

# Periodic Beam Optics

## EXERCISE: A basic brick of optical systems : “FODO CELL”

Consider the following optical sequence, along a straight axis:

a drift of length  $l/2$ ; a focusing thin-lens with focal distance  $f$ ; a drift of length  $l$ ; a defocusing thin-lens with the same focal distance  $f$ ; a drift of length  $l/2$ .

Take  $f$  a positive quantity.

### 1. Theory

- 1.1/ Express the  $2 \times 2$  transfer matrix  $T$  of this FODO cell, in terms of  $l$  and  $f$ .
- 1.2/ Verify that the determinant of  $T$  is 1.
- 1.3/ At what condition on  $f$  and  $l$  is this optical system periodically stable?
- 1.4 / Express the betatron phase advance over this cell, in terms of  $f$  and  $l$ .

### 2. Computer work: Simulate the above FODO cell in zgoubi.

Take  $l = 2\text{m}$ , frozen. The strength of the lens,  $1/f$ , can be varied, we will start with  $f=l$ .

- 2.1/ Compare  $T$  from theory and from MATRIX or TWISS.
- 2.2/ What are the cell phase advance values, horizontal and vertical? Compare with theory.
- 2.3/ Check the stability limit  $f > l/2$ .
- 2.4/ Find the strengths ( $K = G/B\rho$ ) of the F and D lenses to get phase advances of  $0.27 \times 2\pi$  and  $0.1 \times 2\pi$ , respectively horizontal and vertical. FIT can be used for that. Theory can be used, as well.
- 2.5/ Launch 40 particles on a horizontal periodic invariant (use OBJET[KOBJ=8], with the previously found periodic  $\alpha_{x,y}$ ,  $\beta_{x,y}$ ). Transport that bunch through the cell:
  - plot initial and final phase spaces, on a common graph (use FAISTORE);
  - graphically, verify that the ellipse parameters coincide with the periodic  $\alpha_{x,y}$ ,  $\beta_{x,y}$ ;
  - plot the trajectories  $Y(s)$  through the cell (use DRIFT[split] for fine resolution; use IL=2 to log particle data);
  - from this graph data, get an estimate of  $Y_{\min}$  and  $Y_{\max}$ . Give their exact locations  $s(Y_{\min})$  and  $s(Y_{\max})$ .