Homework 8.

Problem 1. 10 points. In number of occasions, it is useful to stretch bunch using two frequency RF system. Consider a storage ring negative η_τ and the RF system operating at two frequencies, the fundamental and the 3rd harmonics:

$$\frac{dE\cdot}{ds} = \frac{e}{C}\left(V_1 \cdot \sin\left(h_{rf} \cdot k_o \cdot \tau\right) + V_3 \cdot \sin\left(3h_{rf} \cdot k_o \cdot \tau + \varphi_3\right)\right)$$

Find at what ratio between the voltages and phase of third harmonic the frequency of small oscillations turns into zero. For this case, find stationary points on the phase diagram, draw characteristic phase-space trajectories (approximately is fine) and show the direction of the motion by arrows.

Problem 2. 4x5 points.

For a single frequency RF system with Hamiltonian with α indicating an energy loss/gain,

$$\left\langle \mathcal{H}_s \right\rangle = \eta_\tau \frac{\pi_\tau^2}{2} + \frac{1}{C}\frac{eV_{RF}}{p_o c}\frac{\cos\left(k_o h_{rf} \tau\right)}{k_o h_{rf}} + \alpha \cdot \tau; \quad \eta_\tau < 0.$$

1. Define the stationary points (RF phases) in the phase space and indicate level of α when stationary points are no longer exists.
2. Draw phase space trajectories for $\alpha = \frac{1}{2} \cdot \frac{1}{C}\frac{eV_{RF}}{p_o c}$. Show the direction of the motion by arrows.
3. Define the depth of the "RF bucket", e.g. the difference between the maximum and minimum π_τ staying within a single RF separatrix (e.g. being localized). Express it through the RF voltage, the slip factor and the value of stationary phase. Note – consider the central separatrix around $\tau = 0$.
4. Find period of the oscillation as function of $\left\langle \mathcal{H}_s \right\rangle$ inside the central separatrix (around $\tau = 0$).