PHY 554. Homework 3.

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1 (10 point): localized orbit correction

The closed orbit can be locally corrected by using steering dipoles. A commonly used algorithm is based on the "three-bumps'' method, where three steering dipoles are used to adjust local-orbit distortion.

Let θ_1 , $\theta_2$and θ_3 be the three bump angles. For the orbit distortion to be localized between first and third dipoles, show that these angles must be related by

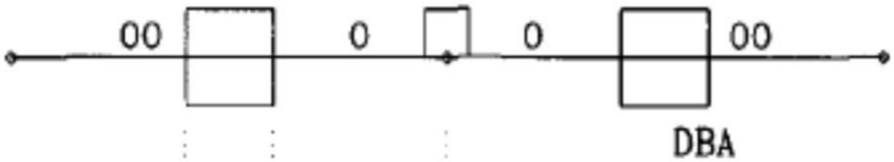
$$\theta_2 = -\theta_1 \sqrt{\frac{\beta_1}{\beta_2}} \frac{\sin \psi_{31}}{\sin \psi_{32}}, \qquad \theta_3 = \theta_1 \sqrt{\frac{\beta_1}{\beta_3}} \frac{\sin \psi_{21}}{\sin \psi_{32}},$$

where β_1, β_2 and β_3 are the beta functions at local bumps and ψ_{ij} is the phase advance between ith and jth steering dipoles.

Show under what condition, the "three-bumps" method can become "two-bumps" method, i.e., only two steering dipoles are used for local orbit distortion.

Hint: when can $\theta_2 = 0$ and what will be the relation between θ_1 and θ_3

2 (10 point): Double Bend Achromat (DBA)



A system with D = D’ = 0 at both start and end is called an achromat system. A DBA section contains two dipoles for orbit bending and one quadrupole in between for optics matching. It has been widely used in designing low emittance storage rings.

To match the dispersion after the DBA, we need to impose a symmetric condition. For half of the DBA section, we have

$$\begin{pmatrix} D_c \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -1/(2f) & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L & L\theta/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Thus the D’ at the center of the DBA section is 0 and D=D’=0 at the end of the DBA. Find the focal length of the quadrupole (f) required for this condition in terms of L(dipole length), L_1(drift between dipole and quadrupole) and θ (dipole angle). And write down the resulting dispersion D_c at the middle of the quadrupole.

3 (10 point) Combined function magnets

The dispersion function in a combined function magnet satisfies

$$D'' + K_x(s)D = \frac{1}{\rho}$$

1) Show that the solution for constant Kx=K>0 is

$$D = a\cos\sqrt{K}s + b\sin\sqrt{K}s + 1/\rho K$$

let D_0 and D_0’ be the initial dispersion function and its derivative at s = 0. Show that solution can be expressed as

$$\begin{pmatrix} D(s) \\ D'(s) \\ 1 \end{pmatrix} = M \begin{pmatrix} D_0 \\ D'_0 \\ 1 \end{pmatrix}$$

, where transfer matrix

$$M = \begin{pmatrix} \cos\sqrt{K}s & \frac{1}{\sqrt{K}}\sin\sqrt{K}s & \frac{1}{\rho K}(1-\cos\sqrt{K}s) \\ -\sqrt{K}\sin\sqrt{K}s & \cos\sqrt{K}s & \frac{1}{\rho\sqrt{K}}\sin\sqrt{K}s \\ 0 & 0 & 1 \end{pmatrix}$$

2) Show that the transfer matrix for constant Kx=K<0 is

$$M = \begin{pmatrix} \cosh\sqrt{|K|}s & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}s & \frac{1}{\rho|K|}(-1+\cosh\sqrt{|K|}s) \\ \sqrt{|K|}\sinh\sqrt{|K|}s & \cosh\sqrt{|K|}s & \frac{1}{\rho\sqrt{|K|}}\sinh\sqrt{|K|}s \\ 0 & 0 & 1 \end{pmatrix}$$

3) Show that the transfer matrix for a pure sector dipole Kx=0 is

$$M = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) \\ -(1/\rho)\sin\theta & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix}$$

4) Using thin lens(small angle) approximation, show that the transfer matrices M for quadrupole and dipole become

$$M_{\text{quad}} = \begin{pmatrix} 1 & 0 & 0 \\ -1/f & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad M_{\text{dipole}} = \begin{pmatrix} 1 & \ell & \ell\theta/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}$$