Homework 7, Due Oct 31, 2016

1. (3 points) Calculate the relative relation between $\Delta V/V$, $\Delta P/P$ and $\Delta E/E$. Note V is the velocity of the particle, P is the amplitude of the momentum and E is the energy of the particle.

2. (4 points) In class, we transform the longitudinal map

$$\begin{aligned} \delta_{n+1} - \delta_n &= \frac{eV}{\beta^2 E_0}(\sin\phi_n - \sin\phi_s) \\ \phi_{n+1} - \phi_n &= 2\pi h\eta\delta_{n+1} \end{aligned}$$

   to longitudinal effective Hamiltonian. Actually we can also establish the one turn matrix for longitudinal motion if assume $\phi_n = \phi_s + \Delta_n$, where $|\Delta_n| \ll 1$. Find this matrix for $(\delta_{n+1}, \Delta_{n+1})$ from (δ_n, Δ_n) . Find the tune for this map, by assuming the tune is very close to zero, which is true in ring accelerator.

3. (4 points) For the example in class, find the synchrotron tune for both 100GeV case and 15GeV proton ring. The relative parameter is the cavity has 5MV voltage, 360 harmonic. Compaction factor $\alpha_c = 0.002$. The RF phase is zero or π. How does the number change if the same ring is for 3GeV electron beam.

4. (9 points) Let us calculate the synchrotron radiation related problem in NSLS II. NSLS II adopts DBA lattice (separate function magnets). Here are the parameters:

Table 1: NSLS II parameters

Parameters	Values
Energy [GeV]	3.0
Circumference [m]	780
Number of dipoles	60
Dipole field [T]	0.4
Beam current [A]	0.5
RF frequency [MHz]	499.68
Harmonic number	1320

   From the design parameters, we can calculated the following parameters:

- In DBA lattice, dispersion D and dispersion slope D' are zero at one end of dipoles and non-zero at the other end of the dipole. Find dispersion function inside the dipole magnet.
- What is the compaction factor α_c of the ring?
- The energy loss due to the dipole field.

- If the accelerating phase of the RF cavity is $\pi/6$, at least how much voltage is required? How much is the power needed?
- Actually the RF voltage is about 3MV. Find the longitudinal tune of NSLS II
- What is the critical radiation frequency of the dipole radiation.
- Find the partition number $\bar{D}$ due to synchrotron radiation in dipole.
- Find the longitudinal damping rate α_E and compare with the period of longitudinal oscillation.
- Find the equilibrium energy spread of NSLS II.