Review of Beam Diagnostics

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Beam transport

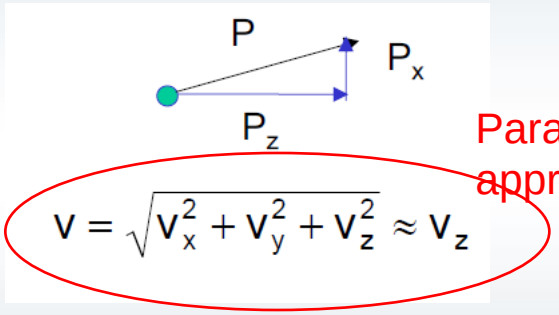
- Each particle is defined by position and momentum:

$$\vec{x} = (x, p_x, y, p_y, z, p_z)$$

- More convenient is to use position and divergence

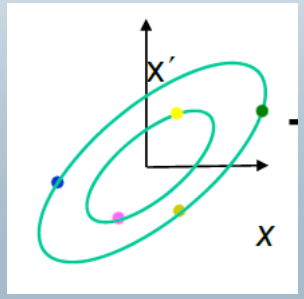
$$\vec{x} = (x, p_x, y, p_y, z, p_z)$$

$$\vec{x} = (x, x' = \frac{p_x}{p_z}, y, y' = \frac{p_y}{p_z}, z, \frac{\Delta p}{p})$$



Paraxial approximation

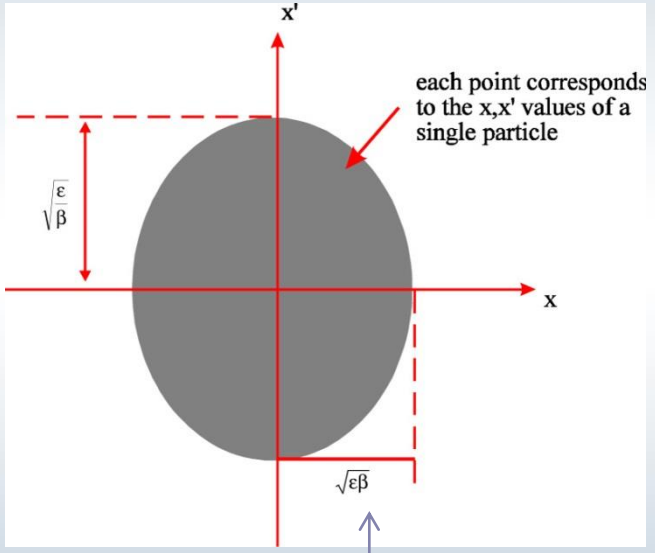
- Assuming no coupling the transverse motion can be represented by two dimensional vectors u=(x,x') and v=(y,y')



Phase-space

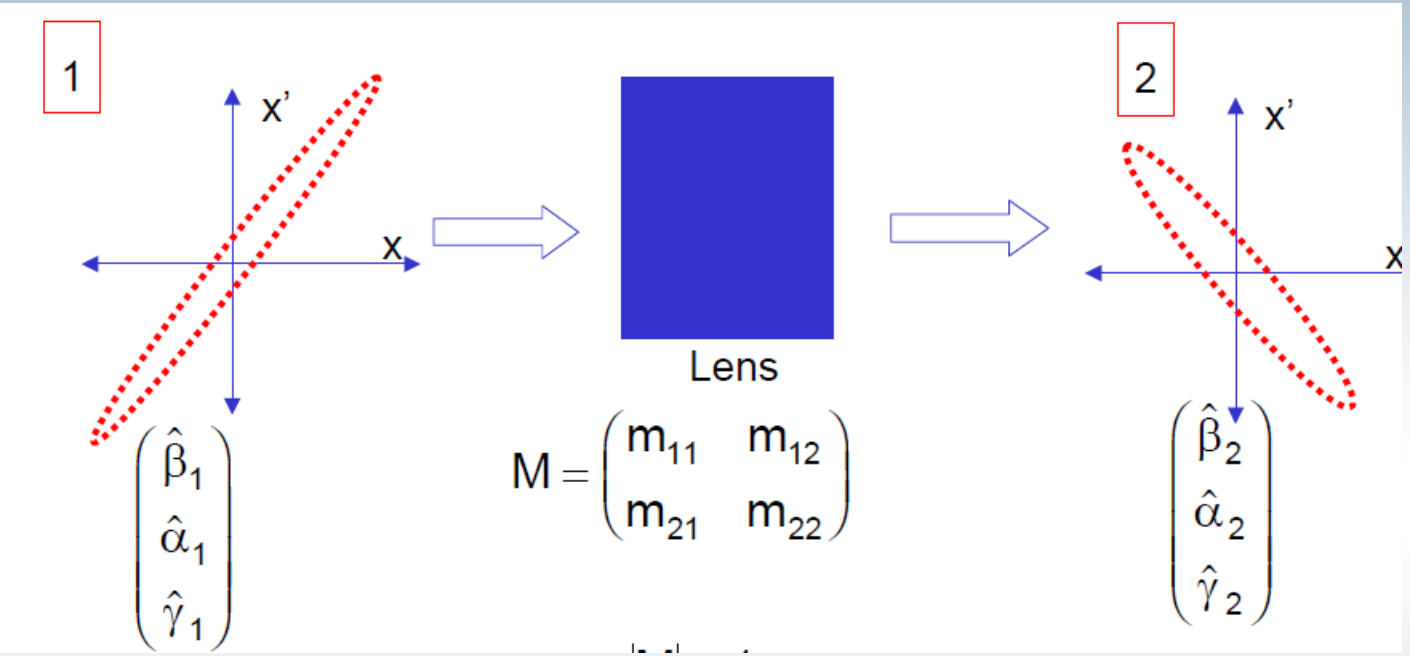
Beam transport

- A good approximation for the beam distribution in phase-space is an ellipse
- Four parameters describe it:
  - β related to beam size
  - α related to tilt
  - γ related to the previous two
  - ε is related to the area of the ellipse and is used to gauge the beam quality
- Equation for ellipse: $\hat{\gamma}x^2 + 2\hat{\alpha}xx' + \hat{\beta}x'^2 = \varepsilon_x$



Beam size

Beam phase-space in a drift



$$\begin{pmatrix}\hat{\beta}_2\\ \hat{\alpha}_2\\ \hat{\gamma}_2\end{pmatrix}=\begin{pmatrix}m_{11}^2 & -2m_{11}m_{12} & m_{12}^2\\ -m_{11}m_{21} & 1+2m_{12}m_{21} & -m_{12}m_{22}\\ m_{21}^2 & -2m_{21}m_{22} & m_{22}^2\end{pmatrix}\begin{pmatrix}\hat{\beta}_1\\ \hat{\alpha}_1\\ \hat{\gamma}_1\end{pmatrix}$$

- Beam size in 2:

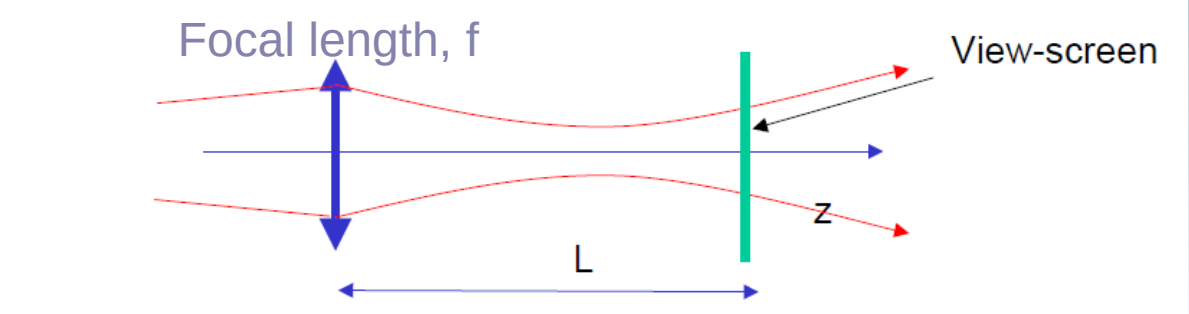
$$x_{rms}=\sqrt{\bar{\beta}2\varepsilon_x}=\sqrt{(m_{11}^2\bar{\beta}1-2m_{11}m_{12}\bar{\alpha}1+m_{12}^2\bar{\gamma}1)\varepsilon_x}$$

Transport matrix

$$\begin{pmatrix} \hat{\beta}_2 \\ \hat{\alpha}_2 \\ \hat{\gamma}_2 \end{pmatrix} = \begin{pmatrix} m_{11}^2 & -2m_{11}m_{12} & m_{12}^2 \\ -m_{11}m_{21} & 1+2m_{12}m_{21} & -m_{12}m_{22} \\ m_{21}^2 & -2m_{21}m_{22} & m_{22}^2 \end{pmatrix} \begin{pmatrix} \hat{\beta}_1 \\ \hat{\alpha}_1 \\ \hat{\gamma}_1 \end{pmatrix}$$

- Can be proven, but not during lecture…

Beam phase-space in a drift



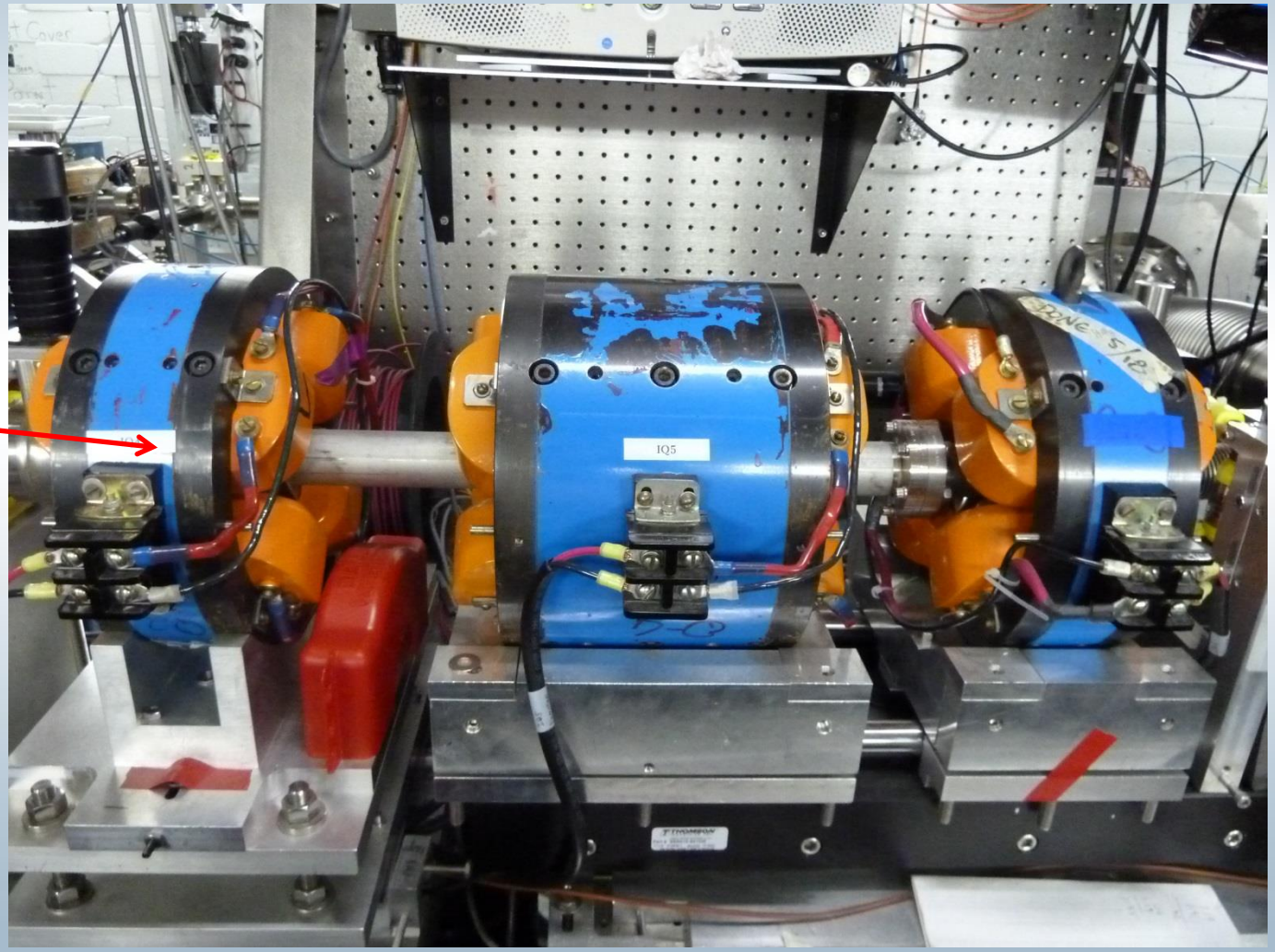
- For a thin lens quadrupole and drift, the transfer matrix M is:

$$M = \begin{pmatrix} m11 & m12 \\ m21 & m22 \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{L}{f} & L \\ -1/f & 1 \end{pmatrix}$$

$$x_{rms} = \sqrt{\bar{\beta}2\varepsilon_x} = \sqrt{(m_{11}^2\bar{\beta}1 - 2m_{11}m_{12}\bar{\alpha}1 + m_{12}^2\bar{\gamma}1)\varepsilon_x}$$

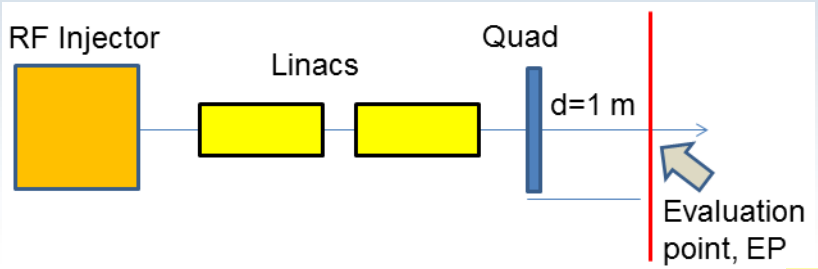
$$x_{rms} = \sqrt{\hat{\beta}_2\tilde{\varepsilon}_x} = \sqrt{\left([1 - L/f]^2\,\hat{\beta}_1 - 2[1 - L/f]L\hat{\alpha}_1 + L^2\hat{\gamma}_1\right)\tilde{\varepsilon}_x}$$

Example: ATF

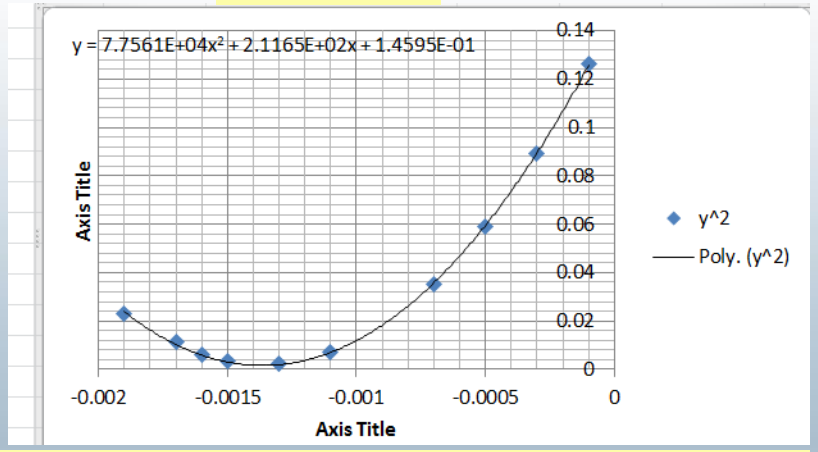


In your simulation…

- Vary quad strength and record images on screen

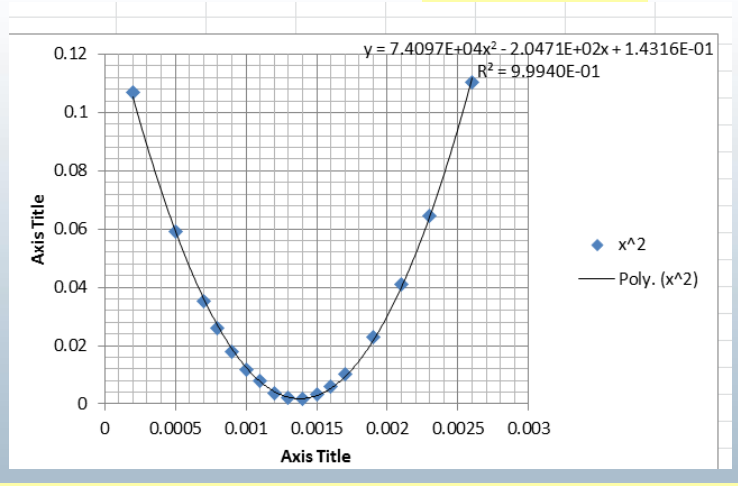


Y Size



Emittance from fit: 0.84 um

X Size

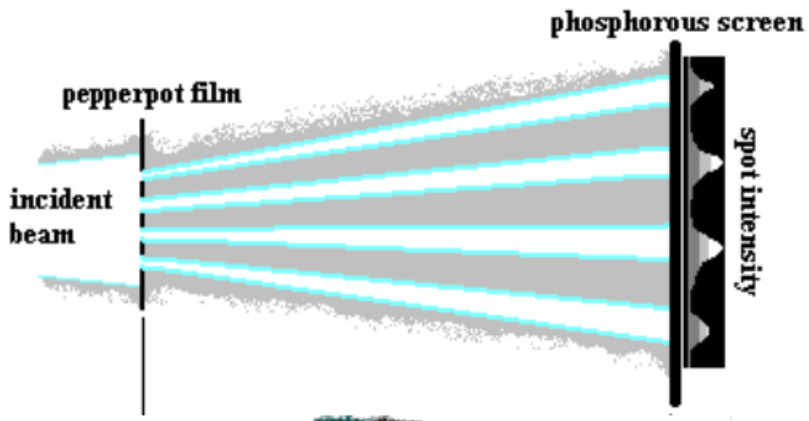


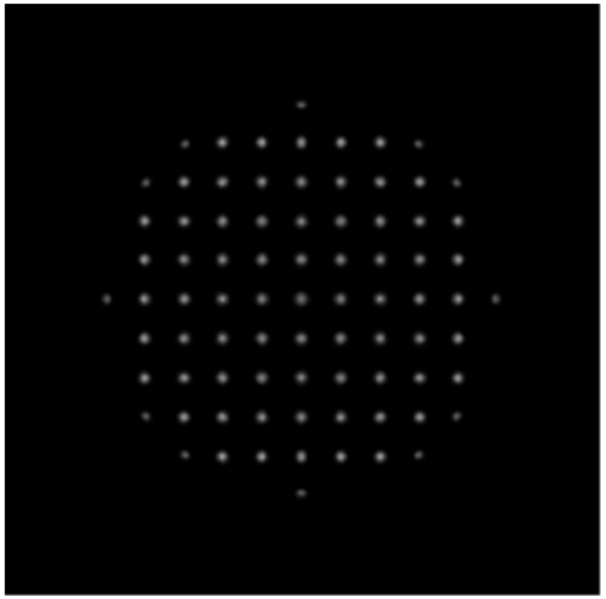
Emittance from fit: 0.87 um

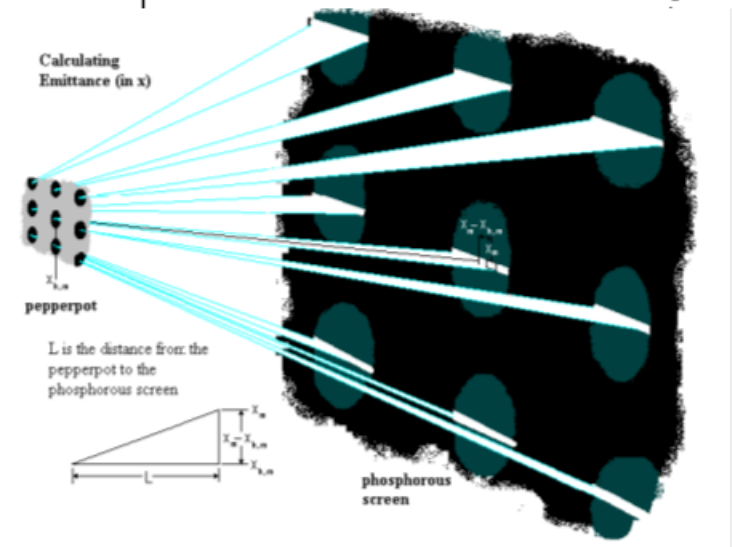
Quad scan technique is limited…

- Make *a priori* assumption that the phase-space distribution is an ellipse
- Ignores space-charge effects
- Alternative options?
- Pepper-pot technique…
- Phase-Space Tomography (My PhD dissertation)…

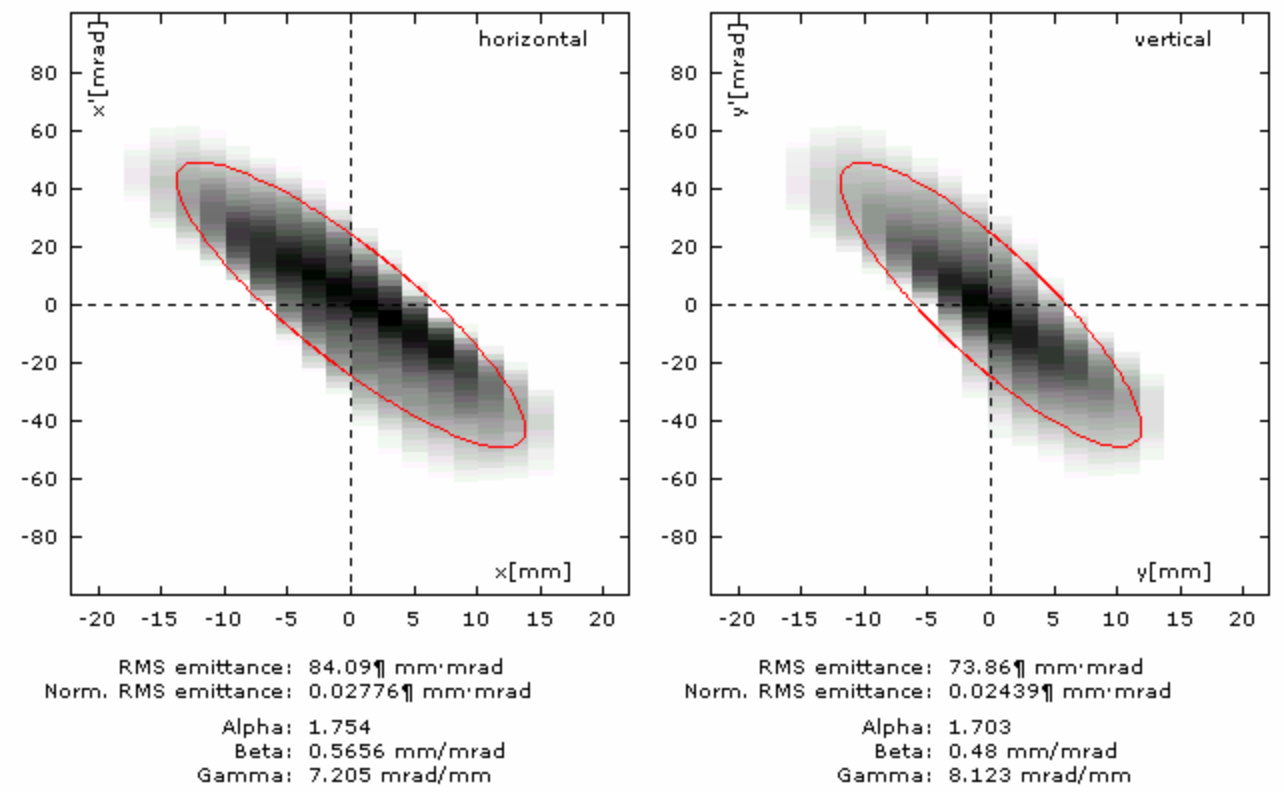
Pepper pot technique (1)







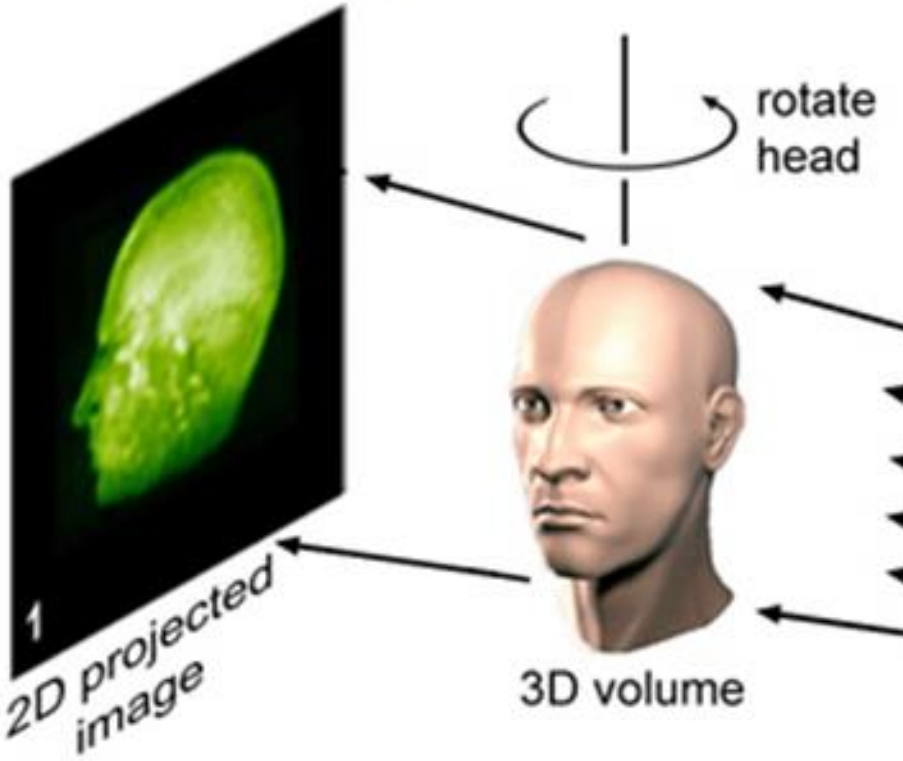
Pepper pot technique (2)



- Resolution sometimes can be poor
- Problematic for very small beams

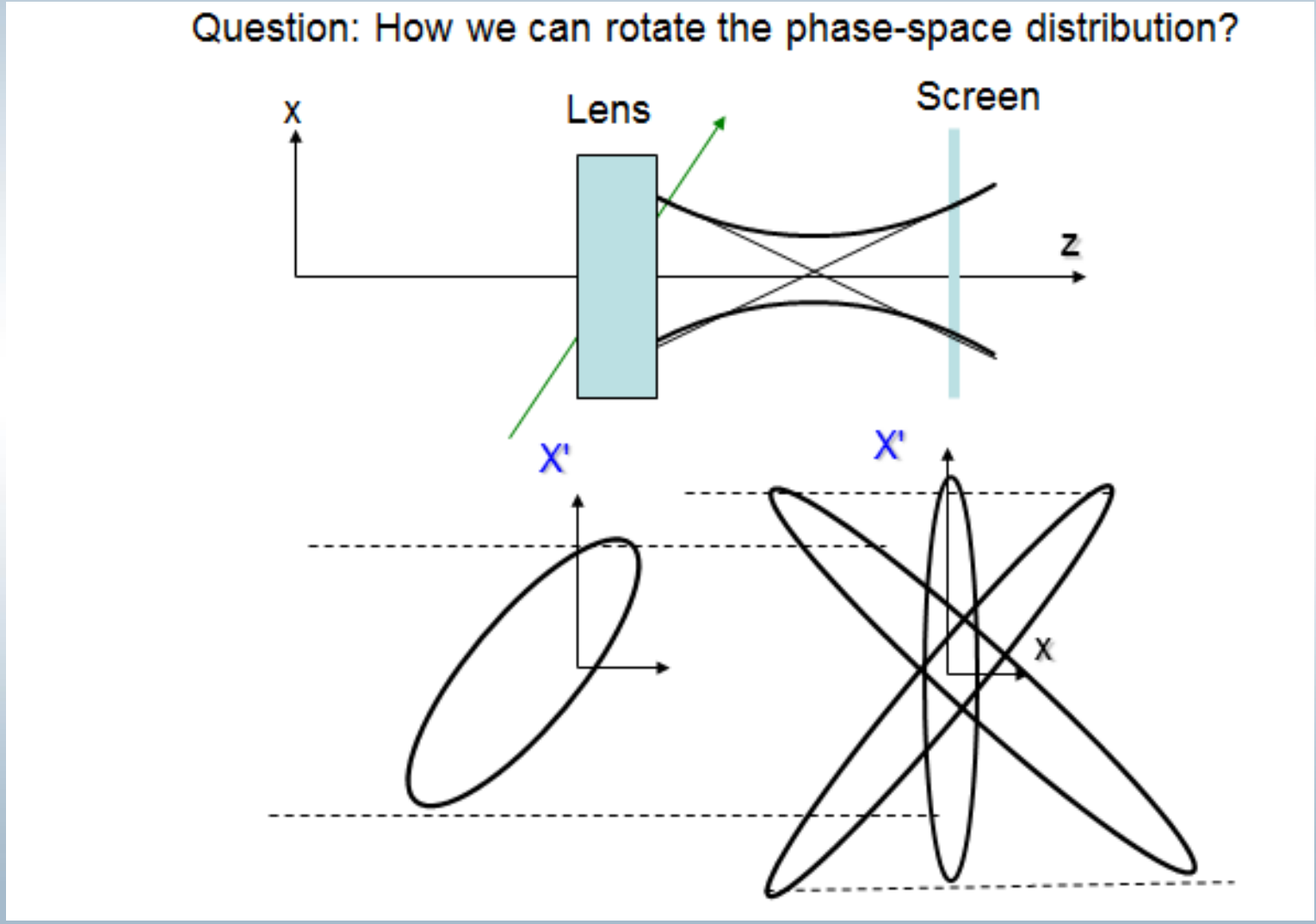
Introduction to tomography

- An object in n-dimensional space can be recovered from a sufficient number of projections onto (n-1)-dimensional space

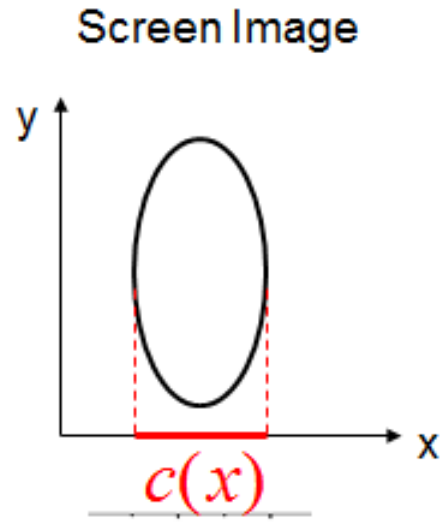


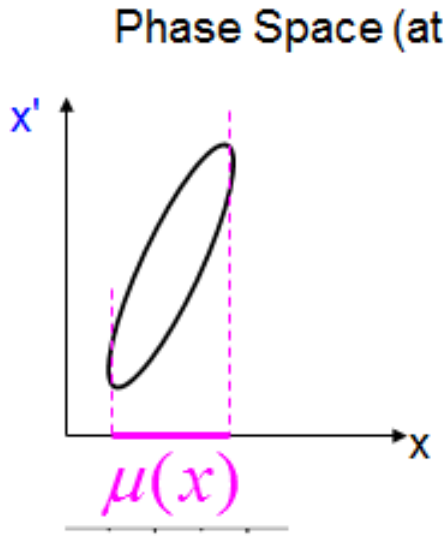
Radon (1917)

Introduction to tomography



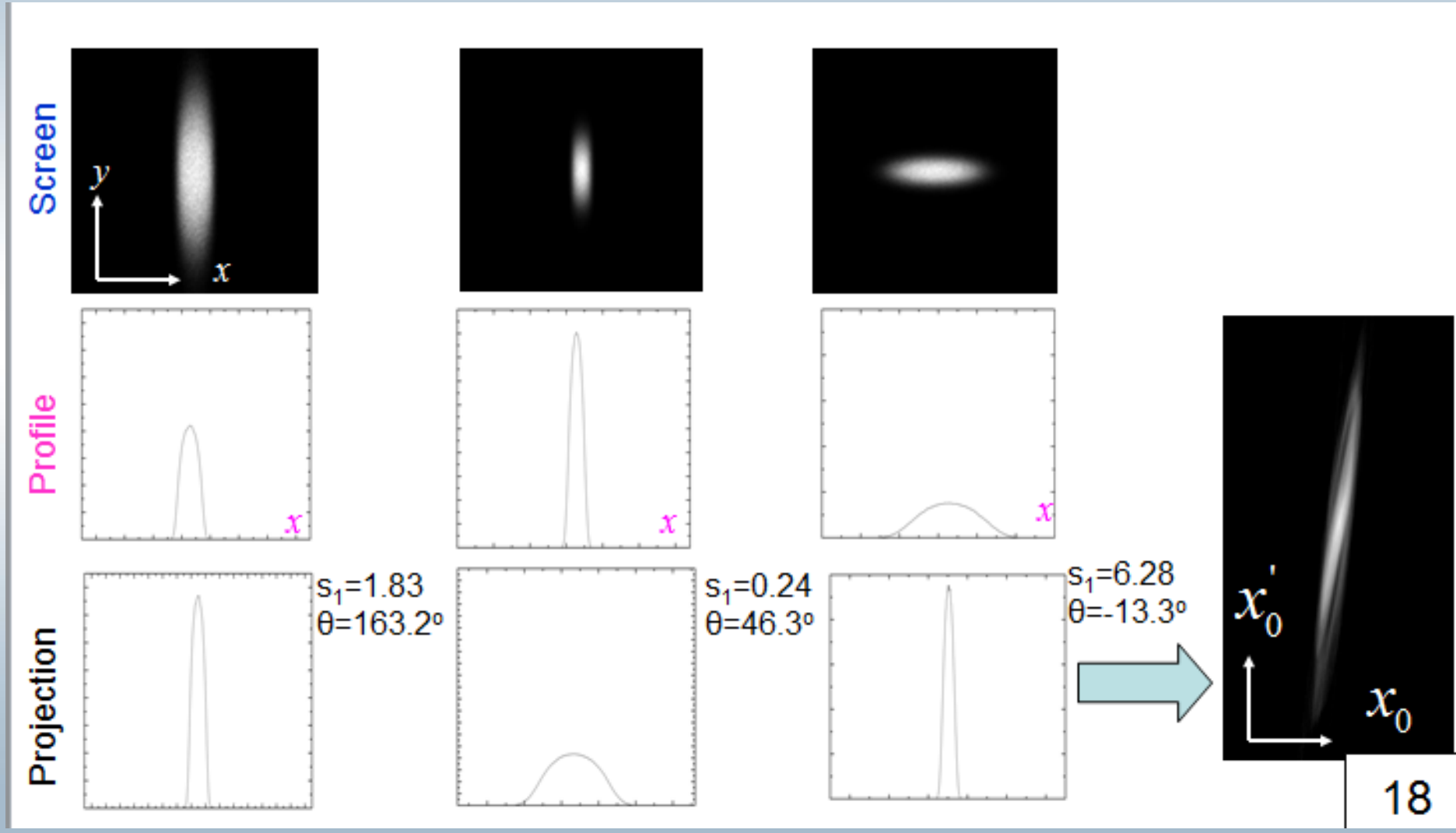
Beam profiles in phase-space





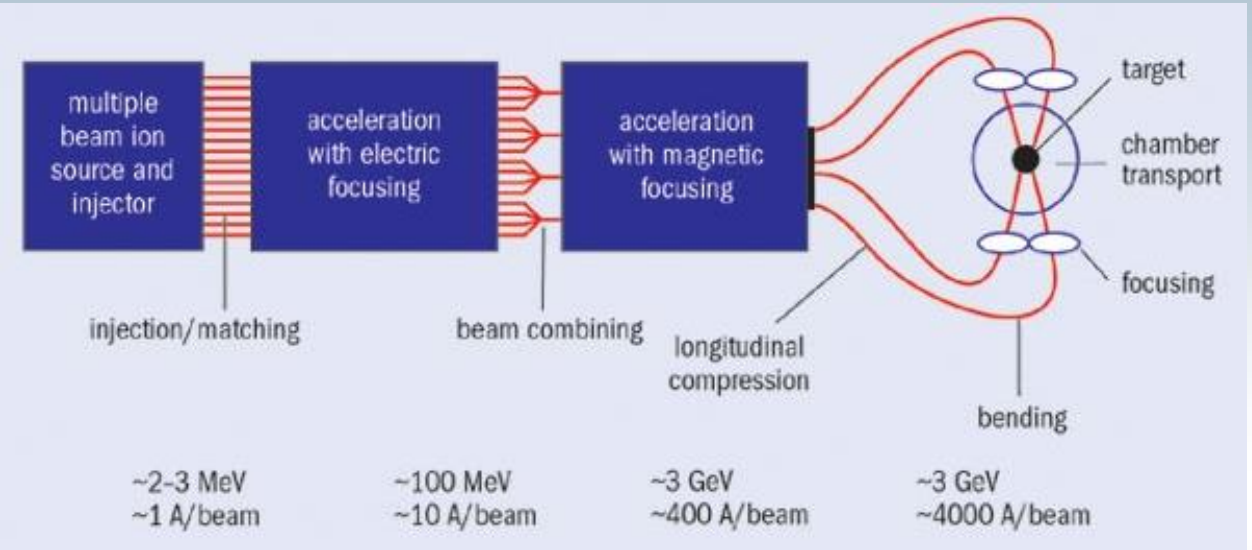
$$c_\theta(x) = \iiint f(x, x', y, y')\,dx'\,dy\,dy' = \mu_\theta(x)$$

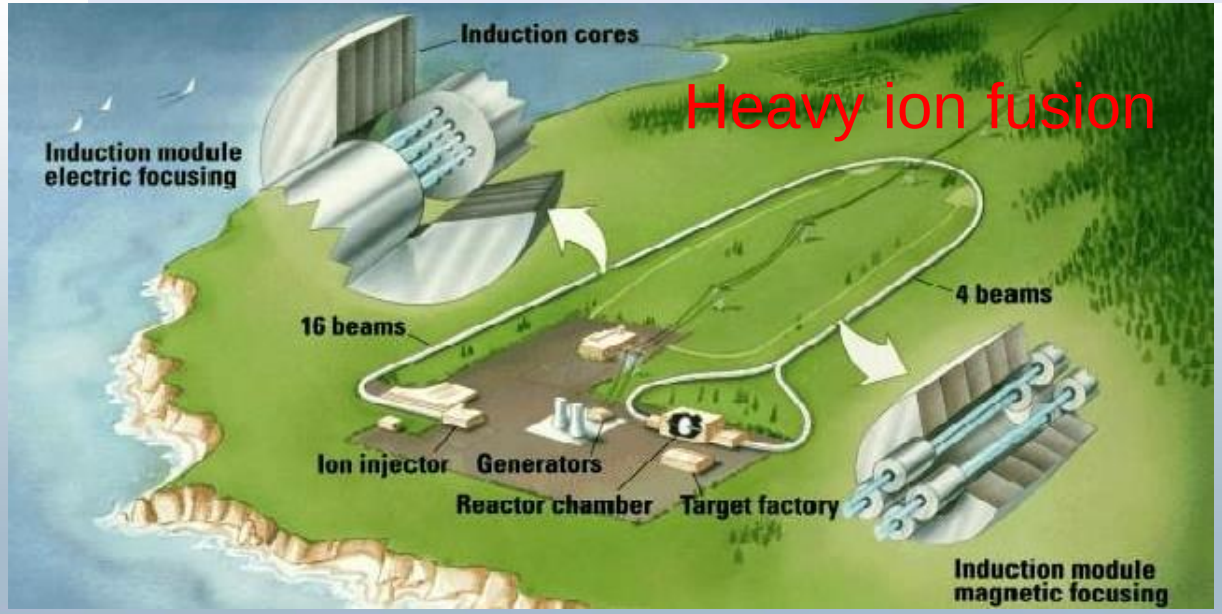
Beam profile example



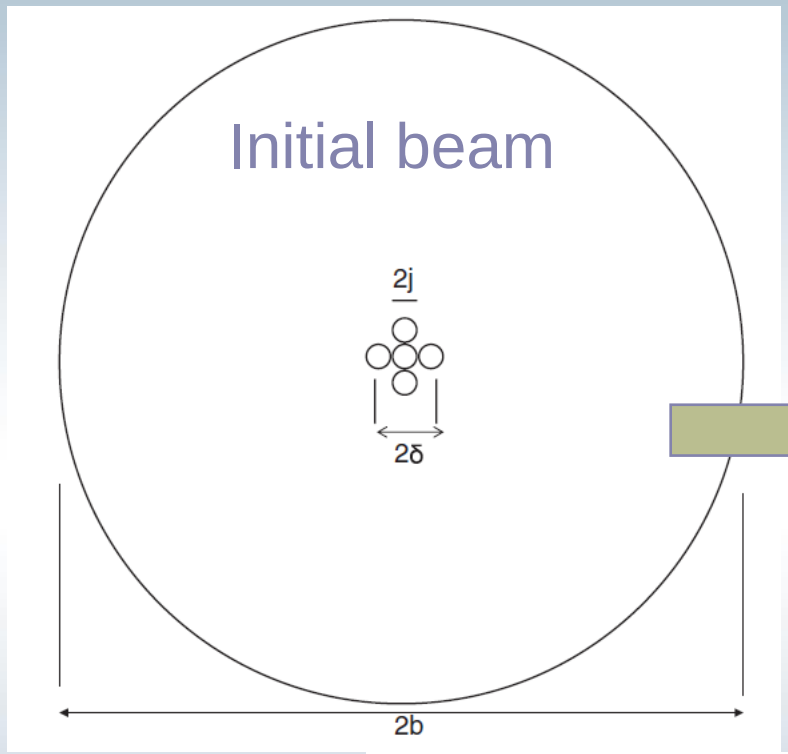
Example

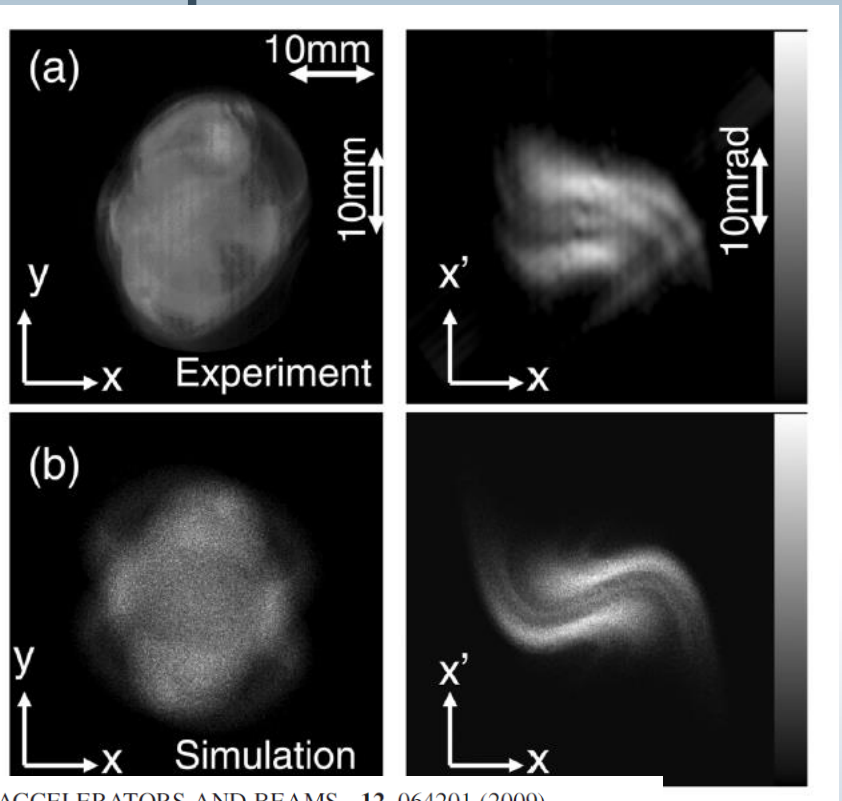
Example of complex beams





Reconstruction of complex beams





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Experimental and numerical study of phase mixing of an intense beam

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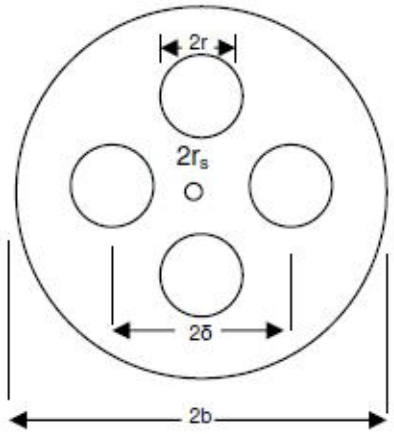
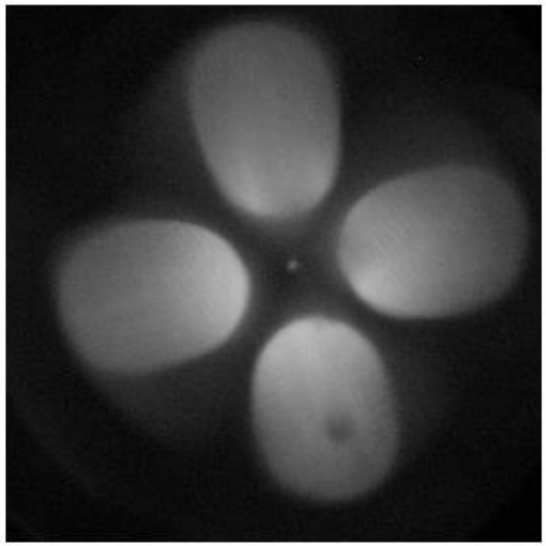
(Received 28 July 2008; published 1 June 2009)

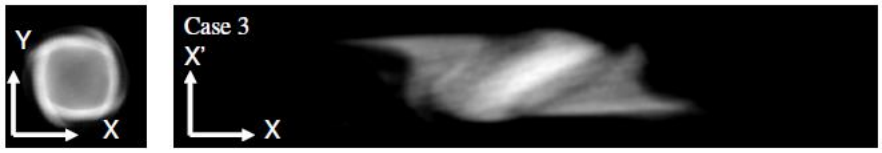
We study, experimentally and numerically, the relaxation of an initially nonuniform intense beam in an alternating-gradient transport line. A nonlinear distribution consisting of five interacting beamlets is created and tracked for longer than seven plasma periods with the help of tomographic phase-space mapping. Emittance growth is initially rapid, but slows down as the nonuniform distribution homogenizes in a few plasma periods. Both growth rates are found to depend on the beam current.

Another example of complex beams



2δ = 8.0 mm
2b = 40 mm
2r = 2.88 mm
$2r_s$= 0.2 mm





(c)