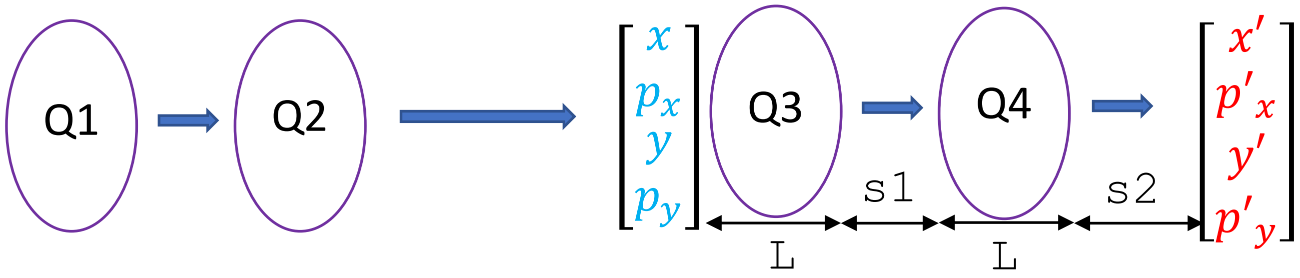
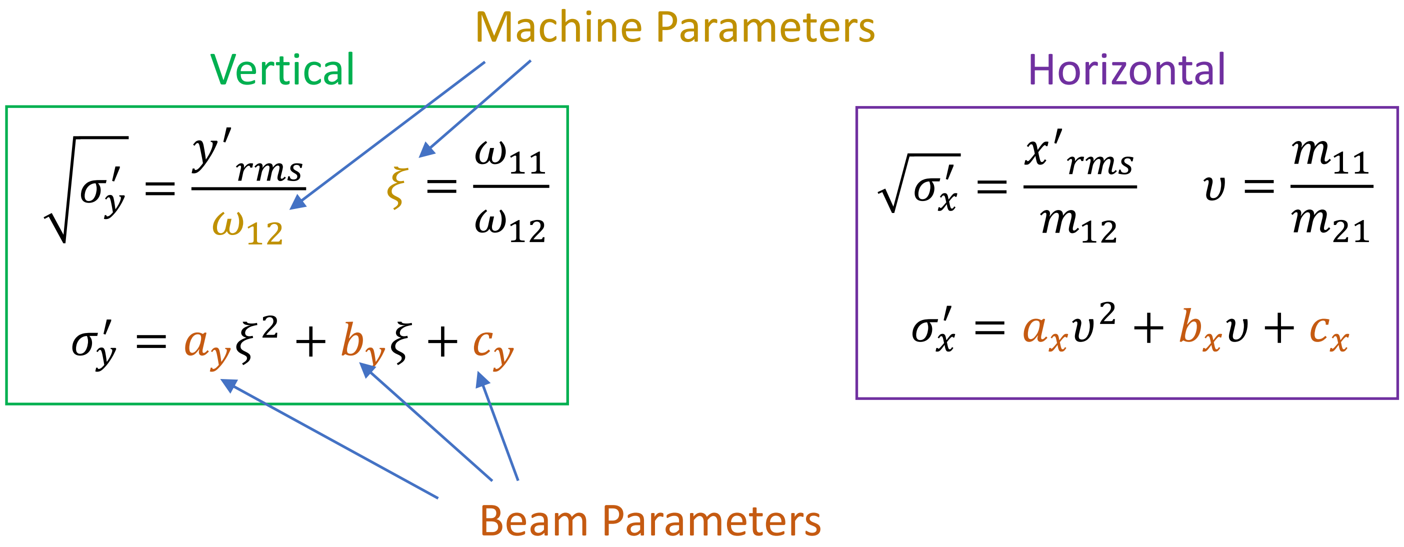
Slice Emittance Measurement

KS

01/8/2022



$$\therefore \begin{bmatrix} \boxed{\begin{matrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{matrix}} & 0 \\ 0 & \boxed{\begin{matrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{matrix}} \end{bmatrix} \begin{bmatrix} x \\ p_x \\ y \\ p_y \end{bmatrix} = \begin{bmatrix} x' \\ p'_x \\ y' \\ p'_y \end{bmatrix}$$



Machine Parameters

Vertical

$$\sqrt{\sigma'_y} = \frac{y'_{rms}}{\omega_{12}} \qquad \xi = \frac{\omega_{11}}{\omega_{12}}$$

$$\sigma'_y = a_y\xi^2 + b_y\xi + c_y$$

Horizontal

$$\sqrt{\sigma'_x} = \frac{x'_{rms}}{m_{12}} \qquad \upsilon = \frac{m_{11}}{m_{21}}$$

$$\sigma'_x = a_x\upsilon^2 + b_x\upsilon + c_x$$

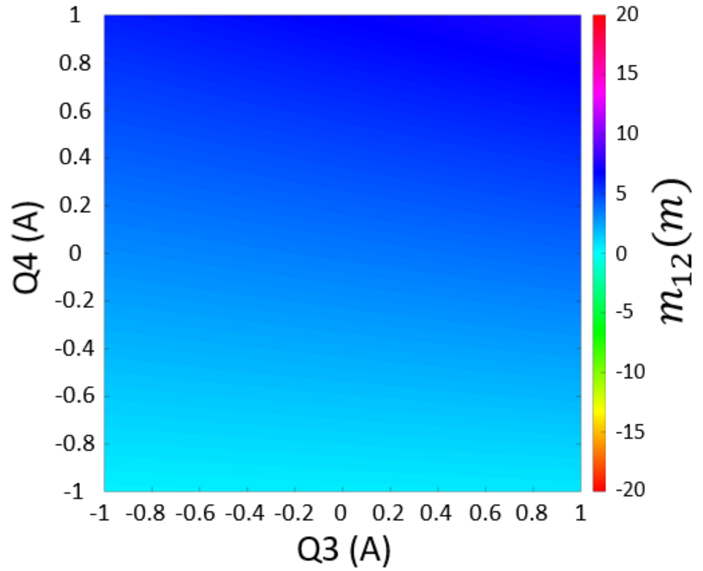
Beam Parameters

Vertical & Horizontal
Machine Parameters are related

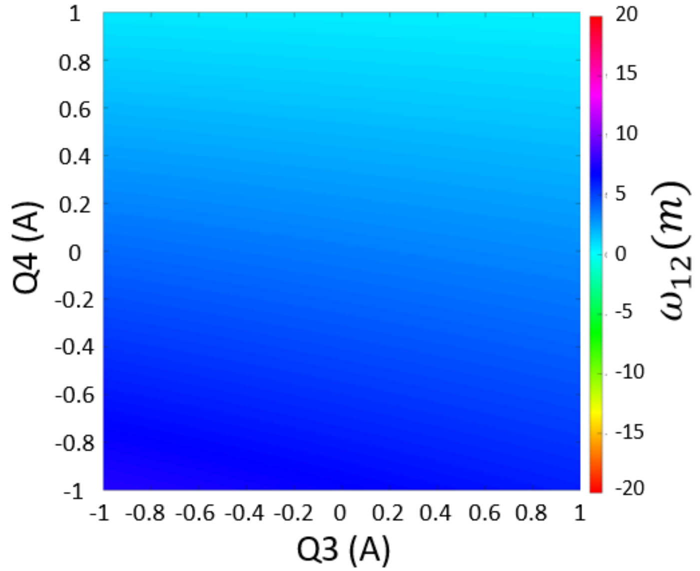
$$\omega_{12}(Q3, Q4) = m_{12}(-Q3, -Q4)$$

$$\xi(Q3, Q4) = \upsilon(-Q3, -Q4)$$

180 deg rotation



(b)



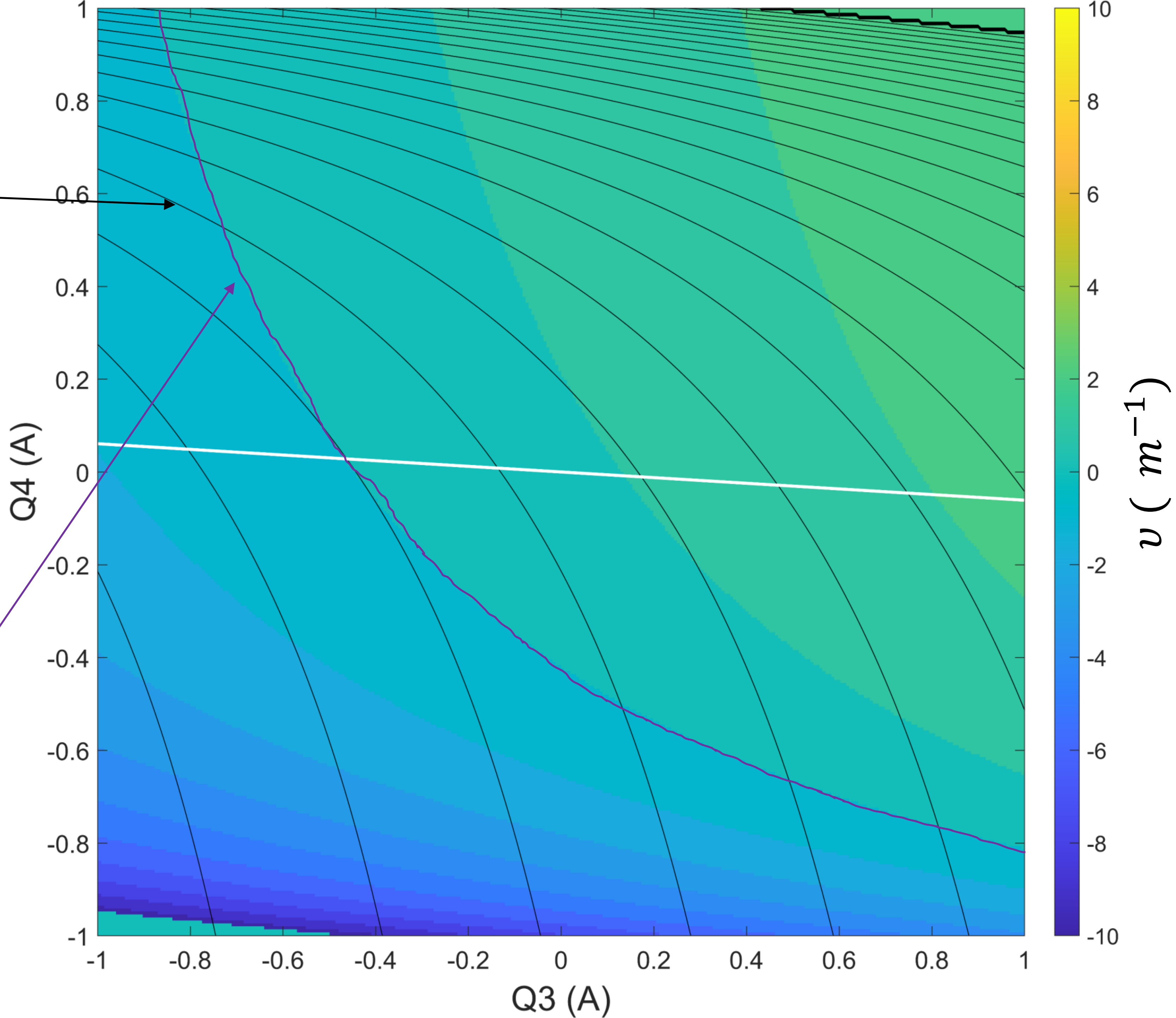
(b)

$$\sigma'_y = a_y\xi^2 + b_y\xi + c_y$$

ξ is constant

⇩

σ'_y is constant



$$\sigma'_x = a_x\upsilon^2 + b_x\upsilon + c_x$$

υ is constant

⇩

σ'_x is constant

ξ is constant

⇓

σ'_y is constant ⇒ $\omega_{12}(Q3,Q4)\Big|_{\xi=const.} = \omega_{12}(Q4)$

⇓

Only a machine property

$$\frac{d\,ln(y'_{rms})}{dQ4}\Bigg|_{\xi=const.} = \frac{1}{\omega_{12}}\frac{d\omega_{12}}{dQ4}\Bigg|_{\xi=const.}$$

$$\frac{1}{\omega_{12}}\frac{d\omega_{12}}{dQ4}\Bigg|_{\xi=const.} \approx \boxed{\frac{1}{aQ_4+b}} + \boxed{c}$$

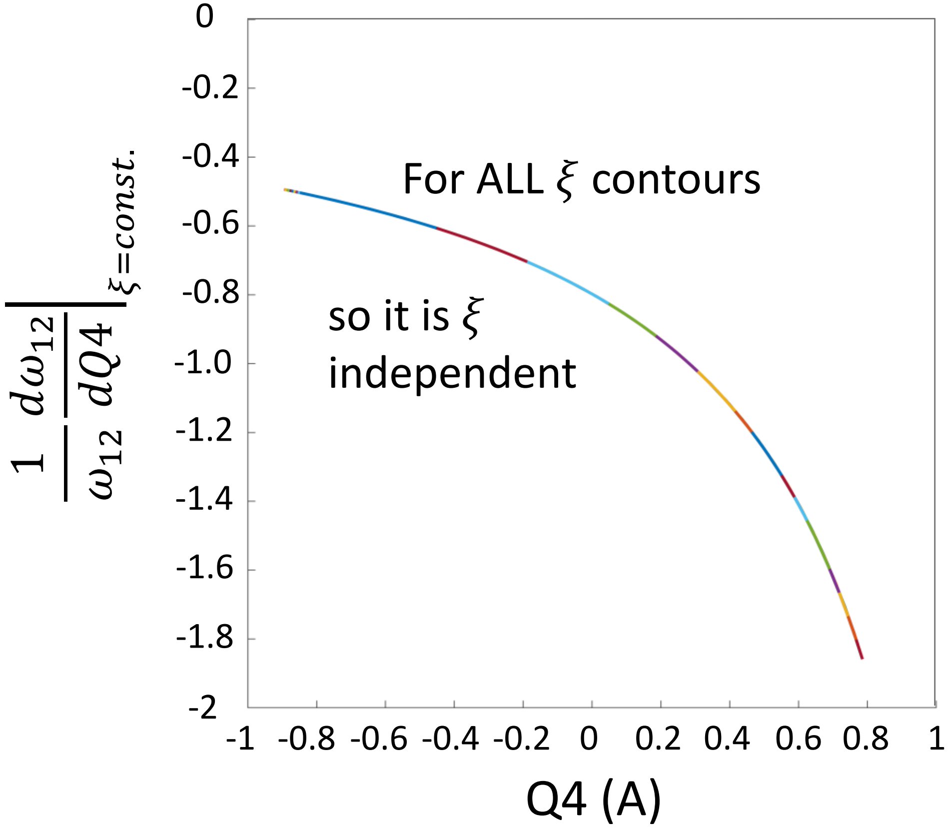
From thin lens

Second order term of ω_{12} O($Q4^2$)

⇓

$$y_{rms}\Big|_{\xi=const.} = y_0\left(\frac{a}{b}Q_4+1\right)^{\frac{1}{a}} e^{c\cdot Q_4}$$

Where Q*Qk = k; $y_0 = y_{rms}(\xi, Q_4 = 0)$

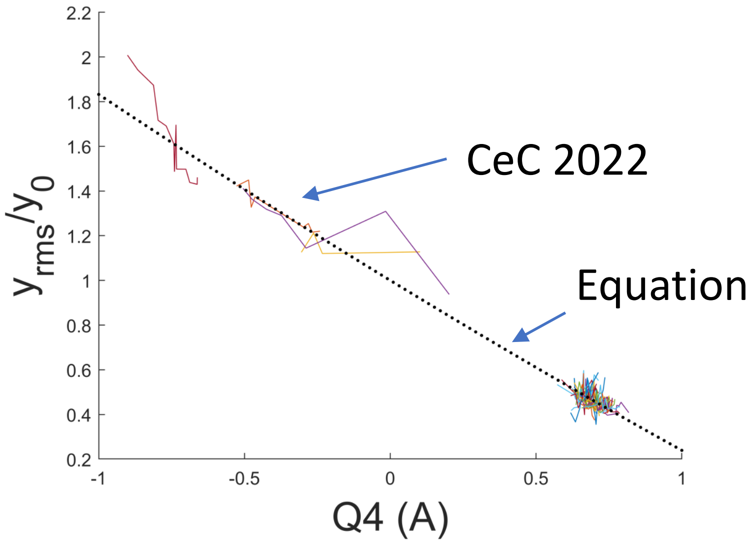


For ALL ξ contours

so it is ξ independent

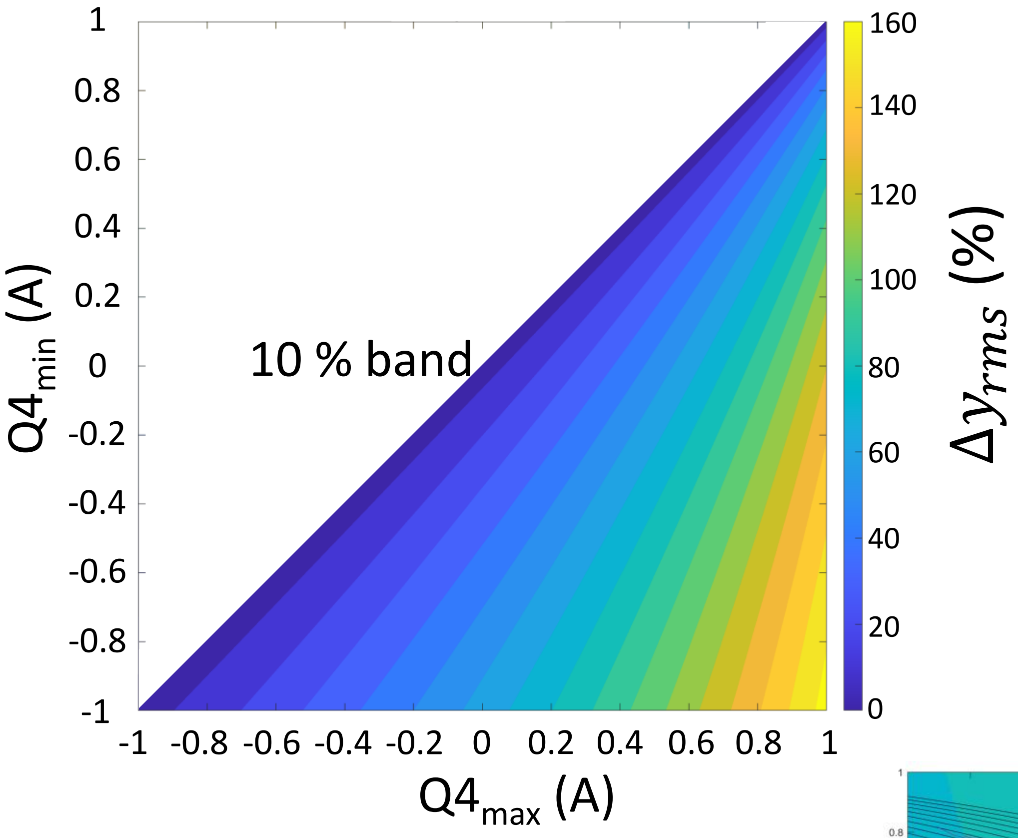
Thin lens	Thick lens
a = 1;	a = 0.9679;
b = -1.2534[A];	b = -1.3080[A];
c = 0[A^{-1}];	c = -0.0333[A^{-1}];
* b =(s1+s2)/(s1*s2*L*Qk)	

$$y_{rms}\Big|_{\xi=const.} = y_0\left(\frac{a}{b}Q_4+1\right)^{\frac{1}{a}} e^{c\cdot Q_4}$$



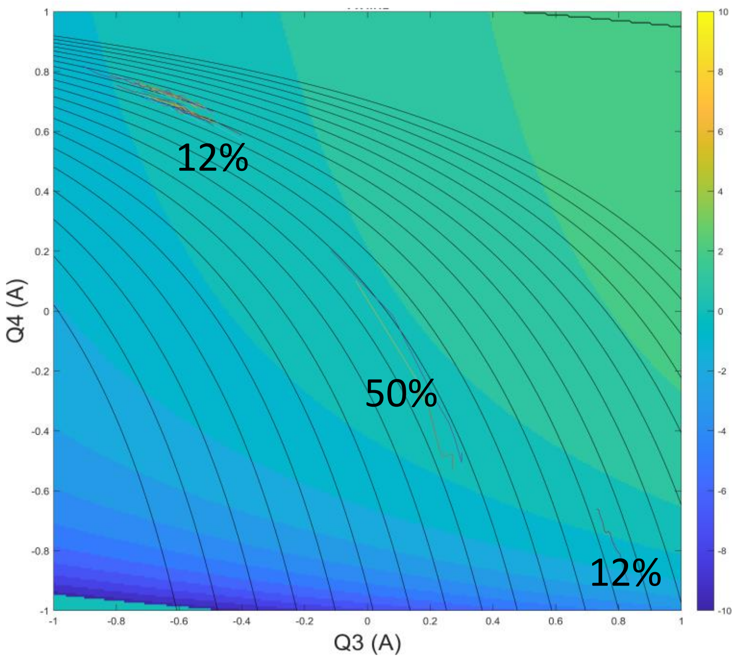
CeC 2022

Equation



10 % band

So Q4 range will defines time resolve resolution



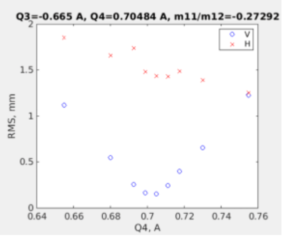
12%

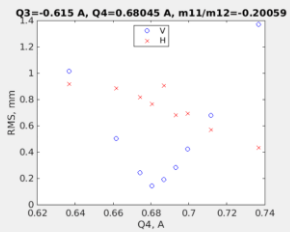
50%

12%

In the search of vertical focusing

Pick a Q3 then scan Q4 for minimum y .
Next, change Q3 and scan Q4 for minimum y again.





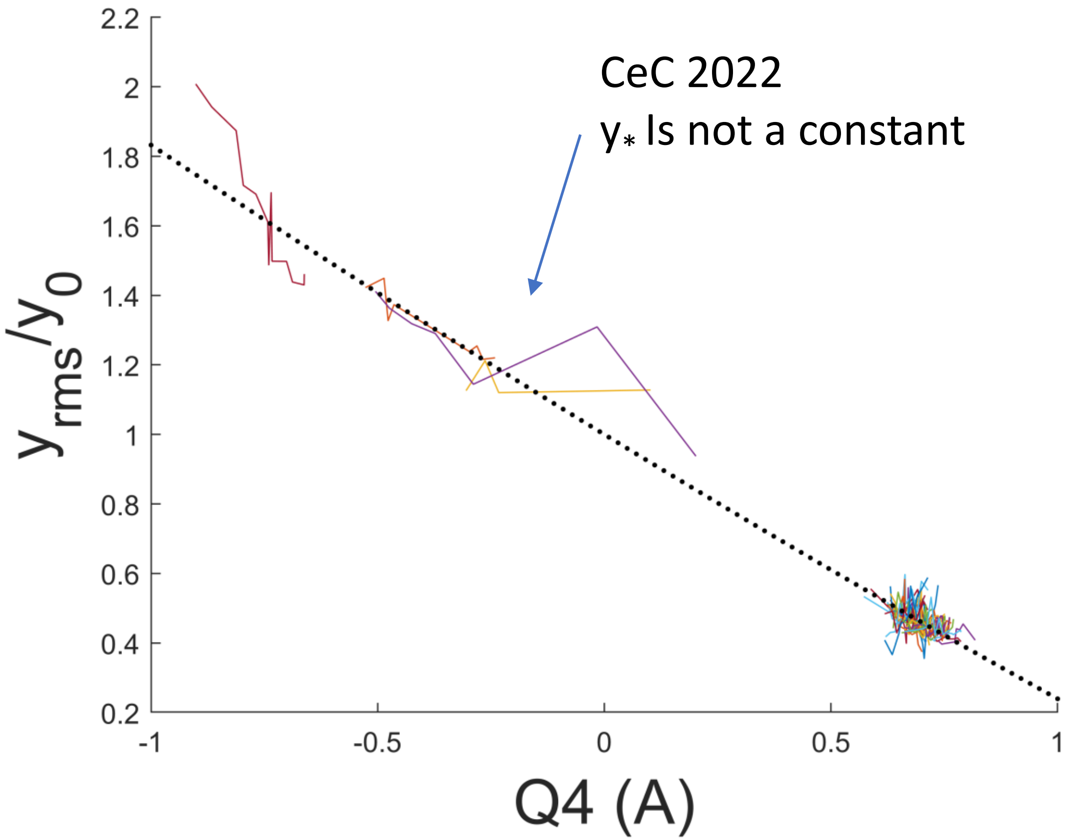
$$Q3_1 \longrightarrow \{\ldots\ Q4_1\ Q4_2\ \boxed{Q4_3}\ \ldots\} \longrightarrow y_{*1} = y(Q3_1,\ Q4_3)$$

$$Q3_2 \longrightarrow \{\ldots\ \boxed{Q4_1}\ Q4_2\ Q4_3\ \ldots\} \longrightarrow y_{*2} = y(Q3_2,\ Q4_1)$$

$$\vdots$$

y_{*1} is the minimum of $y(\text{Q31},\ \{Q_4\})$

y_{*2} is the minimum of $y(\text{Q32},\ \{Q_4\})$



CeC 2022
y_* Is not a constant

But why $y_{*1} = y_{*2} = \cdots ?$ ← **No! They Don't** for [Drift][t-Quad] system

$$y_* = \frac{\varepsilon}{\beta^2} s^2$$

Define vertical beam focal point y = y_* , when :

$$\left.\frac{\partial y(Q_3,\ Q_4)}{\partial Q_3}\right|_{Q_4=const.} = 0 \quad \text{or} \quad \left.\frac{\partial y(Q_3,\ Q_4)}{\partial Q_4}\right|_{Q_3=const.} = 0$$

So y_* is also a function of $(Q_3,\ Q_4)$

$$\Rightarrow \quad \left.\frac{\partial y}{\partial Q_3}\right|_{Q_4=const.} = 0$$

$$\left.\frac{\partial y}{\partial \xi}\frac{\partial \xi}{\partial Q_3}\right|_{Q_4=const.} = 0$$

$$\because \frac{\partial \xi}{\partial Q_3} \neq 0 \quad \therefore \left.\frac{\partial y}{\partial \xi}\right|_{Q_4=const.} = 0$$

$$\because y_{rms}\Big|_{\xi=const.} = y_0\left(\frac{a}{b}Q_4 + 1\right)^{\frac{1}{a}} e^{c\cdot Q_4}$$

$$\therefore y(\xi,\, Q_4) = y_0(\xi)L(Q_4)$$

In here:

ξ and Q_4 is independent.

Since Changing Q_4, Q_3 will also change to keep ξ constant

$$\frac{\partial y}{\partial \xi}(Q_4, \xi) = L(Q_4) \times \frac{\partial y_0(\xi)}{\partial \xi}$$

$$\because L(Q_4) \neq 0$$

$$\therefore \frac{\partial y_0(\xi)}{\partial \xi} = 0 \Rightarrow \left.\frac{\partial y}{\partial \xi}\right|_{Q_4=const.} = 0$$

focal point y = y_*

$\xi = \xi_*$ $\Rightarrow$ $\mathrm{y} = y_*$ Independent of Q_4

At ξ_* contour $\Rightarrow$ $y_{rms}(Q_3, Q_4)$ in focus BUT $y_{rms} = y_0(\xi_*)L(Q_4)$

At y_* contour $\Rightarrow$ $y_{rms}(Q_3, Q_4)$ Out of focus (not a minimum) BUT $y_{rms} = y_*(Q_{3*}, Q_{4*})$ is constant

- focuses beam vertically with Q3 Q4
- Test 3 points σ_x' along the ξ contour
- Set Q3 Q4 according to the changes of υ that requires
- Change Q1 Q2 to re-focuses beam vertically
- Essentially, we shifted ξ contour to where contains the minimum σ_x'

$$\frac{\langle x^2 \rangle}{m_{12}^2} = \sigma_h = \varepsilon\beta \cdot \Delta\nu^2 + \frac{\varepsilon}{\beta}$$

$$\left.\frac{d\upsilon}{dQ4}\right|_{\xi=const.} = 0$$