Homework 13. Due October 26

Problem 1. 5 points. Beam envelope in straight section.

For a one-dimensional motion consider beam propagating in a straight section starting as s_o and having length L. Let's eigen vector (beam envelope) at s_o is given by:

$$Y(s_o)=\begin{bmatrix} w_o(s) \\ w_o'(s)+\dfrac{i}{w_o(s)} \end{bmatrix};$$

$$\beta_o(s_o)\equiv w_o^2(s_o);\ \ \alpha_o(s_o)=-\frac{\beta'(s_o)}{2}\equiv -w_o(s_o)w_o'(s_o); \qquad (1)$$

(a) Propagate the eigen vector along the straight section. Show that β-function can be expressed as

$$\beta(s)=\beta^*+\frac{(s-s^*)^2}{\beta^*};$$

where β^*, s^* can be found from initial conditions (1). Hint, use derivative of β-function to find s^*. β^* is frequently used in colliders to describe the beam envelope in detectors.

(b) Calculate the (betatron) phase advance acquired in the straight section. Express it using β^*, s^*. Write expression for x(s) and x'(s). Show that x'=const.

(c) What is the maximum possible phase advance in a straight section (e.g. when s_o,L are unlimited)?