Study of microbunching instability in an EIC MBEC cooling system

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Version 2

Monday 19th July, 2021

1 Introduction

The strong hadron cooling system for EIC uses chicanes in the electron path to convert the energy modulations in the electron beam into the density modulations. It is known that the combination of the chicane's dispersion with the CSR wakefields in the bending magnets can lead to a microbunching instability of the beam [1]. Such an instability may be detrimental to the quality of the beam, and could interfere with the cooling process. In this note, I evaluate the strength of the instability for the parameter of the BNL EIC project using theoretical analysis of Ref. [1].

In this note I use the Gaussian system of units.

2 Parameters of the chicane and its lattice functions

The parameters of the electron beam are listed in Table 1.

Electron beam energy	160 MeV
RMS length of the electron beam, σ_z	4 mm
RMS relative energy spread of the electron beam, σ_η	1.0×10^{-4}
Peak electron beam current, I_e	30 A
Electron beam charge, Q_e	1 nC
Normalized electron beam emittance, ϵ	1 μm

Table 1: Parameters of the electron beam in MBEC cooling section.

The electron chicane consists of four rectangular bends of length 0.15 m with bending angles ±8.0 degrees. The bends are placed back-to-back with no drifts between them, with the chicane length equal to 0.6 m. The initial values of the lattice functions at the entrance to the chicane are: $\beta_{x0} = 3.138$ m, $\beta_{y0} = 19.773$ m, $\alpha_{x0} = 10.290$, $\alpha_{y0} = 34.081$, $D_x = D_y = D'_x = D'_y = 0$. The bending radius for these parameters is $R = 1.08$ m. Some of the optical functions for the chicane are shown in Figs. 1-6 with $s = 0$ corresponding to the chicane entrance.

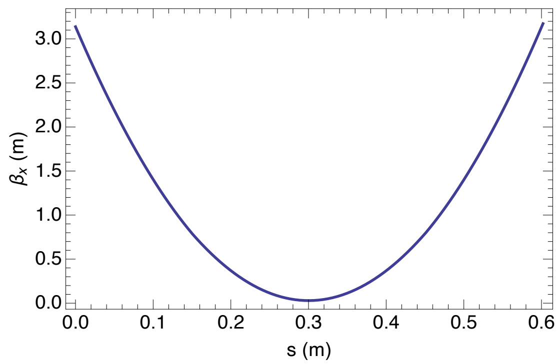

Figure 1: The function $\beta_x(s)$.

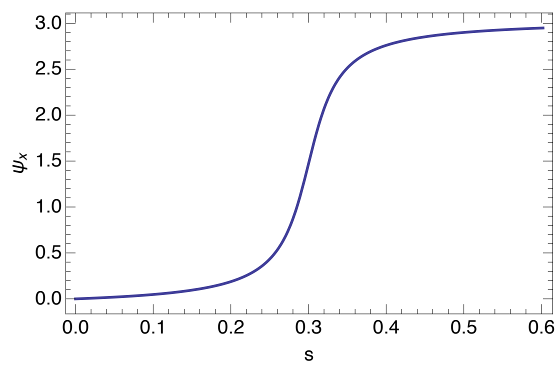

Figure 2: The phase advance $\psi_x(s)$.

3 Estimates of the instability gain factor

Before presenting the results of the solution of an integral equation for the gain function derived in Ref. [1], I give a qualitative analysis of the instability using simple physics arguments. I begin with the expression for 1D CSR impedance

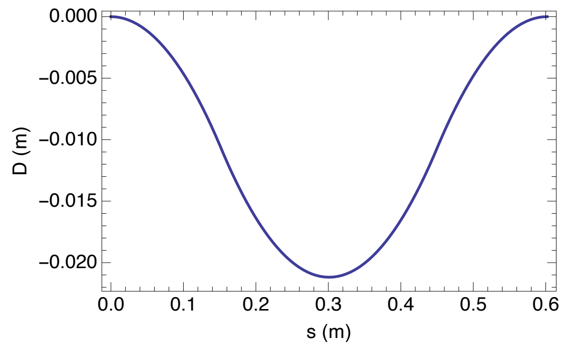

Figure 3: Dispersion function $D(s)$.

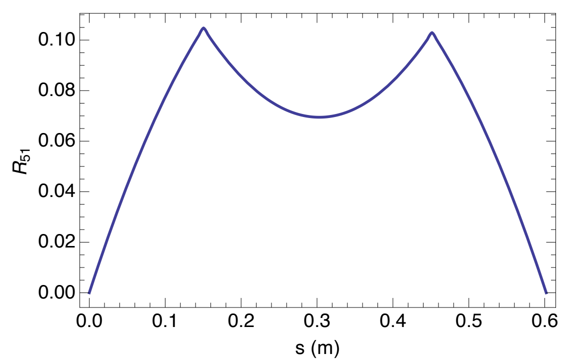

Figure 4: Function $R_{51}(s)$ as defined in Ref. [1].

as a function of frequency ω,

$$Z_{\text{CSR}}(\omega) = 3^{1/6}\left(1+\frac{i}{\sqrt{3}}\right)\Gamma\left(\frac{2}{3}\right)\left(\frac{\omega}{\omega_H}\right)^{1/3}\frac{\omega_H}{c^2}L, \tag{1}$$

where $\omega_H = pc/\gamma eB$ is the cyclotron frequency, B is the magnetic field, p is the particle momentum, Γ is the gamma function, and L is the path length.

Let us assume that there is a sinusoidal linear density modulation in the beam with amplitude n_1 (this is a perturbation of the number of particles per unit length) with the wavelength $\lambda = 2\pi/k$ much smaller than the bunch length. Associated with this density modulation, there a current modulation with the amplitude $I_1 = en_1v \approx en_1c$. The impedance (1) produces a sinusoidal voltage on the beam $V_1 = Z_{\text{CSR}}(\omega)I_1$ with the same period λ and modulates the beam energy (here $\omega = ck$). The relative energy modulation is $\eta = eV_1/\gamma mc^2$. Finally,

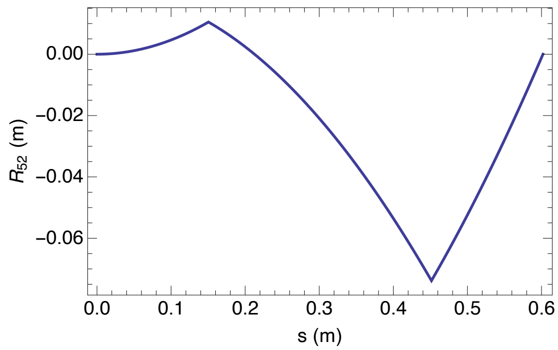

Figure 5: Function $R_{52}(s)$.

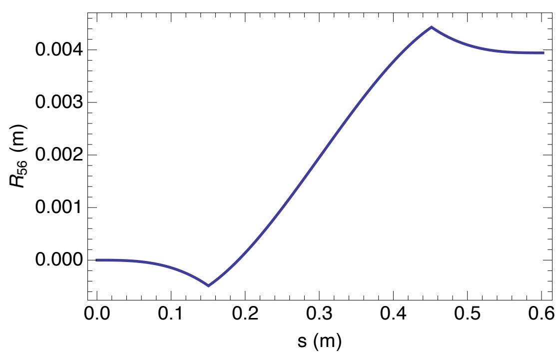

Figure 6: Function $R_{56}(s)$. The final value of $R_{56} = 4$ mm.

longitudinal slippage of the particles of different energy due to R_{56} leads to creation of a new density modulation, $n_2 = kn_0\eta R_{56}$, where $n_0 = I_0/ec$ is the number of particles per unit length in the beam (below I will use the peak value of this quantity in the bunch with I_0—the peak current). Collecting all these formulas, I find for the amplification factor G defined as a ratio n_2/n_1 the following formula

$$G(k) = \frac{n_2}{n_1} = \frac{eI_0kR_{56}}{\gamma mc^2} Z_{\mathrm{CSR}}\left(\frac{k}{c}\right). \tag{2}$$

For the wavelength $\lambda = 1$ μm, with the peak current $I_0 = 30$ A, $R_{56} = 4$ mm, and L estimated as a length of the chicane, $L = 0.6$ m, I find a large amplification factor of $G = 28$. However the gain factor decays with the wavelength as $\lambda^{-4/3}$ and, for example, for $\lambda = 10$ μm $G = 1.3$.

The above estimates completely ignore the beam emittance and the beam energy spread which tend to suppress the instability.The suppression due to the emittance effect comes into the gain G as a following factor:

$$e^{-(k^2\epsilon/2\beta_{x0})(\beta_{x0}^2 R_{51}^2+R_{52}^2)}. \tag{3}$$

For the estimate of the effect I read-off from Figs. 4 and 5 approximate values $R_{51} \sim 0.06$ and $R_{52} \sim -5$ cm. Substituting these values into Eq. (3) and using parameters from Table 1 I find the suppression parameter (3) equal to 10^{-331}, which means that the gain factor for $\lambda = 1$ μm with account of the emittance is basically zero. The suppression becomes weaker for longer wavelength, but as was pointed out above the instability strength diminishes with the increased wavelength.

There is another suppression effect due to the energy spread in the beam,

$$e^{-(k^2\sigma_\eta^2/2)R_{56}^2}. \tag{4}$$

Estimating this factor for $\lambda = 1$ μm with $R_{56} \sim 4$ mm I find that this factor is equal to 0.04, and hence also plays a role in the suppression.

4 Solution of the integral equation

More accurate quantitative results can be obtained by solving the integral equation for the gain factor derived in [1]. I carried out these calculations for the beam parameters and the chicane described in the previous sections. The chicane was split in 100 slices and the integral equation (30) from Ref. [1] was solved numerically. The result of this solution is shown in Figs. 7 and 8 as a gain factor G dependence versus the wavelength of the perturbation λ. These

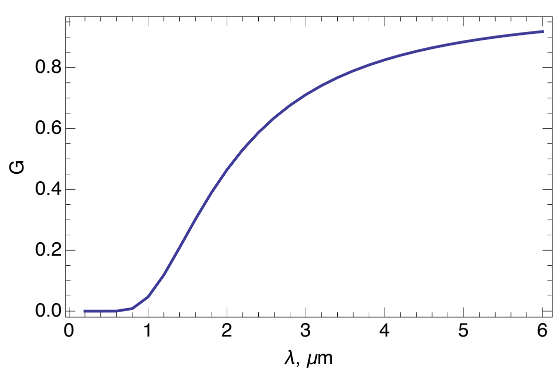

Figure 7: The amplification factor G as a function of the wavelength λ.

results show that the amplification is strongly suppressed at small wavelengths

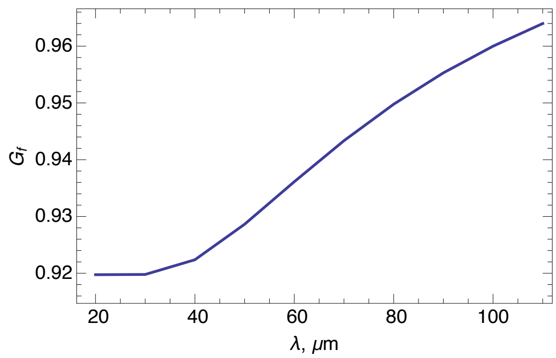

Figure 8: The amplification factor G as a function of the wavelength λ.

due to the emittance and energy spread effects in agreement with the estimates of section 2. At large wavelengths, as shown in Fig. 8, there is no amplification because of the small CSR impedance.

5 Summary

Calculations show that the CSR driven microbunching instability is strongly suppressed by the emittance and energy spread effects, and does not lead to noise amplification in the MBEC cooler for EIC.

6 Acknowledgments

The author thanks I. Pogorelov for independent calculation of the lattice parameters of the chicane. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under Award Number DE-SC0020592.

References

[1] S. Heifets, G. Stupakov, and S. Krinsky. "Coherent synchrotron radiation instability in a bunch compressor". In: *Phys. Rev. ST Accel. Beams* 5 (6 June 2002), p. 064401. DOI: 10.1103/PhysRevSTAB.5.064401. URL: https://link.aps.org/doi/10.1103/PhysRevSTAB.5.064401.