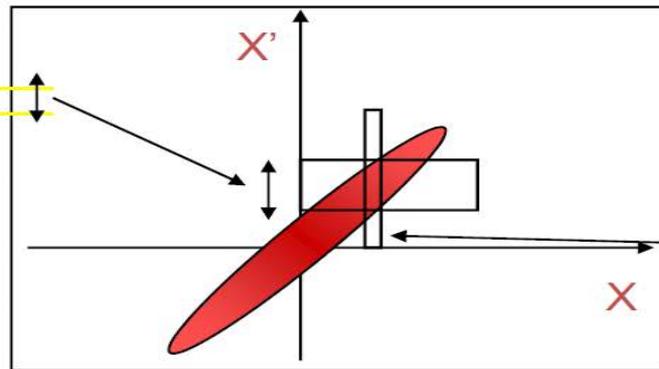
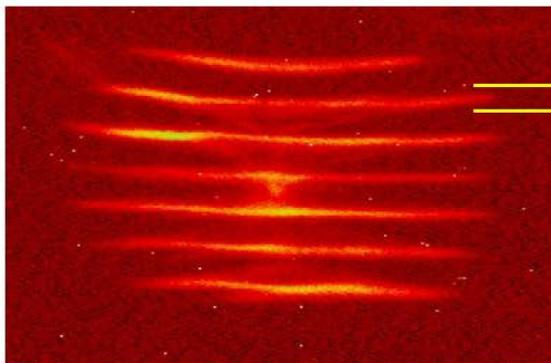
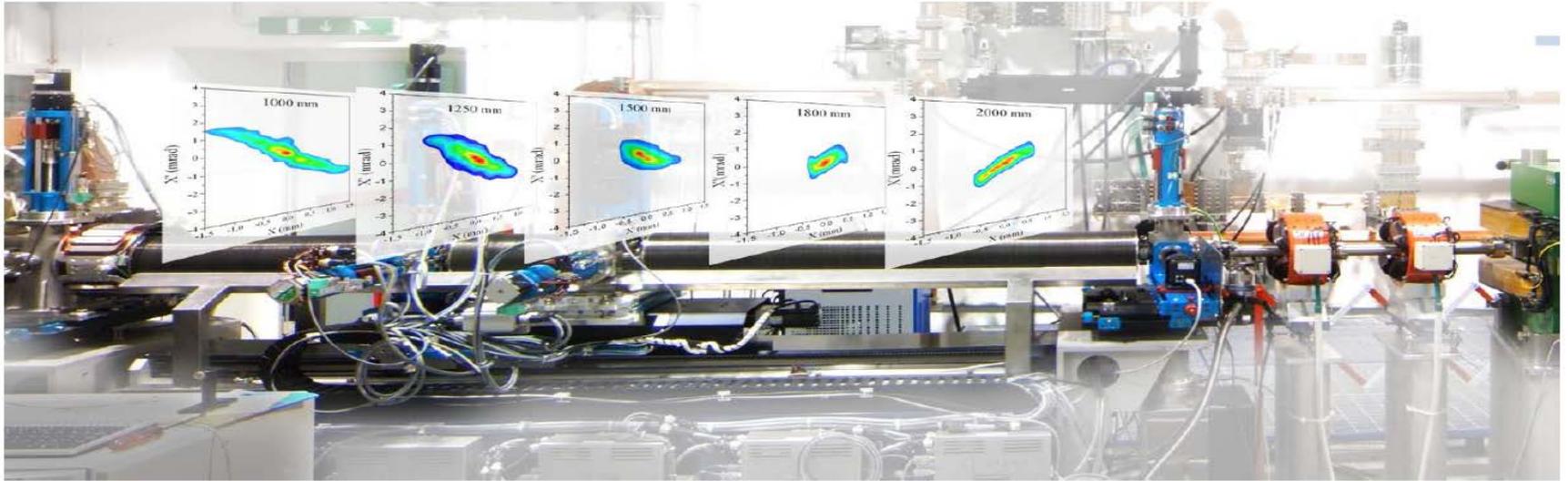
From width and position of slit image mean beam angle and divergence of slice at position  $u$  is readily computed.

By moving slit across the beam complete distribution in  $x, x'$  space is reconstructed.

Conditions for good resolution:  $v \gg s$

*Required multiple shots to reconstruct the phase space.*

# Phase space reconstruction at SPARC (LNF) Multi slit



Measurements could be performed for single bunch in one shot in single location

# Things to do in ATF control room (emittance measurements)

1. Learn how to measure emittance using 4 screens.
2. Measure emittance for different RF gun solenoid current settings Collect several data for each solenoid settings (minimum 10 solenoid settings).

***HW1: Plot these data. Find the optimum solenoid settings and minimum emittance. Conduct error analysis.***

3. For some solenoid settings (1-3) perform quadrupole scan.

***HW2: Plot profile vs quadrupole current. Calculate emittance (see next slide) for each solenoid settings (units?).***

***HW3: Based on quadrupole scan find quadrupole calibration coefficient  $K[1/(m \cdot A)]$  to convert quadrupole current to quadrupole focus strength ( $1/f[m]=K \cdot I[A]$ ). For given energy***

# Quadrupole scan

The quadrupole scan technique is a standard technique used in accelerator facilities to measure the transverse emittance. It is based on the fact that the squared rms beam radius ( $x_{rms}^2$ ) is proportional to the quadrupole “strength” or inverse focal-length  $f$  squared, so

$$x_{rms}^2 = \langle x^2 \rangle = A \left( \frac{1}{f^2} \right) - 2AB \left( \frac{1}{f} \right) + (C + AB^2) \quad (1)$$

where A, B, C are constants and  $f$  is the focal length defined as

$$\frac{1}{f} = \kappa l, \quad (2)$$

$$k \left[ \frac{1}{cm^2} \right] = \frac{G \left[ \frac{\text{Gauss}}{cm} \right]}{\text{Brho} [\text{Gauss} \cdot \text{cm}]}$$

here  $\kappa$  is the magnet focusing strength in units of 1 over length squared and  $l$  is the effective length of the magnet.

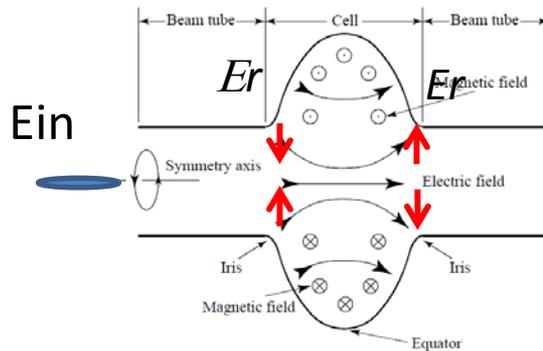
The emittance can be estimated according to

$$\varepsilon = \frac{\sqrt{AC}}{d^2} \quad (3)$$

where  $d$  is the distance from the magnet you scan to the point you calculate the beam rms radius.

# Questions from the last class

- Focusing due to entrance and exit of RF field
  - More energy gain => stronger focusing



Eout

$$\Delta p_r = \frac{e}{c} \int E_r dz$$

$$r'_{in} = \frac{\Delta p_{in_r}}{p_{in_z}}$$

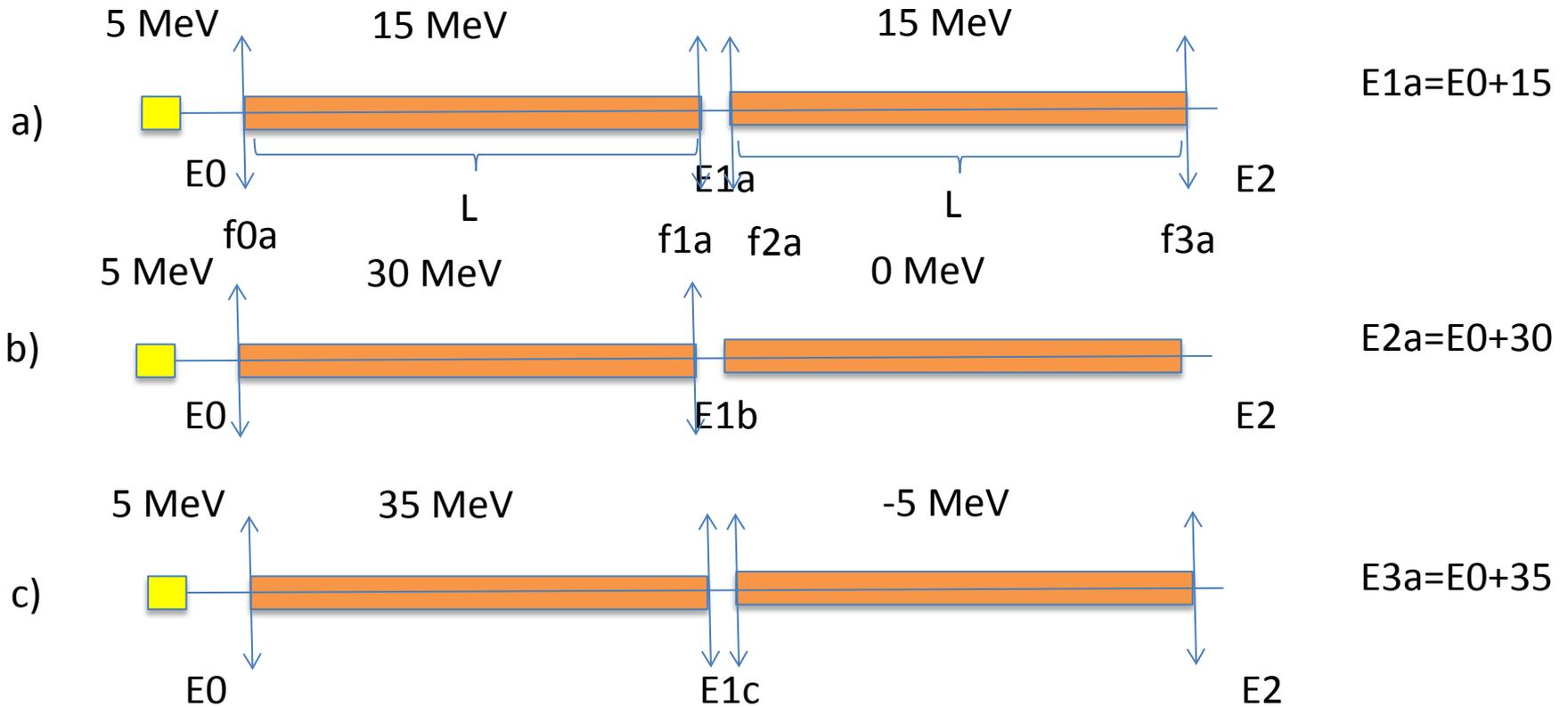
$$r'_{out} = \frac{\Delta p_{out_r}}{p_{out_z}}$$

$$\Delta p_{in_r} \sim -\Delta p_{out_r}$$

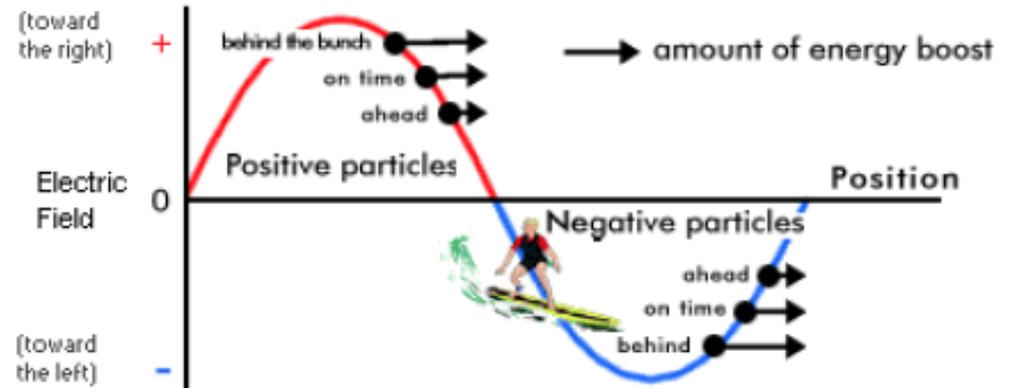
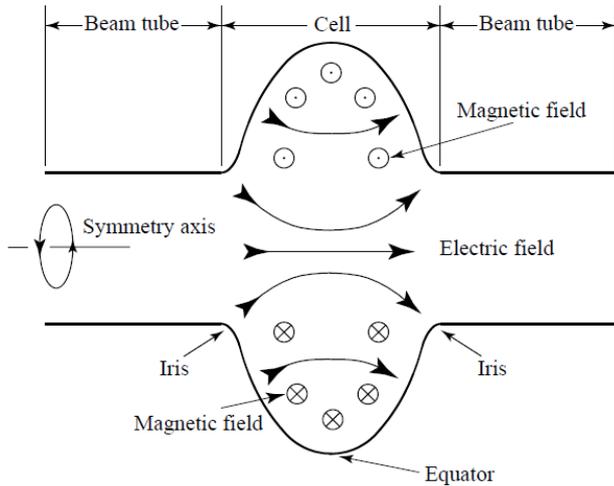
$$E_{out} = E_{in} + \Delta E$$

Accel. cavity: Entrance kick (focus) is larger than exit kick (defocus) => as a result **focusing**  
 Deaccel. cavity: Entrance kick (defocus) is smaller than exit kick (focus) => as a result **focusing**

# What is the difference in different Linac configurations?



# RF Field acceleration:



The RF field must be synchronous (correct phase relation) with the beam for a sustained energy transfer.

$$E_z(z, t) = E(z) \cos\left(\omega t - \int_0^z k(z) dz + \phi\right),$$

For efficient particle acceleration, the phase velocity of the wave must closely match the beam velocity. If we consider a particle of charge  $q$  moving along  $+z$  direction with a velocity at each instant of time equal the phase velocity of the traveling wave, then the electric force on the particle is given by

$$F_z = q E(z) \cos \phi$$

Energy gain

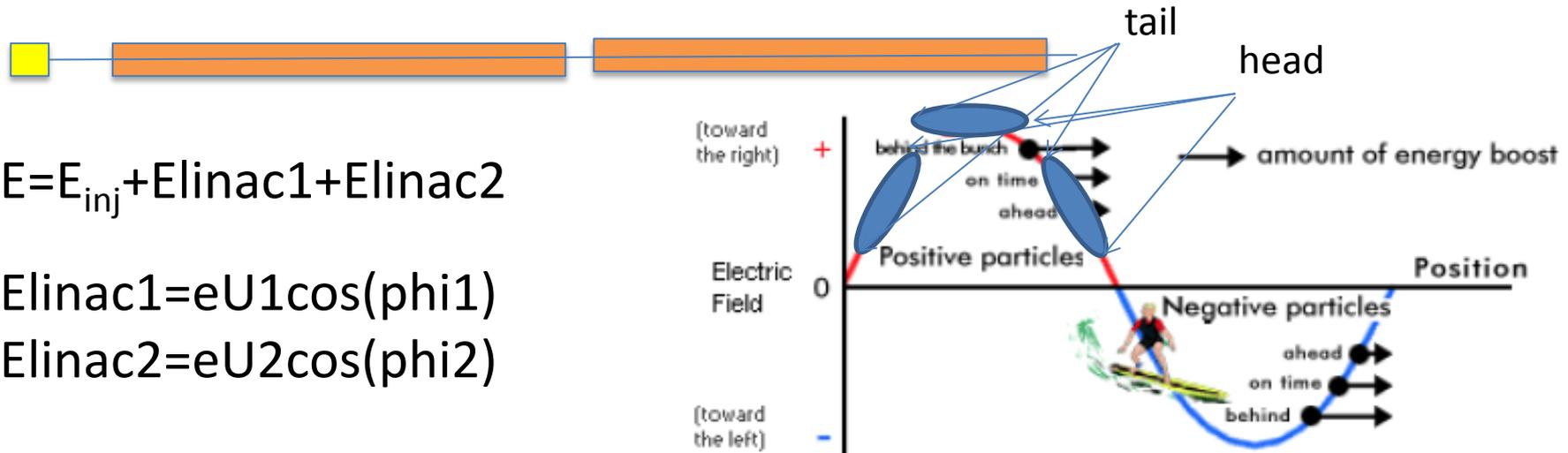
$$\Delta \mathcal{E} = q \int_{-L/2}^{L/2} E(0, z) \cos[\omega t(z) + \phi] dz$$

# Multi linacs acceleration

5 MeV

36 MeV

36 MeV



$$E = E_{inj} + E_{linac1} + E_{linac2}$$

$$E_{linac1} = eU_1 \cos(\phi_1)$$

$$E_{linac2} = eU_2 \cos(\phi_2)$$

If there is enough voltage provided by one linac.

The final energy can be reached by combination different phases.

For ATF:

$U_1 = U_2 = 36 \text{ MV}$ ,  $E_{inj} = 5 \text{ MeV}$

$E_{final} = 35 \text{ MeV}$



phi1	phi2
65.4	65.4
65.4	-65.4
0.0	99.6
0.0	-99.6
90.0	33.6
-90.0	-33.6
33.6	90.0
-33.6	-90.0

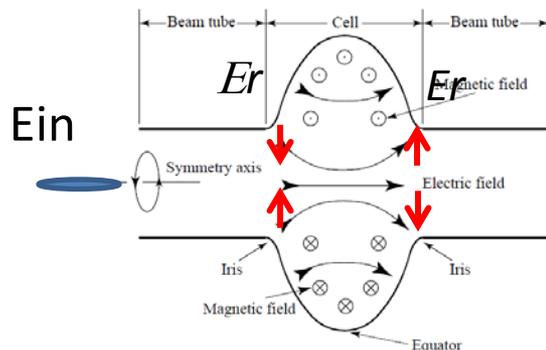
Why one operation could be better then others?

# Few things to remember

- Space charge force depends on energy
  - Higher energy => less space charge effects

$$\sigma_x''(\zeta, s) + \kappa_\beta^2 \cdot \sigma_x(\zeta, s) = \frac{r_e \lambda(\zeta)}{2\gamma^3 \sigma_x(\zeta, s)} + \frac{\epsilon_{n,x}^2}{2\gamma \sigma_x^3(\zeta, s)}$$

- Focusing due to entrance and exit of RF field
  - More energy gain => stronger focusing



$$\Delta p_r = \frac{e}{c} \int E_r dz$$

$$r'_{in} = \frac{\Delta p_{in_r}}{p_{in_z}}$$

$$r'_{out} = \frac{\Delta p_{out_r}}{p_{out_z}}$$

$$\Delta p_{in_r} \sim -\Delta p_{out_r}$$

$$E_{out} = E_{in} + \Delta E$$

Accel. cavity: Entrance kick (focus) is larger than exit kick (defocus) => as a result **focusing**  
 Deaccel. cavity: Entrance kick (defocus) is smaller than exit kick (focus) => as a result **focusing**

# PHY 542

## COMPUTATIONAL EXERCISE – RF Linac

Exercise: RF linac acceleration

1. Open file *ATF\_LINAC.in*. Find acceleration linac line description. There are two linacs. Make sure that the both cavities gradient is sufficient to accelerate e-beam on 36 MeV by each cavity. Change adjust maximum gradient (maxE parameter).  
*(hint: set acceleration phase to 0 in both linacs and run ASTRA for this project)*
2. Search for optimum linac set points for fix energy gain 30 MeV. Set up linac acceleration gradient 16 MV/m. Set the same phase for both linac to accelerate 15 MeV each. ( $\phi=65$  deg). Find final energy spread and emittance.
3. Repeat step 2 for different linac phases:
  - a. Linac Phase1=65 LinacPhase2=-65
  - b. Linac\_Phase1=34 Linac Phase2=90
  - c. Linac Phase1=90 LinacPhase2=34 (have you got the same energy?)
  - d. Linac Phase1=0 LinacPhase2=100
4. What linacs phase settings provide minimum **emittance**?
5. What linacs phase settings provide minimum **energy spread**?

Same exercise without space charge:

6. Try turn off space charge and repeat steps 2-5.
7. Why final emittance is different without space charge?