Homework 2. PHY 564

Problem 1. 10 points Motion of non-radiating charged particle in constant uniform magnetic field is a well known spiral:

$$\frac{d\vec{p}}{dt} = \frac{e}{c}\left[\vec{v}\times\vec{H}\right] = \frac{e}{c}H\left[\hat{e}_x v_y - \hat{e}_y v_x\right]; \vec{H} = \hat{e}_z H$$

$$\mathrm{E} = c\sqrt{m^2c^2 + \vec{p}^2} = conts;\ \gamma = const; v = const;$$

$$p_z = const;\ z = v_{oz}t + z_o;$$

$$p_x^2 + p_y^2 = const;\ p_x + ip_y = p_\perp e^{i\varphi(t)} = m\gamma v_\perp e^{i\varphi(t)}$$

simple substitution gives:

$$m\gamma v_\perp \frac{de^{i\varphi(t)}}{dt} = \frac{e}{c}\left[\vec{v}\times\vec{H}\right] = -i\frac{e}{c}Hv_\perp e^{i\varphi(t)}$$

$$r_\perp = x + iy = i\omega m\gamma v_\perp \frac{de^{i\varphi(t)}}{dt}$$

$$\varphi(t) = \omega t + \varphi_o;\ \omega = -\frac{eH}{m\gamma c}$$

and trajectory: $z = v_{oz}t + z_o; x + iy = v_\perp / \omega \cdot e^{i\omega t}$. Do not forget to apply Re or Im to all necessary formulae. Use analytical extension of the Lorentz transformation to complex values by going into a reference frame with x-velocity going approaching infinity $\beta \Rightarrow \infty; \chi \to 0; \chi\beta \to 1$. Show that transverse electric field becomes a magnetic field (with an imaginary value) and visa versa. Follow this path and transfer 4-coordinates to that frame. Use analytical extension of *exp, sin, cos* to complex values and transform the solution above in that for motion in constant magnetic field. Compare it with known solution is your favorite EM book .

Problem 2. 4 points

Find maximum energy of a charged particle (with unit charge *e*!) which can be circulating in Earth's larges possible storage ring: the one going around Earth equator with radius of 6,384 km.

First, find it for storage ring using average bending magnetic field of a super-conducting magnet with strength of 10 T (100 kGs).

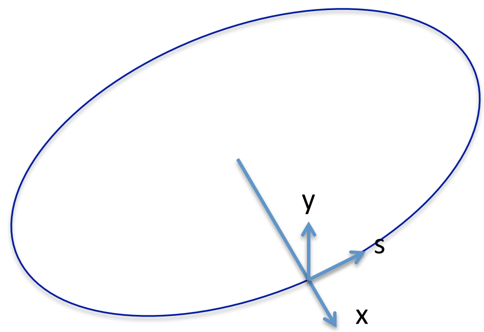
Second, find it for a very strong DC electric dipole fields of 10 MV/m.

Compare these energies with current largest (27 km in circumference) circular collider, LHC, circulating 6.5 TeV (1 TeV = 10^{12} eV).

Hint: assume that particles move with speed of the light. Check the final result for protons having rest mass of 938.27 MeV/c^2

Homework 3

Problem 1. 5 points. Plane symmetry and plane trajectories.



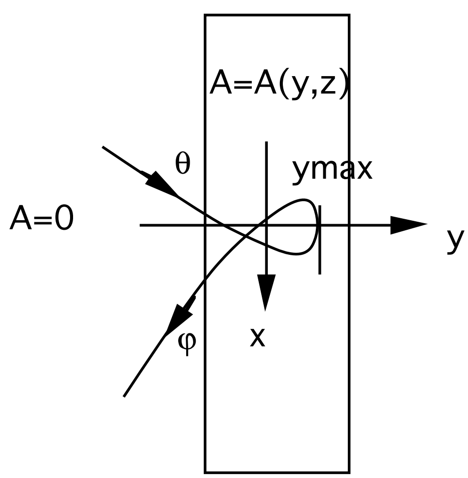
(a) Plane reference orbit (torsion κ=0) requires that total out-of plane force is equal zero. Find ratio between radial (x, horizontal) magnetic field and of-plane (vertical, y) electric field to satisfy this condition. What's happens when both of them are equal zero?

(b) Define full set of condition on EM field providing that all in-plane trajectories (e.g. all trajectories with y=0 and y'=0, but otherwise arbitrary) to stay in-plane, i.e. y=0 is a solutions. Consider that particles have different energies.

Hint: use Lorentz force

Problem 2. 10 points. Magnetic Mirror: An electron propagates through a magnetic field with vector potential . Find an additional invariant of motion caused by independence of vector potential on x. Write explicit expression for px using this invariant. Consider a magnet with mid-plane symmetry (z is perpendicular to the plane of figure) shown below with inside the magnet and outside the magnet. Let's consider an electron entering the magnet in the middle plane with mechanical momentum laying in the x-y plane, making turn in the magnet and coming out.

1. Show that trajectory of electron remains in the plane;
2. Find angle of out-coming trajectory of the electron (reflected angle).
3. Find equation defining depth of penetration of electron inside the magnet using .



Clues: use Lorentz force to find (1), Use canonical momentum to connect mechanical momentum with for (2,3)