1. The energy loss per turn is given by

\[ U_0 = \frac{e^2 \beta^3 \gamma^4}{3 \varepsilon_0 \rho} . \] (1)

With \( \rho = 2.22 m \) and \( \gamma = 1 GeV / 0.511 MeV = 1957 \), eq. (1) yields

\[ U_0 = \frac{e^2 \beta^3 \gamma^4}{3 \varepsilon_0 \rho} = 39.6 KeV = 6.33 \times 10^{-15} J . \] (2)

The critical photon energy is given by

\[ E_c = h \omega_c , \] (3)

where \( h \) is the denoted Planck constant and

\[ \omega_c = \frac{3}{2} \gamma^3 \frac{c}{\rho} \approx 1.512 \times 10^{18} rad / s \] (4)

is the critical angular frequency of the synchrotron radiation. Inserting eq. (4) into eq. (3) yields

\[ E_c = 0.996 KeV = 1.594 \times 10^{-16} J . \] (5)

The total synchrotron radiation power for a beam is given by the 1-turn energy loss of all particles in the ring divided by the time it takes for one circulation (i.e. the revolution period)

\[ P_{\text{beam}} = \left( U_0 \cdot N_{\text{ring}} \right) \frac{1}{T_{\text{rev}}} = \left( U_0 \cdot \frac{I_b}{e} T_{\text{rev}} \right) \frac{1}{T_{\text{rev}}} = U_0 \frac{I_b}{e} . \] (6)

where \( N_{\text{ring}} = I_b T_{\text{rev}} / e \) is the total number of electrons in the ring. Inserting eq. (2) and \( I_b = 200 mA \) into eq. (6) give

\[ P_{\text{beam}} \approx 7.91 KW . \] (7)
Since the two intersection points are on the light-cone opened-up by \( x = (x_0, \vec{x}) \), they satisfy the following equation:

\[
(x_0 - X_0) - \sqrt{(X_1 - x_1)^2 + (X_2 - x_2)^2 + (X_3 - x_3)^2} = 0 ,
\]
and

\[
(x_0 - Y_0) - \sqrt{(Y_1 - y_1)^2 + (Y_2 - y_2)^2 + (Y_3 - y_3)^2} = 0 .
\]

Subtracting eq. (9) with eq. (8) yields

\[
Y_0 - X_0 = \sqrt{(X_1 - x_1)^2 + (X_2 - x_2)^2 + (X_3 - x_3)^2} - \sqrt{(Y_1 - y_1)^2 + (Y_2 - y_2)^2 + (Y_3 - y_3)^2} .
\]

The three points \( \vec{x} \), \( \vec{X} \) and \( \vec{Y} \) form a triangle and since the difference in the length of any two sides of a triangle is always smaller than the length of the third side, it follows from eq. (10)

\[
Y_0 - X_0 \leq \sqrt{(X_1 - Y_1)^2 + (X_2 - Y_2)^2 + (X_3 - Y_3)^2} .
\]

The time it takes for the particle to get from \( \vec{X} \) to \( \vec{Y} \) is given by

\[
\Delta t = \frac{Y_0 - X_0}{c} ,
\]
and hence the average velocity of the particle during its travelling from \( \vec{X} \) to \( \vec{Y} \) is
\[ \langle v_{\text{particle}} \rangle = \frac{\sqrt{(X_1 - Y_1)^2 + (X_2 - Y_2)^2 + (X_3 - Y_3)^2}}{\Delta t} = c \frac{\sqrt{(X_1 - Y_1)^2 + (X_2 - Y_2)^2 + (X_3 - Y_3)^2}}{Y_0 - X_0}. \tag{13} \]

According to eq. (11), the following relation holds

\[ \frac{\sqrt{(X_1 - Y_1)^2 + (X_2 - Y_2)^2 + (X_3 - Y_3)^2}}{Y_0 - X_0} \geq 1, \tag{14} \]

and inserting eq. (14) into eq. (13) yields

\[ \langle v_{\text{particle}} \rangle = c \frac{\sqrt{(X_1 - Y_1)^2 + (X_2 - Y_2)^2 + (X_3 - Y_3)^2}}{Y_0 - X_0} \geq c. \tag{15} \]

Eq. (15) violates special relativity and hence the trajectory of a particle cannot intersect a light-cone twice.
3. The angular distribution of radiation power is given by

\[
\frac{dP(t_r)}{d\Omega} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{4\pi c} \frac{\beta^2}{(1-\beta \cos \theta)^2} \left[1 - \frac{\sin^2 \theta \cos^2 \phi}{\gamma^2 (1 - \beta \cos \theta)^2}\right].
\]  \hspace{1cm} (1)

For \( \frac{1}{\gamma^4} \ll \theta \ll 1 \) and \( \gamma \gg 1 \), we can use the following approximation

\[
1 - \beta \cos \theta \approx 1 - \beta \left(1 - \frac{1}{2} \theta^2 \right) \\
= 1 - \beta + \frac{1}{2} \beta \theta^2 \\
= \frac{1}{\gamma^2 (1 + \beta)} + \frac{1}{2} \theta^2 \\
= \frac{1}{\gamma^2} \left[1 + \frac{1}{2 - (1 - \beta)}\right] + \frac{1}{2} \theta^2,
\]  \hspace{1cm} (2)

and eq. (1) becomes

\[
\frac{dP(t_r)}{d\Omega} \approx \frac{1}{4\pi\varepsilon_0} \frac{2e^2}{\pi c} \frac{\gamma^6 \beta^2}{(1 + \gamma^2 \theta^2)^3} \left[1 - \frac{4\gamma^2 \theta^2 \cos^2 \phi}{(1 + \gamma^2 \theta^2)^2}\right].
\]  \hspace{1cm} (3)

Since the factor inside the square bracket is between 0 and 1, the angular width of eq. (3) is determined by the factor \( \left(1 + \gamma^2 \theta^2\right)^{-3} \), i.e. the radiation power drops substantially when \( \theta \geq \frac{1}{\gamma} \).