

## Home Work PHY 554

**HW 1 (2 points):** Calculate relations between three dimensionless infinitesimal parameters:

$$\frac{dE}{E} \equiv \frac{d\gamma}{\gamma}; \frac{dp}{p} \equiv \frac{d(\beta\gamma)}{\beta\gamma}; \frac{dv}{v} \equiv \frac{d\beta}{\beta}$$

where E is energy, p is momentum and v is velocity of a particle. Hint: use relativistic relations between  $\beta$  and  $\gamma$ .

**HW 2 (5 points):** In class we introduced the map of longitudinal motion in a storage ring

$$\delta_{n+1} = \delta_n + \frac{eV_{rf}}{\beta^2 E_0} (\sin\phi_n - \sin\phi_s); \quad (1)$$

$$\phi_{n+1} = \phi_n + 2\pi h\eta \cdot \delta_{n+1},$$

1. For small oscillation variations of the RF phase about the synchronous phase

$$\varphi = \phi - \phi_s; \quad |\varphi| \ll 1$$

linearize the map (1) by keeping only first order on  $\varphi$  and find one turn transport matrix  $M$  for longitudinal motion:

$$\begin{pmatrix} \varphi \\ \delta \end{pmatrix}_{n+1} = M \begin{pmatrix} \varphi \\ \delta \end{pmatrix}_n$$

2. Using Courant-Snyder parametrization we used for transverse motion find value of  $\cos\mu_s, \beta_s, \alpha_s$  in parametric form (e.g. using  $\sin\mu_s = \sqrt{1 - \cos^2\mu_s}$ ,  $\mu_s = 2\pi Q_s = \cos^{-1}(\cos\mu_s)$ ).

3. Assuming that  $\mu_s \ll 1$ , find analytical expression for synchrotron tune and compare it with that we found in Lecture 9.

**HW 3 (3 points):** (4 points) Use formula developed in lecture, find the synchrotron tunes for 100 GeV and 15 GeV protons in a storage ring for the following parameters (similar to RHIC collider at BNL):

RF voltage,

$$V = 500 \text{ kV}$$

Depending on the sign of the slip factor the synchronous phase is zero or 180 degrees,

$$\phi_s = 0, \pi$$

Harmonic number,

$$h = 360$$

Compaction factor,

$$\alpha_c = 0.002$$