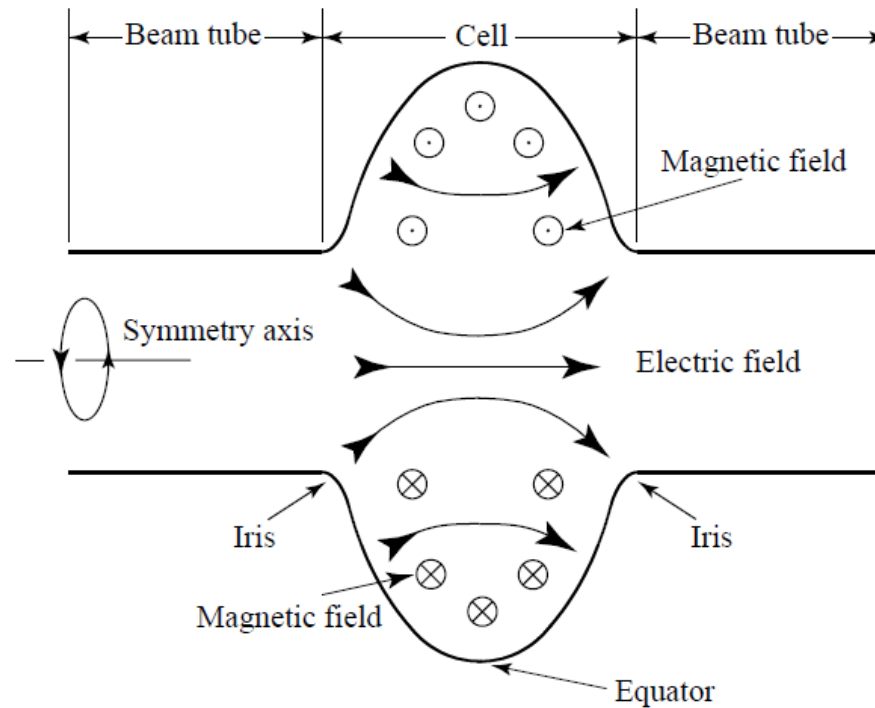


PHY 554

Fundamentals of Accelerator Physics

Lecture 10: Introduction to RF accelerators

Jun Ma



Acknowledgement

Next three lectures use some materials from courses on RF and Superconducting RF (SRF) accelerators taught by Prof. S. Belomestnykh at SBU/BNL and USPAS, which can be found on the following websites:

http://case.physics.stonybrook.edu/index.php/Courses:_P554_Fundamentals_of_Accelerator_Physics,_Spring_2014

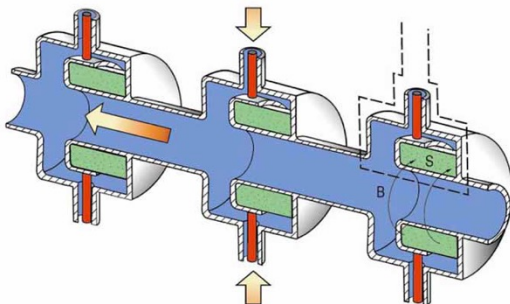
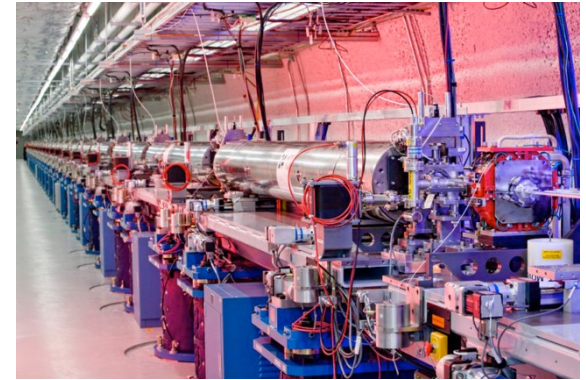
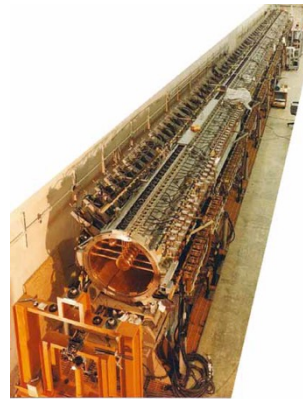
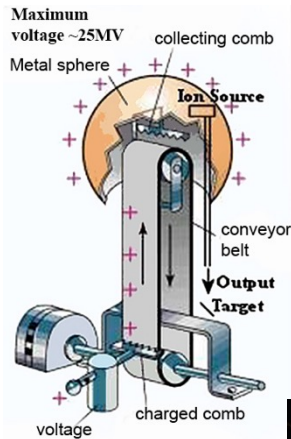
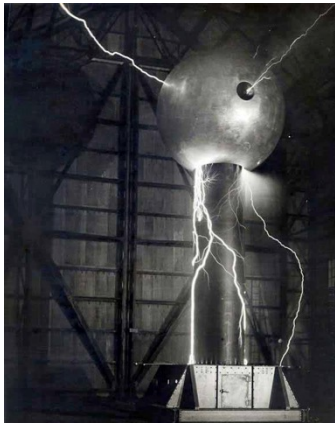
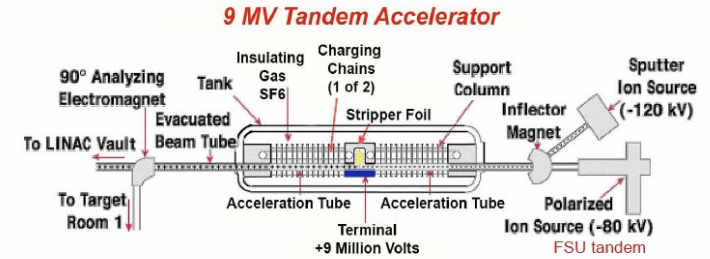
<http://case.physics.stonybrook.edu/index.php/CASE:Courses>

<https://sites.google.com/site/srfsbu11/>

<http://uspas.fnal.gov/materials/materials-table.shtml>

I added additional material as needed

Linear accelerators: from electrostatic to RF

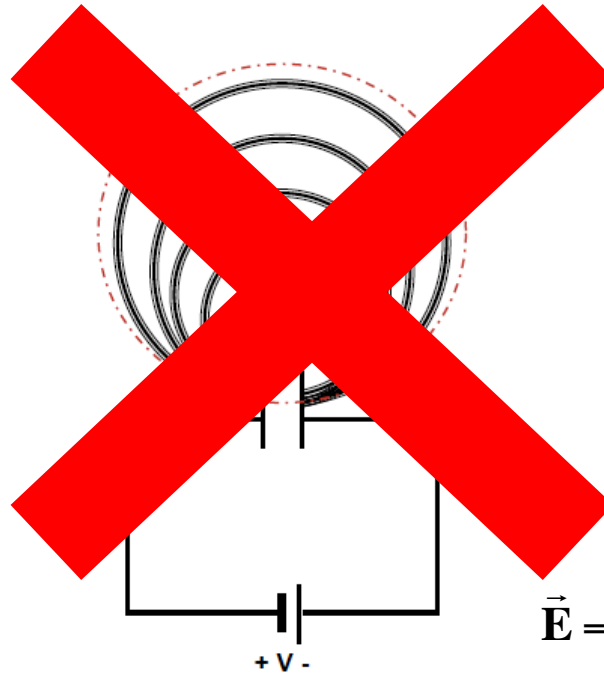


Can one gain the energy again and again by passing through a DC accelerating gap?

Electrostatic: what is the limit ?

Maxwell equations and energy conservation law!

$$\Delta E = e \oint \vec{E} \cdot d\vec{l} = -\frac{e}{c} \frac{\partial}{\partial t} \left(\int \vec{H} \cdot d\vec{s} \right)$$



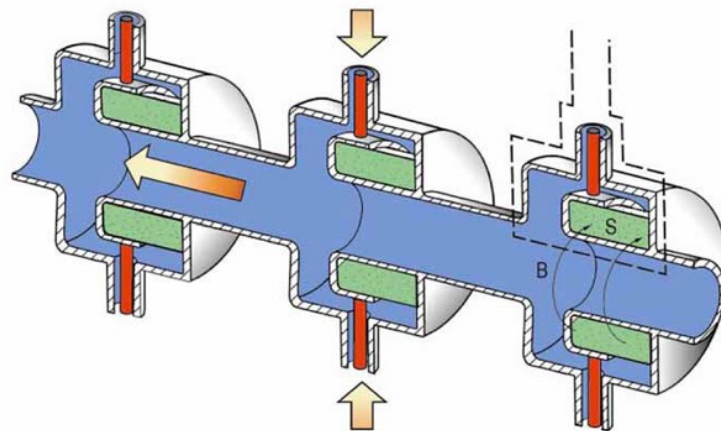
DC

$$\Delta E = e \oint \vec{E} \cdot d\vec{l} = 0$$

$$\vec{E} = -\vec{\nabla} \varphi \rightarrow E(\vec{r}) = E(0) - e\varphi(\vec{r})$$

Can not cheat the Maxwell equations

Induction linacs: linear betatrons

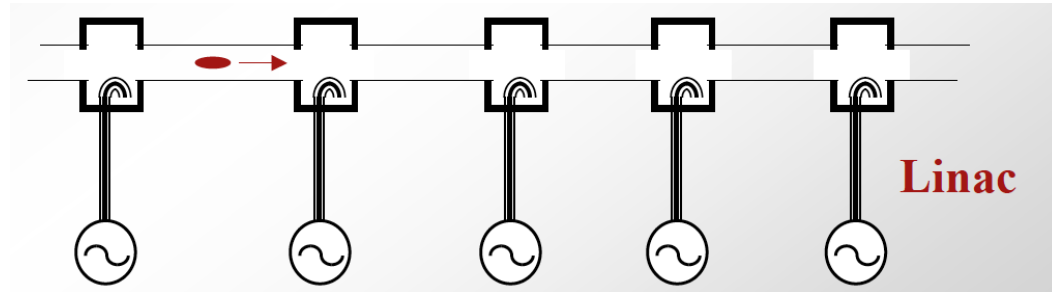


$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \left(\int \vec{H} \cdot d\vec{s} \right)$$

- Useful for high power and high current beams
- Have limited accelerating field
- By nature are pulsed, with relatedly low rep-rate (kHz)

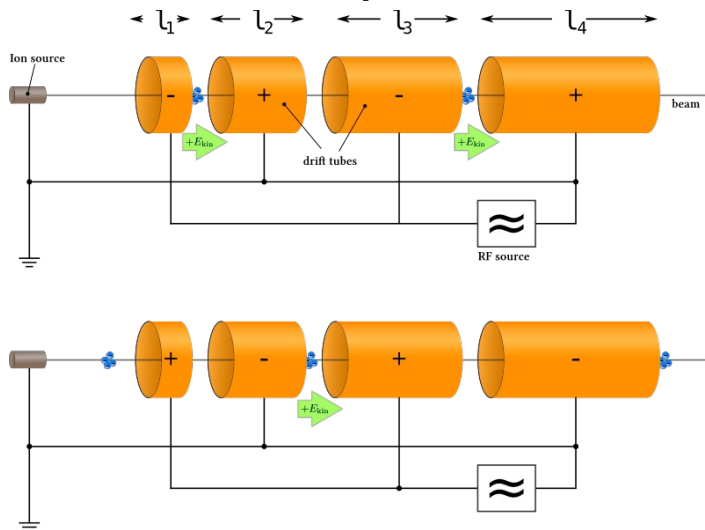
How RF accelerator works

- It has oscillating (typically sinusoidal in time) longitudinal (along the particle's trajectory) electric field
- It also has longitudinal structure (cells) which alternates the direction of the field
- When particle propagates through the RF accelerator, the field direction in each cell is synchronized with the particle arrival and the effect from all cells is added coherently

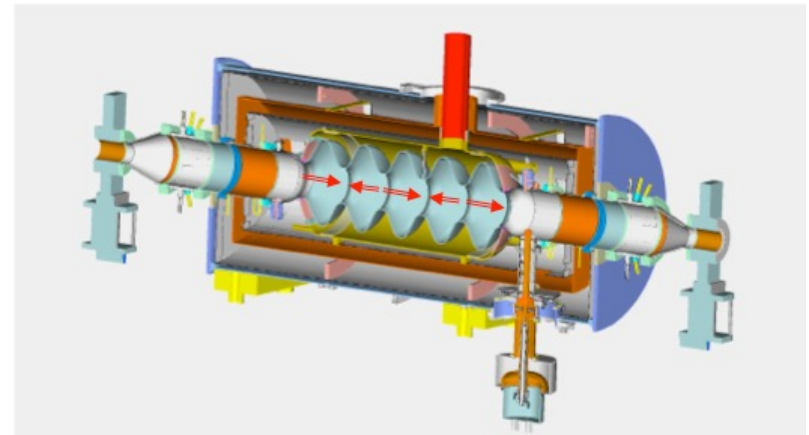


$$\frac{dE}{dt} = e\vec{E} \cdot \vec{v} \quad \rightarrow \quad \text{sign}(\vec{E} \cdot \vec{v}) = \text{const}$$

Wideröe's linac: $\beta = v/c$ is changing

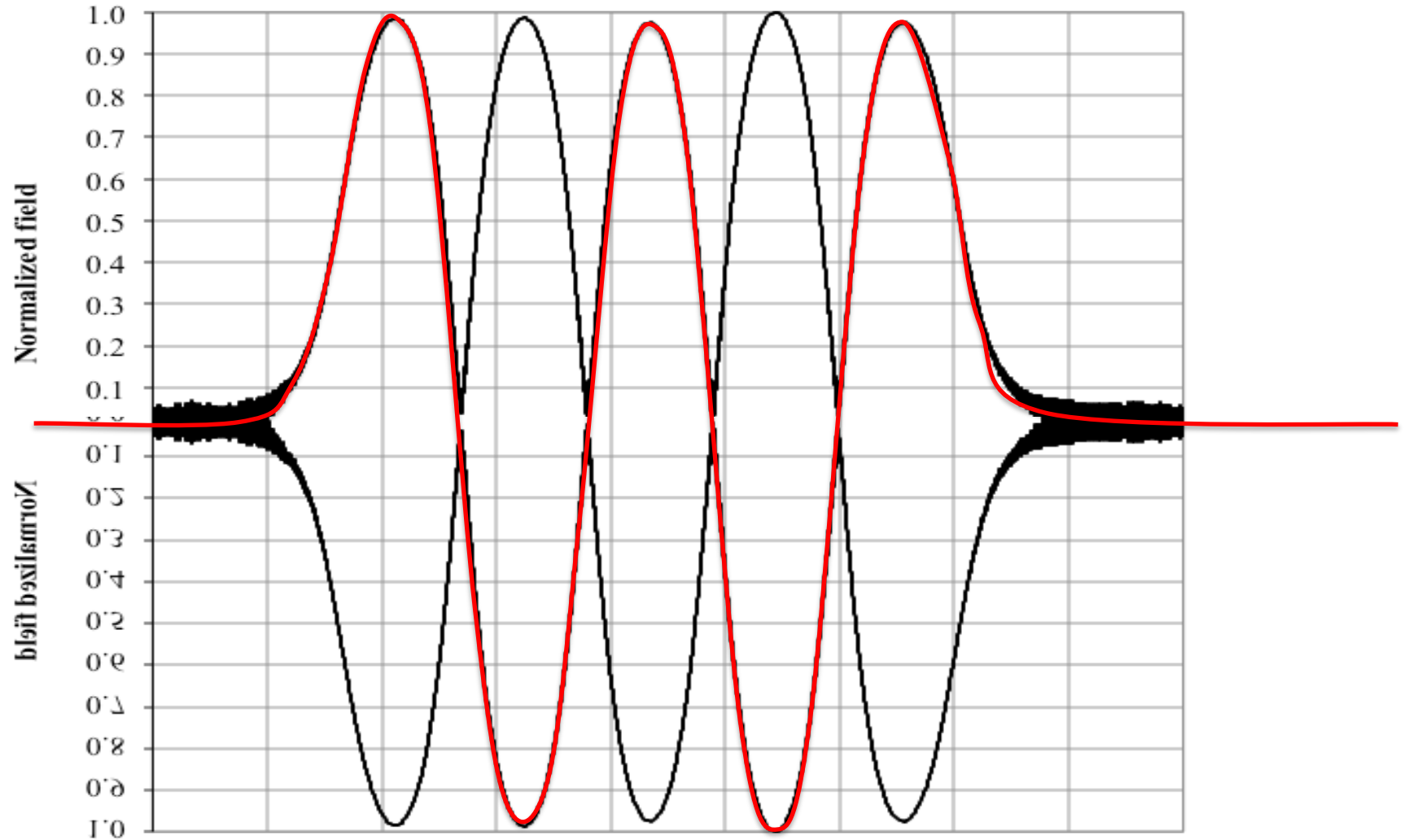


Electron linac

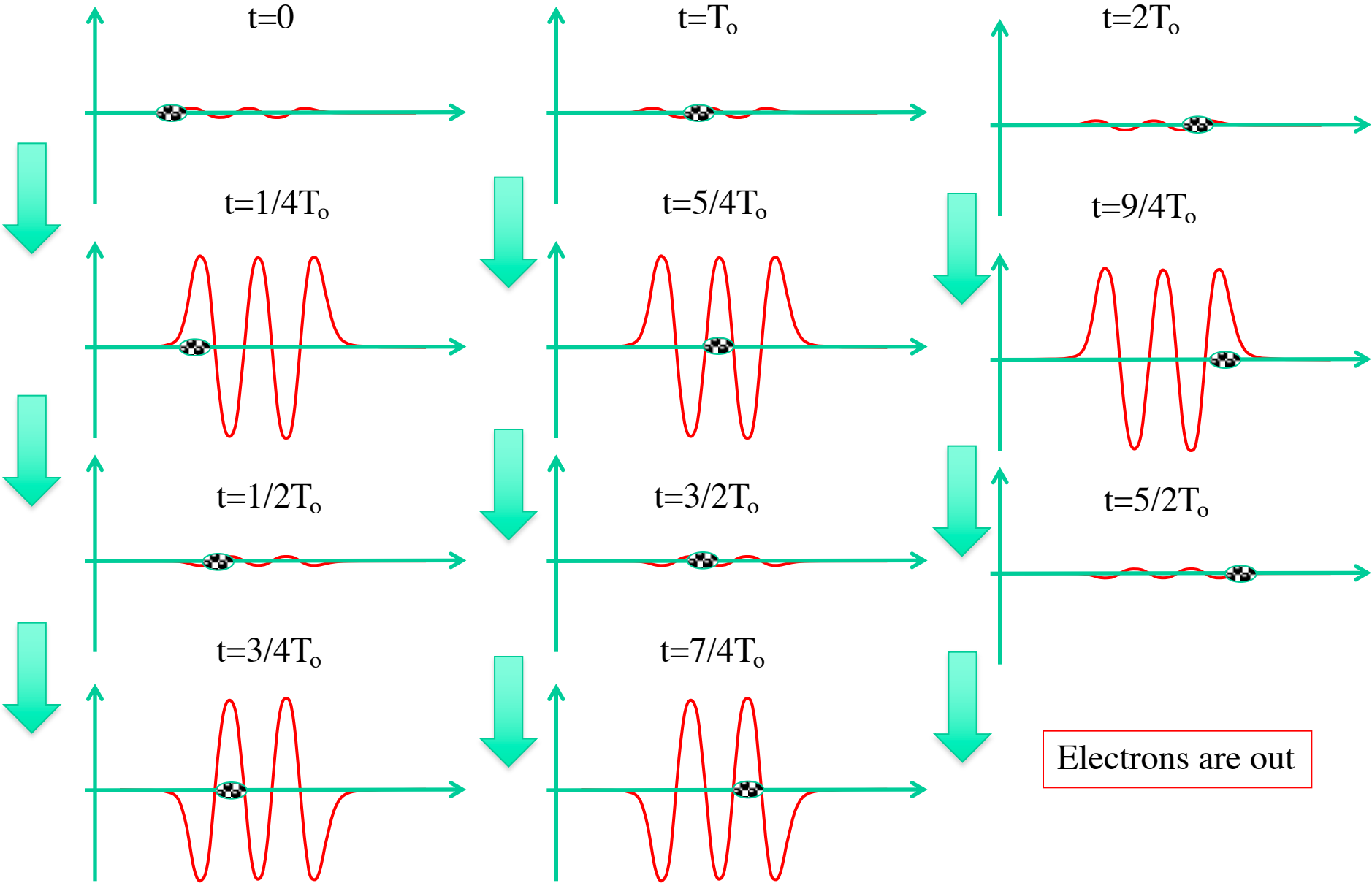


$$\beta = v/c \sim 1$$

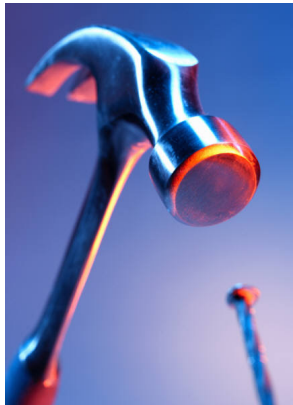
Wave-form in 5-cell cavity



How $\beta=1$ RF linac works? Example of 5-cell cavity



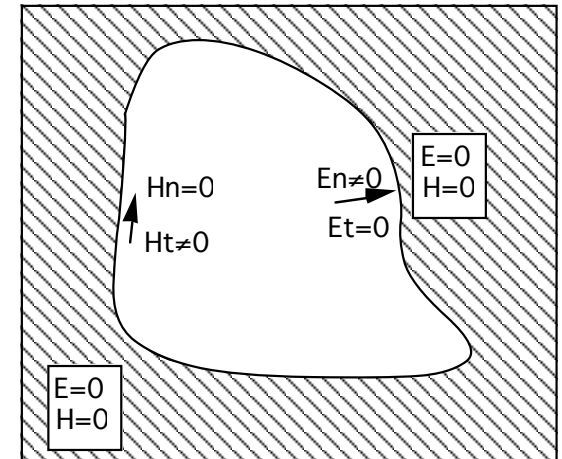
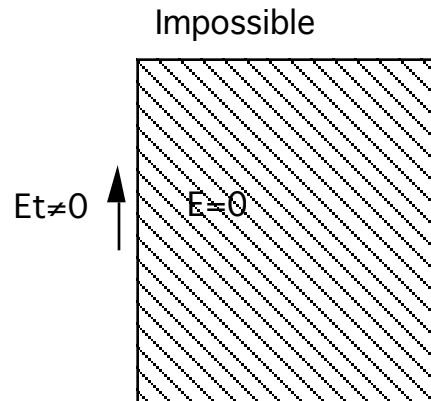
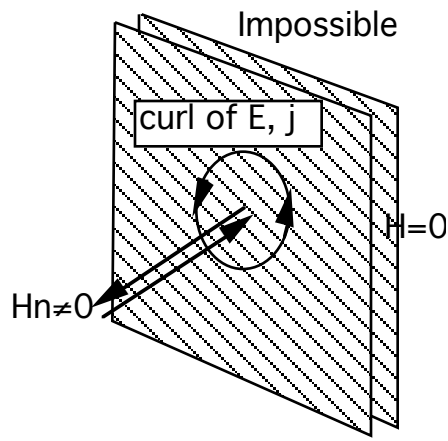
Simple things to remember



- Acceleration in DC electrostatic is limited to the difference in terminal potential (e.g. voltage between the ground and the cathode)
- RF linear accelerators (RF linacs or simply linacs) are not limited in beam energy
- In RF linacs, the coherent addition/subtraction of the energy gain from cell to cell happens by design: period of the electric field oscillation is matched to the travel time of electron between the cells.
- Accurate synchronization of RF linac is important task for any linear accelerator

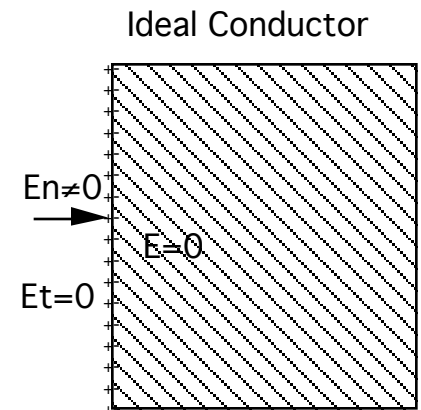
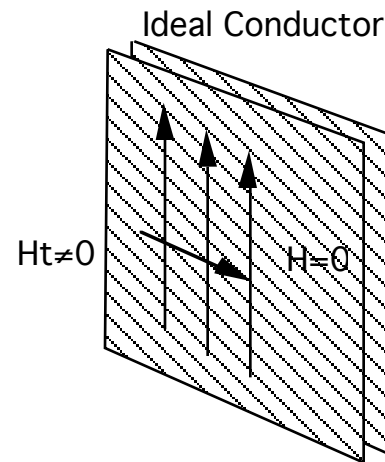
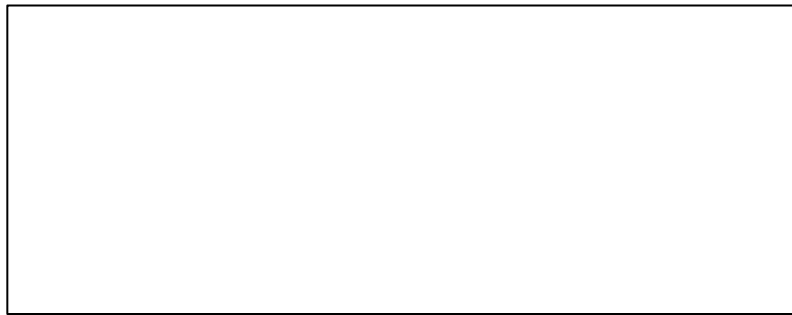
A bit of EM and conducting media

$$\vec{j} = \sigma \vec{E};$$



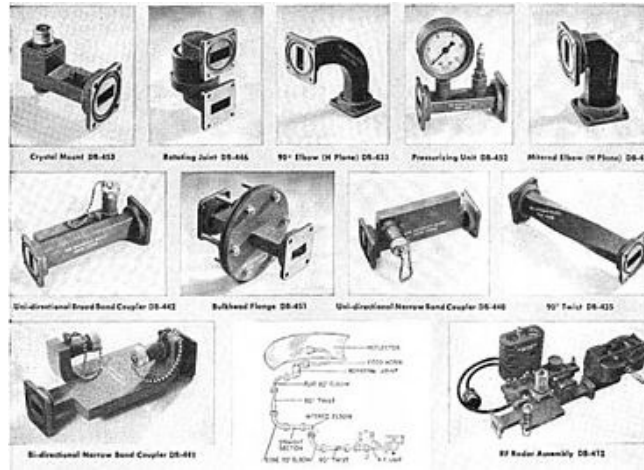
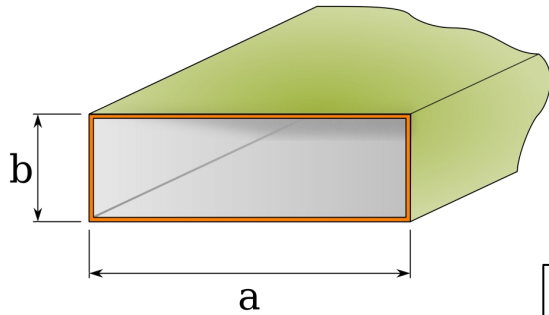
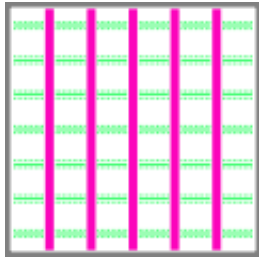
Boundary conditions

- We are considering oscillating EM fields in RF structures
- RF structures are built from highly conducting material, both to contain EM field inside and to provide low losses
- In first approximation we can consider an ideal boundary conditions and take finite conductivity as a perturbation later
- Q-factor: $Q_{\text{room temp}} \sim 10^3-10^5$, $Q_{\text{SRF}} \sim 10^9-10^{10}$

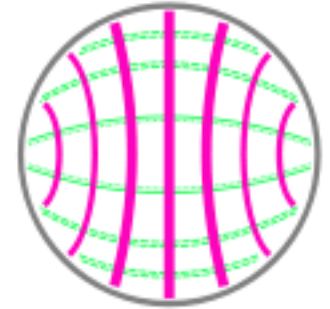


Waveguides

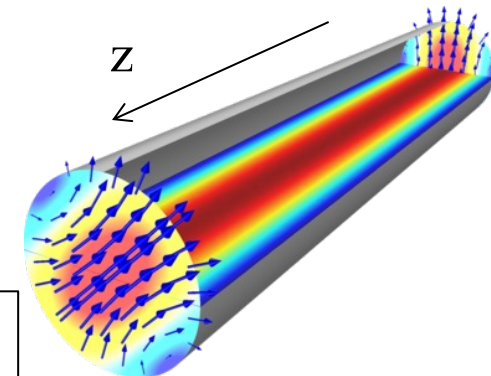
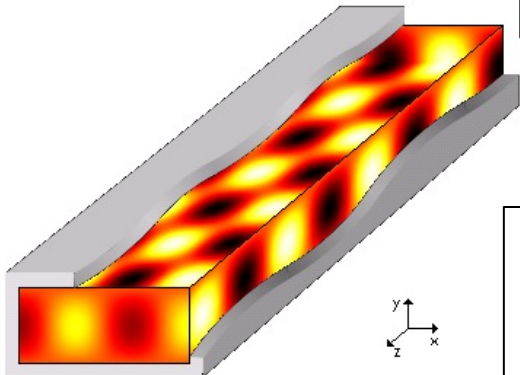
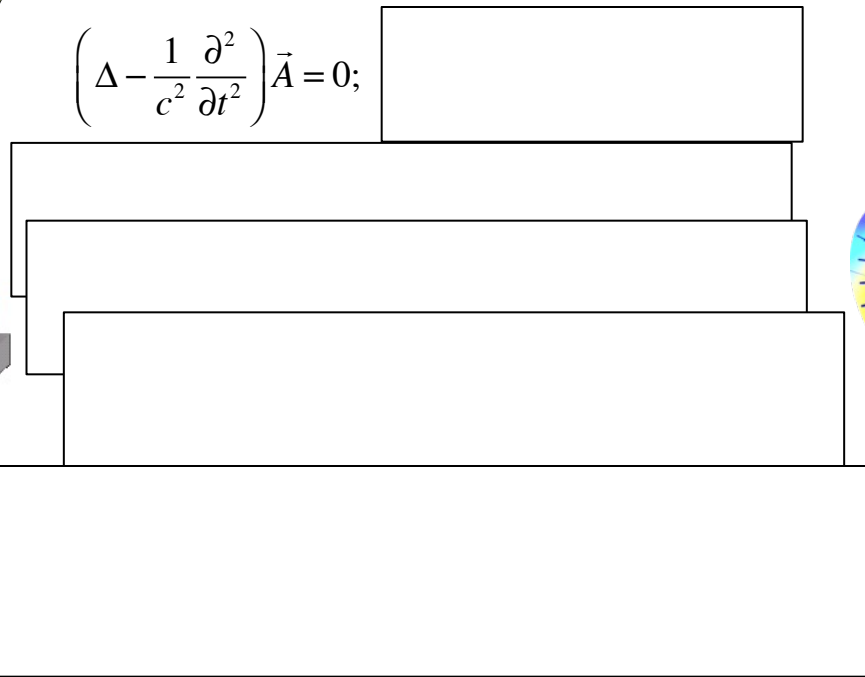
Rectangular



Circular

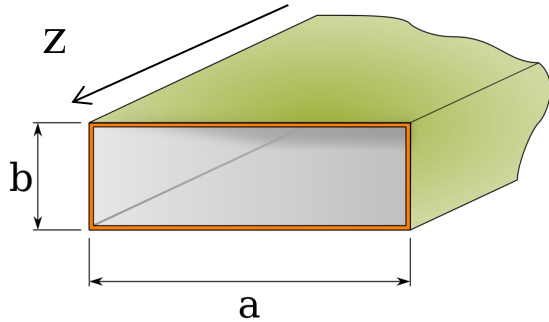


$$\left(\Delta - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \vec{A} = 0;$$



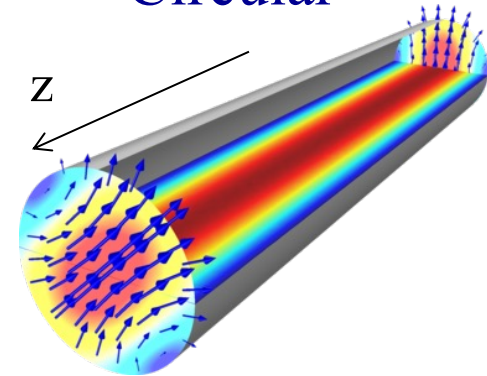
TE and TM waves

Rectangular

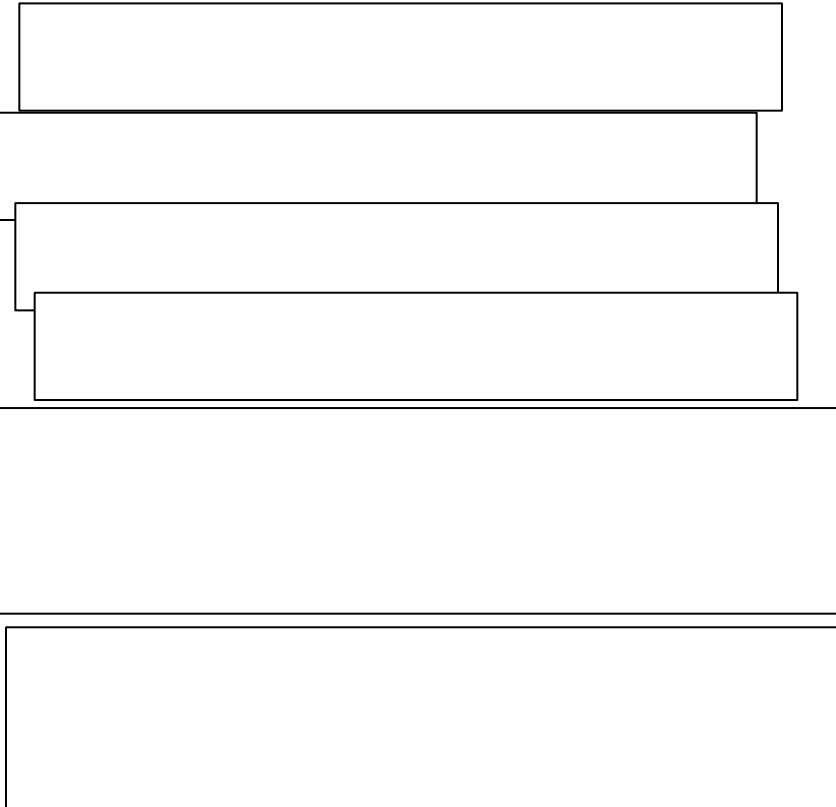
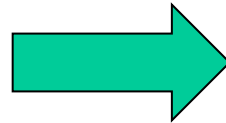


- There is simplification
 - The modes are divided into two types: TE (transverse electric) and TM (transverse magnetic)

Circular

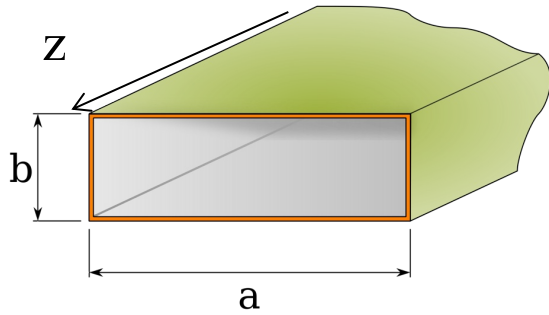


- Last two equations indicated that E_z and B_z fully determine transverse component of the EM field
- It means that we can always consider a linear combination of the fields with $E_z = 0$ everywhere (TE) and $B_z = 0$ everywhere (TM)
- Naturally, when we interested in accelerating particles, we will need TM mode with $E_z \neq 0$.



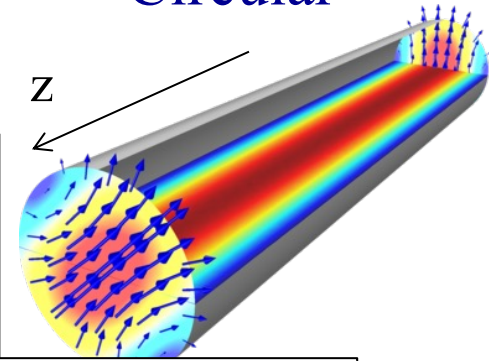
Cut-off frequency

Rectangular

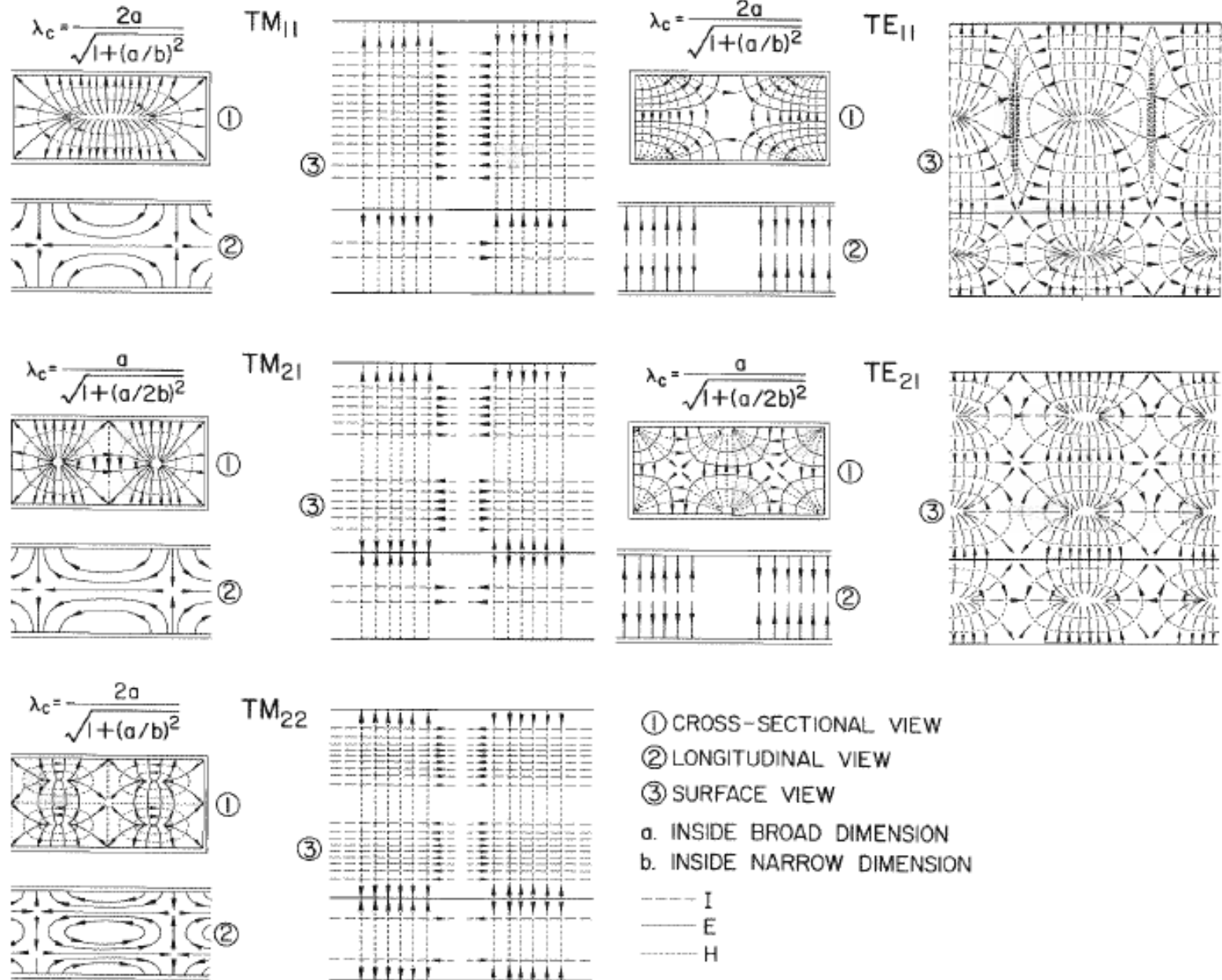


- EM field is a linear combination of modes with $E_z = 0$ everywhere (TE) and $B_z = 0$ everywhere (TM)

Circular

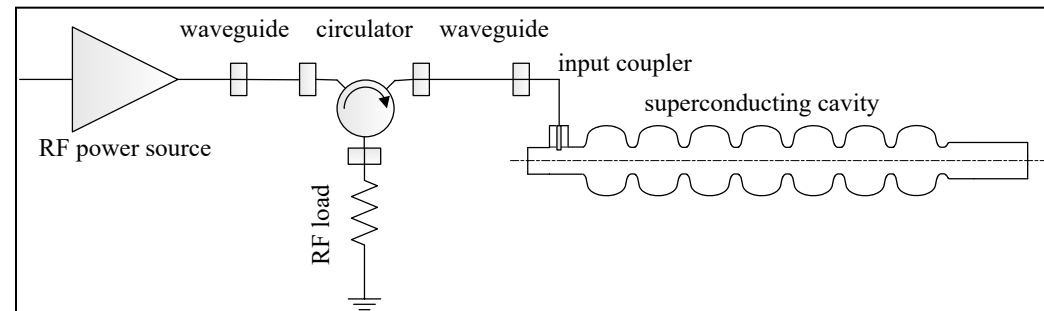
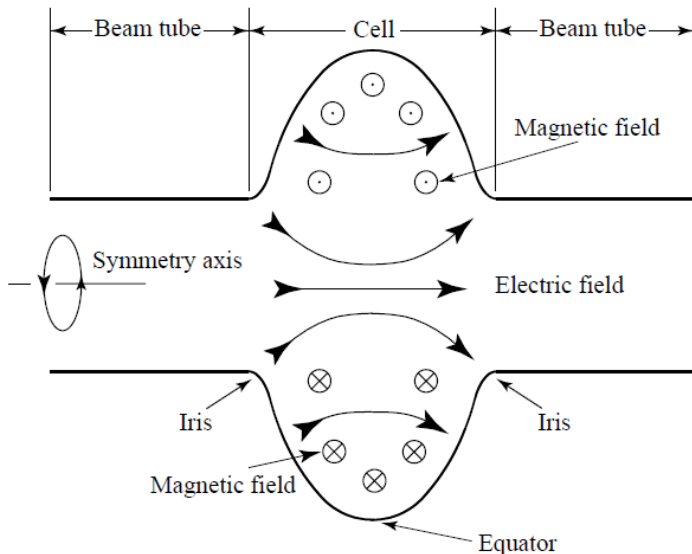
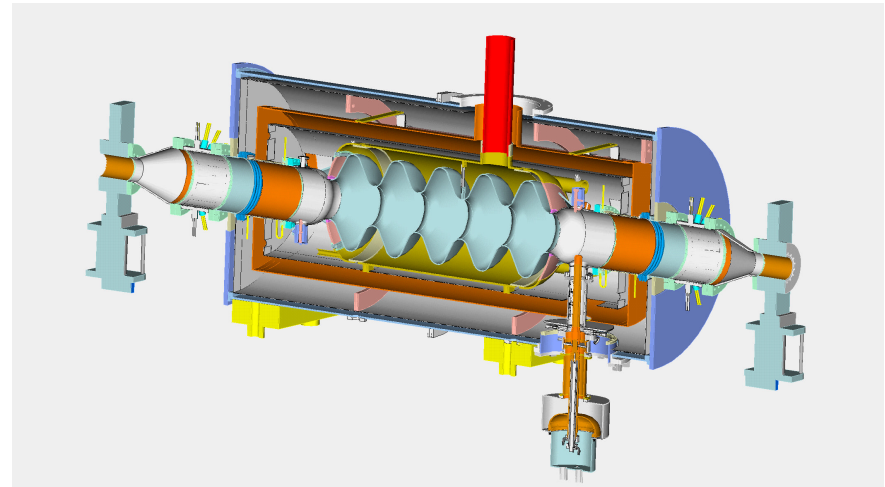
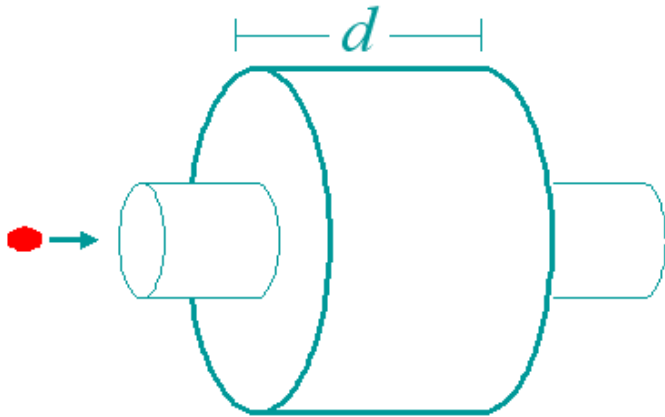


Modes in rectangular waveguide



RF cavities

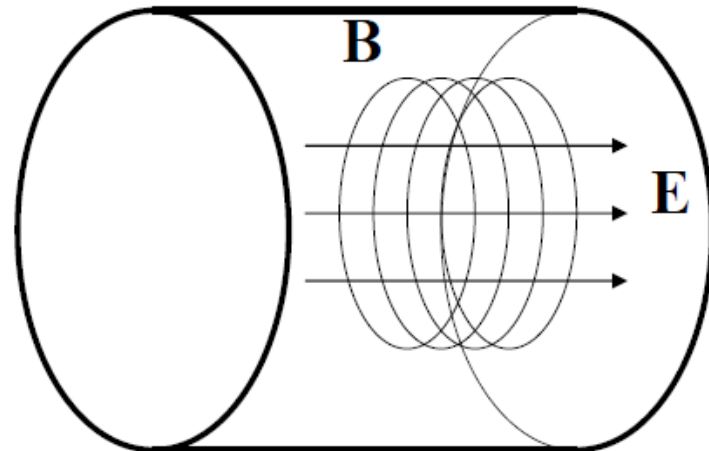
are designed to confine the EM field inside: It means that they operate at frequency below cut-off of the beam-pipes attached to them



RF Cavity Modes:

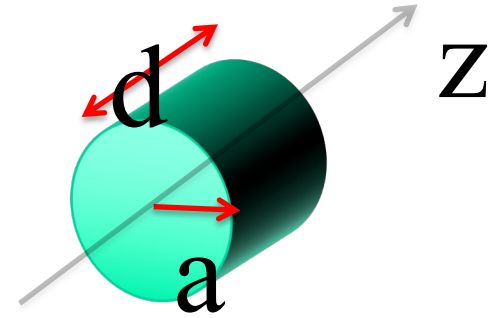
the lowest accelerating is TM_{010} mode

- Fields in the cavity are solutions of the equation
- Subject to the boundary conditions $\hat{n} \times \mathbf{E} = 0, \hat{n} \cdot \mathbf{H} = 0$ $\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \begin{Bmatrix} \mathbf{E} \\ \mathbf{H} \end{Bmatrix} = 0$
- Two extra surfaces ($z=0$ and $z=d$): but this is no problem for TM mode
- An infinite number of solutions (eigen modes) belong to two families of modes with different field structure and eigen frequencies: TE modes have only transverse electric fields, TM modes have only transverse magnetic fields.
- One needs longitudinal electric field for acceleration, hence the lowest frequency TM_{010} mode is used.
- For the pillbox cavity w/o beam tubes
- Note that frequency does not depend of the cavity length! But only its radius.



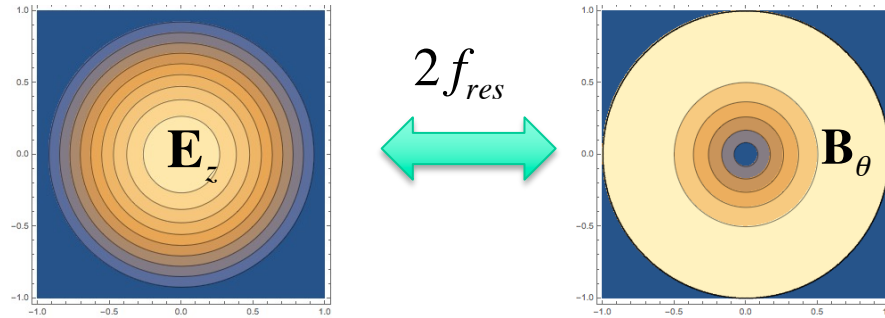
Pillbox Cavity

- Similarly, to a previous exercise, we need to pick TM mode to have non-zero E_z component
- We also select TEM₀₁ waveguide mode and $k_z=0$

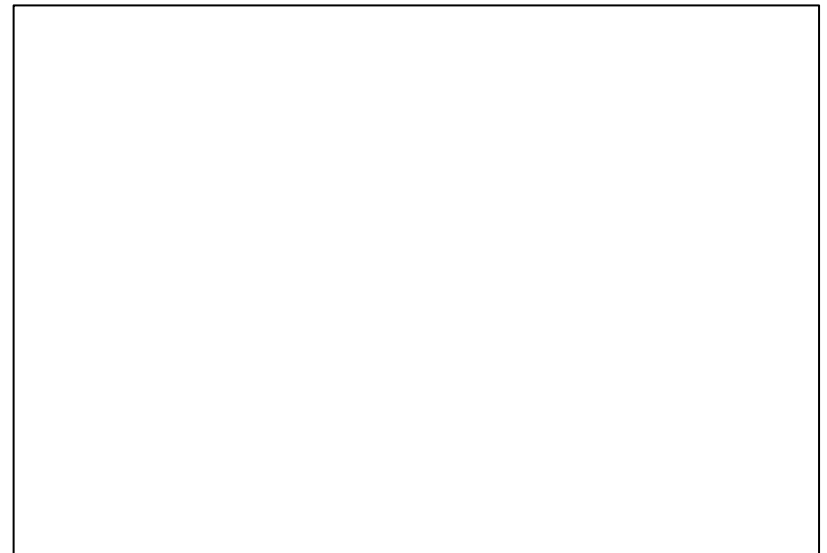


EM Cavity

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_o(\vec{r})\cos(\omega t + \varphi(\vec{r}))$$
$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_o(\vec{r})\sin(\omega t + \psi(\vec{r}))$$
$$\int \vec{\mathbf{E}}_o^2 dV = \int \vec{\mathbf{B}}_o^2 dV \quad (\cdot c^2 \text{ for SI})$$



- Each mode has full analogy with a resonant LC circuit or a mechanical oscillator: energy stored in electric field can be compared to potential energy, and energy stored in magnetic field – to kinetic energy
- Typical energy stored in 5 cell, 700 MHz SRF cavity operating at 20 MV/m is ~ 500 J
- What much more impressive is the intra-cavity power of about 2,000 GW!



Quality factor - definition

- Let's consider a stand-alone cavity without any external couplers

- Energy stored in the cavity
$$W = \int \left(\epsilon_o \frac{\vec{\mathbf{E}}^2}{2} + \mu_o \frac{\vec{\mathbf{H}}^2}{2} \right) dV = \mu_o \int \vec{\mathbf{H}}_o^2 dV$$

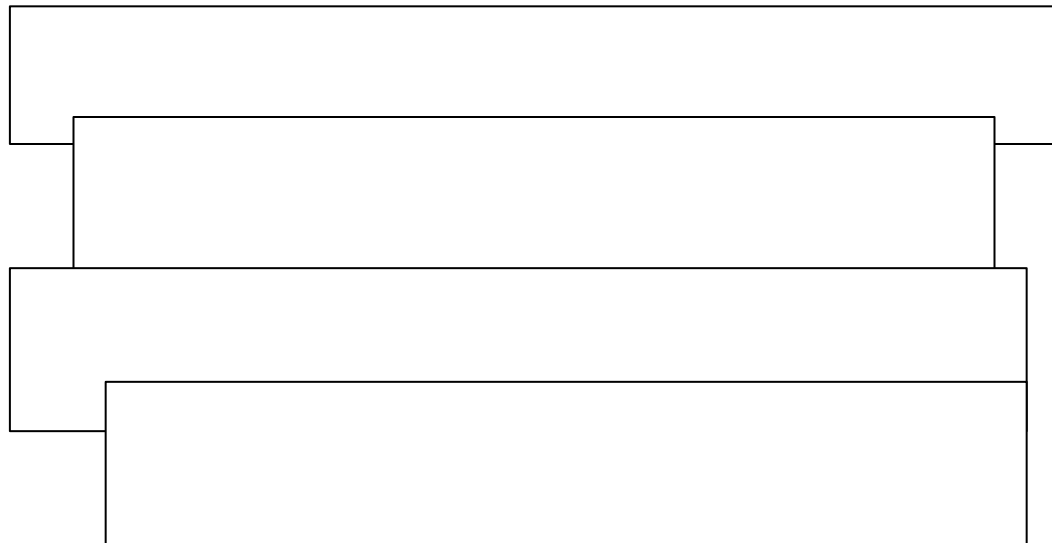
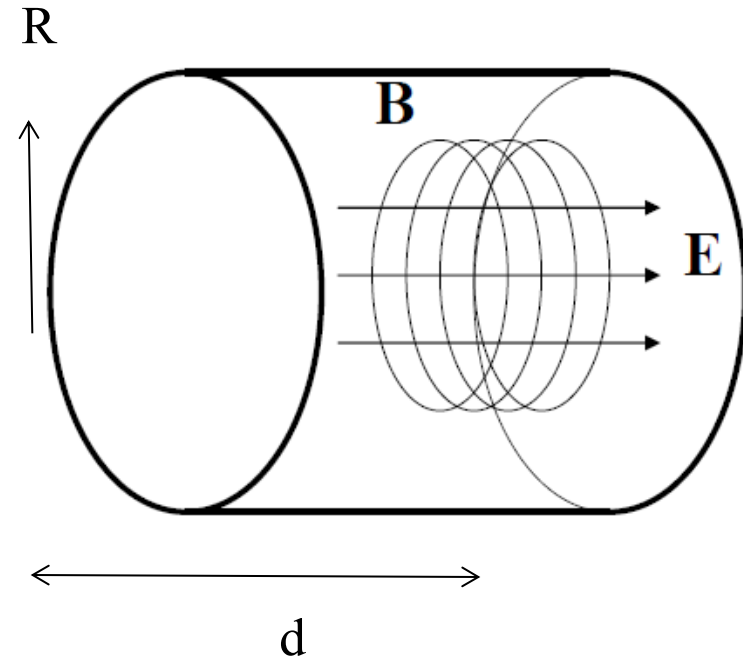
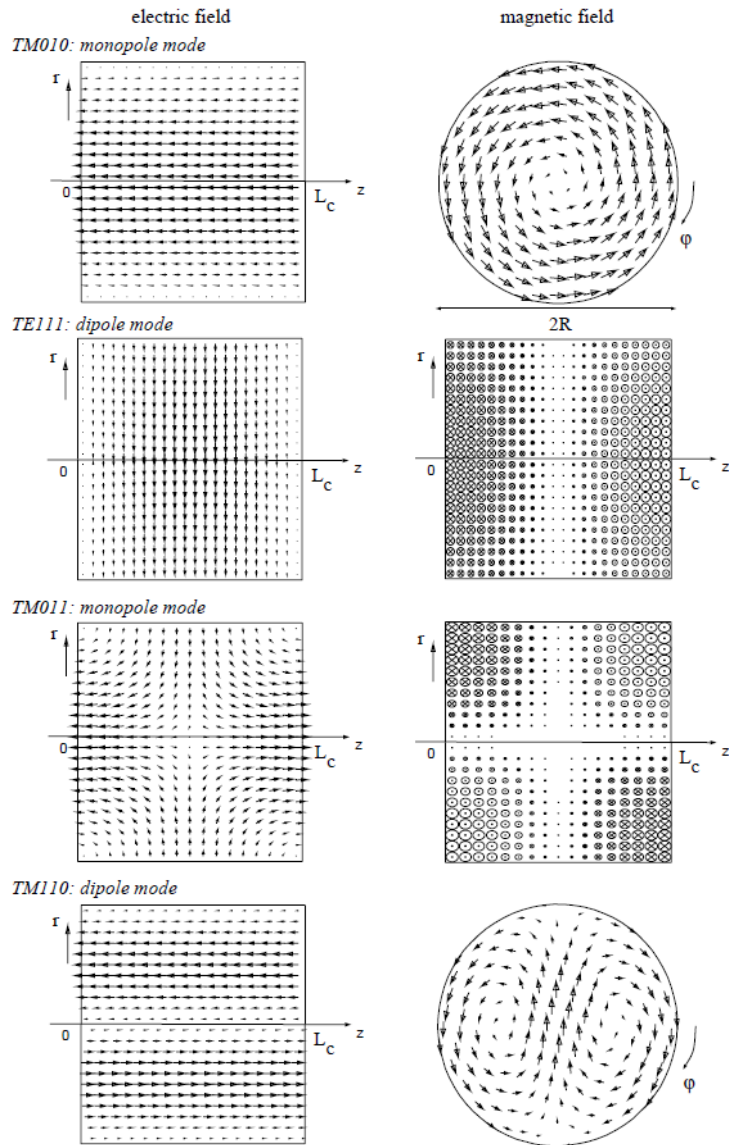
- Losses in the power

$$P_{loss} = \frac{W \omega}{Q}$$

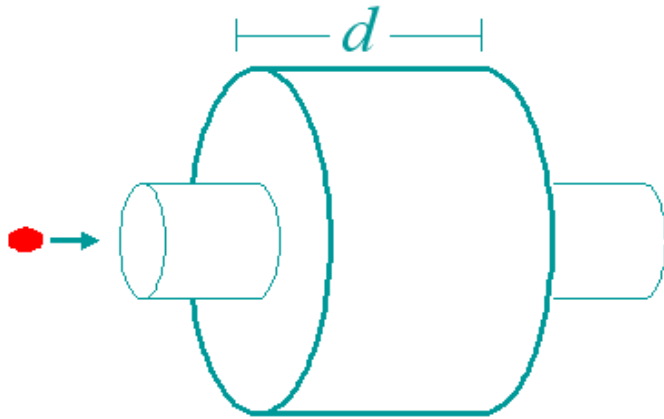
- Quality factor - Q - it is the number of RF oscillation times 2π required for energy inside the cavity to reduce e-fold.

Fundamental and high order modes (HOMs)

Eigenmodes in a Pill-box cavity



Accelerating voltage & transit time



- Assuming charged particles moving along the cavity axis, one can calculate accelerating voltage as

$$V_c = \left| \int_{-\infty}^{\infty} E_z(\rho=0, z) e^{i\omega_0 z/\beta c} dz \right|$$

For the pillbox cavity one can integrate this analytically:

$$V_c = E_0 \left| \int_0^d e^{i\omega_0 z/\beta c} dz \right| = E_0 d \frac{\sin\left(\frac{\omega_0 d}{2\beta c}\right)}{\frac{\omega_0 d}{2\beta c}} = E_0 d \cdot T$$

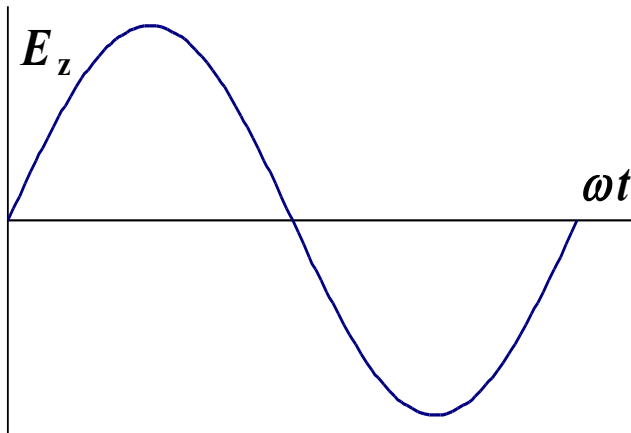
where T is the transit time factor.

- To get maximum acceleration:

$$T_{transit} = t_{exit} - t_{enter} = \frac{T_0}{2} \Rightarrow d = \beta\lambda/2 \Rightarrow V_c = \frac{2}{\pi} E_0 d$$

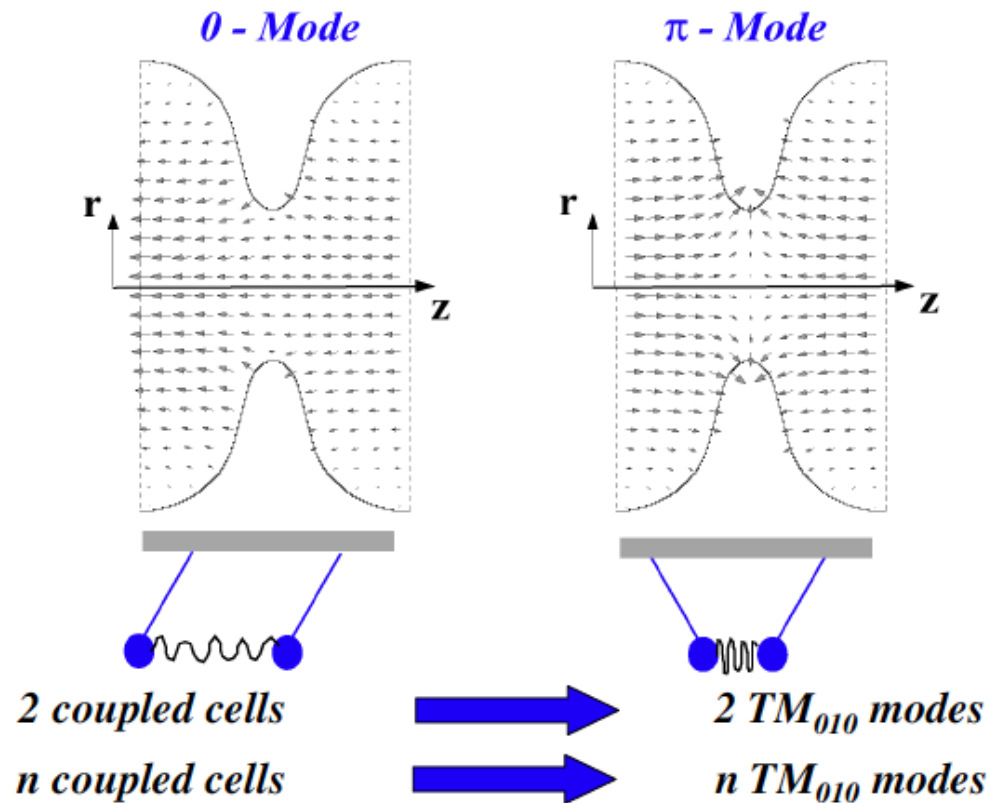
Thus for the pillbox cavity $T = 2/\pi$.

- The accelerating field E_{acc} is defined as $E_{acc} = V_c/d$. Unfortunately the cavity length is not easy to specify for shapes other than pillbox so usually it is assumed to be $d = \beta\lambda/2$. This works OK for multi-cell cavities, but poorly for single-cell ones.

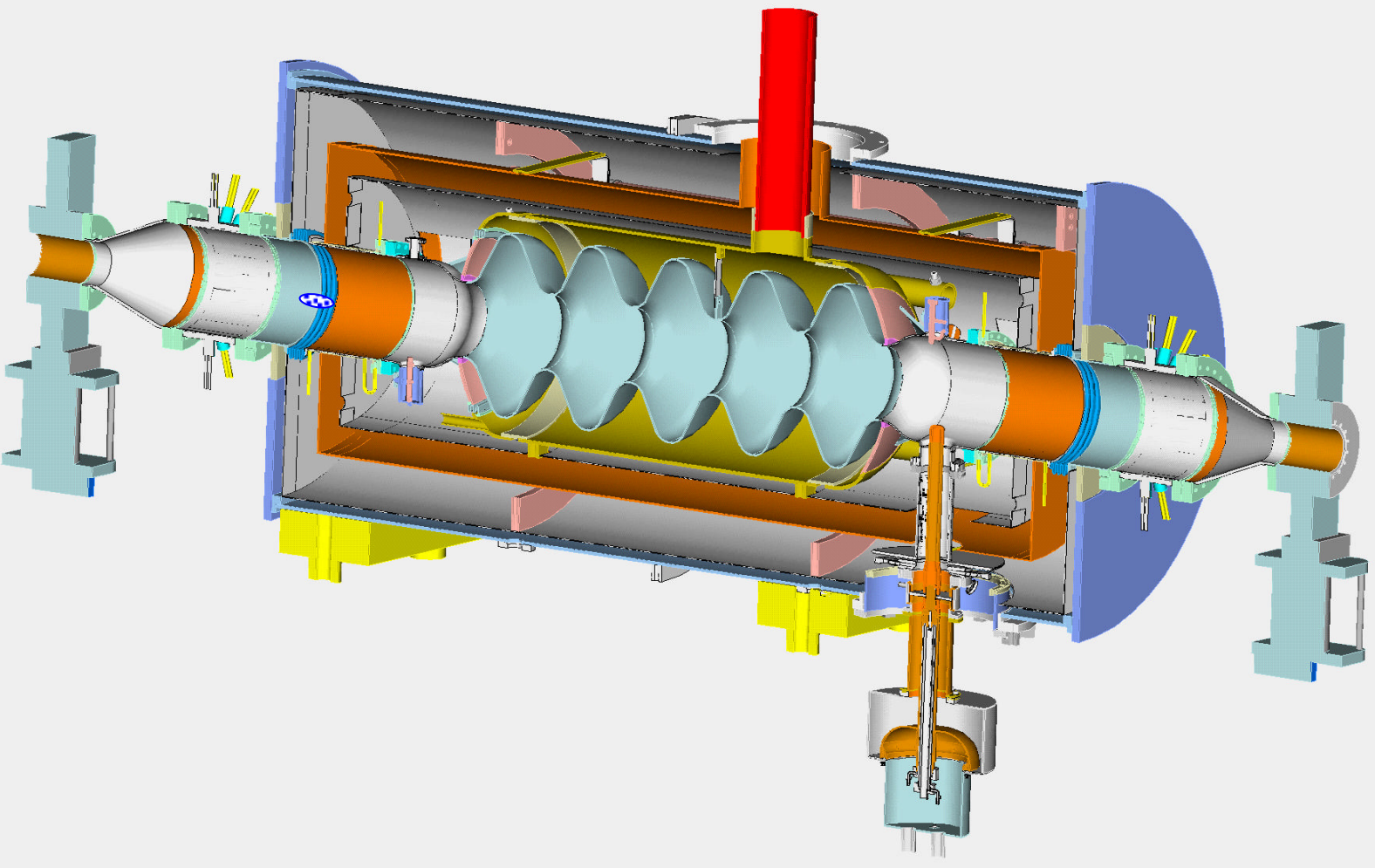


Multicell cavities: coupled oscillators

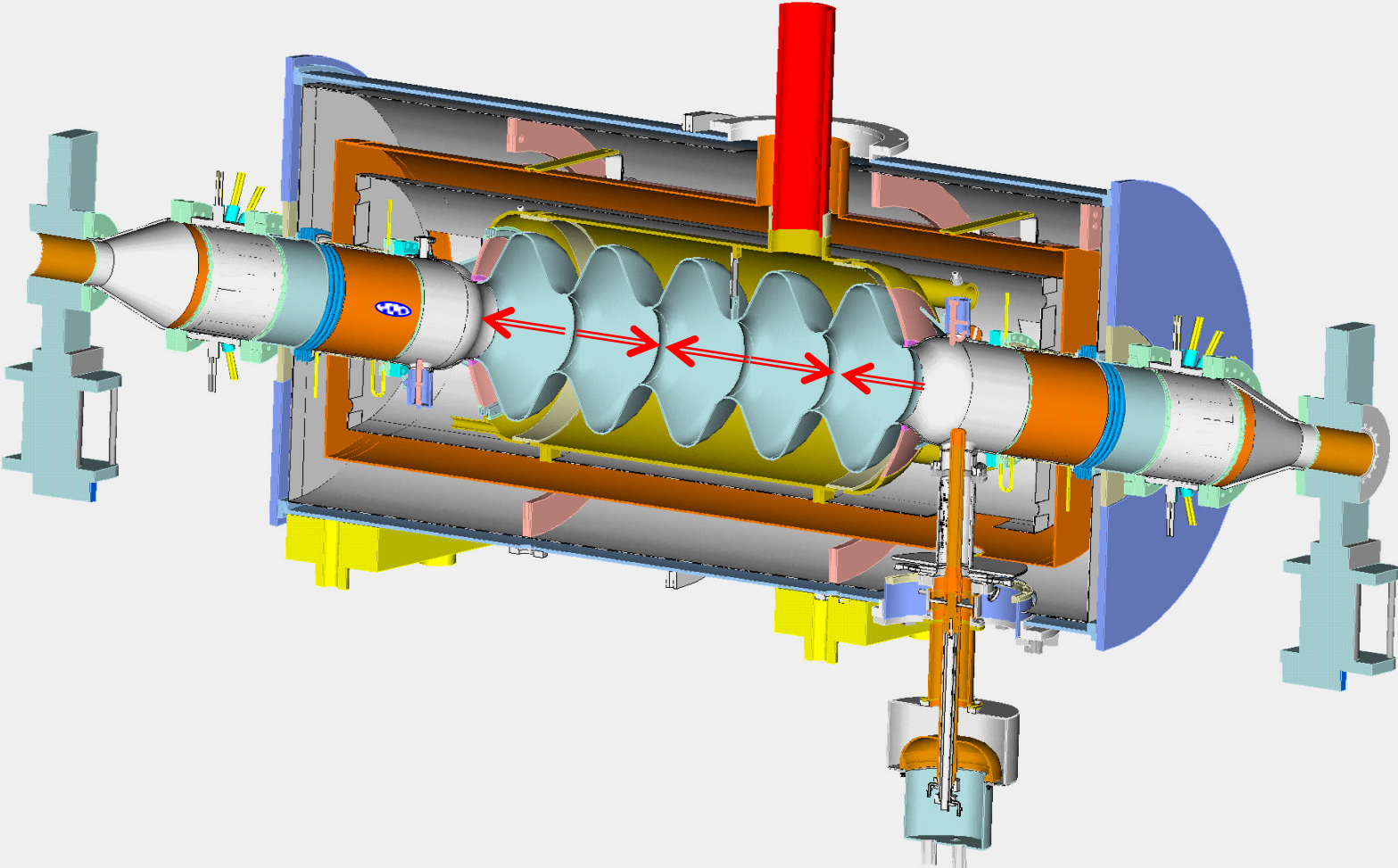
- Several cells can be connected together to form a multicell cavity.
- Coupling of TM_{010} modes of the individual cells via the iris (primarily electric field) causes them to split:



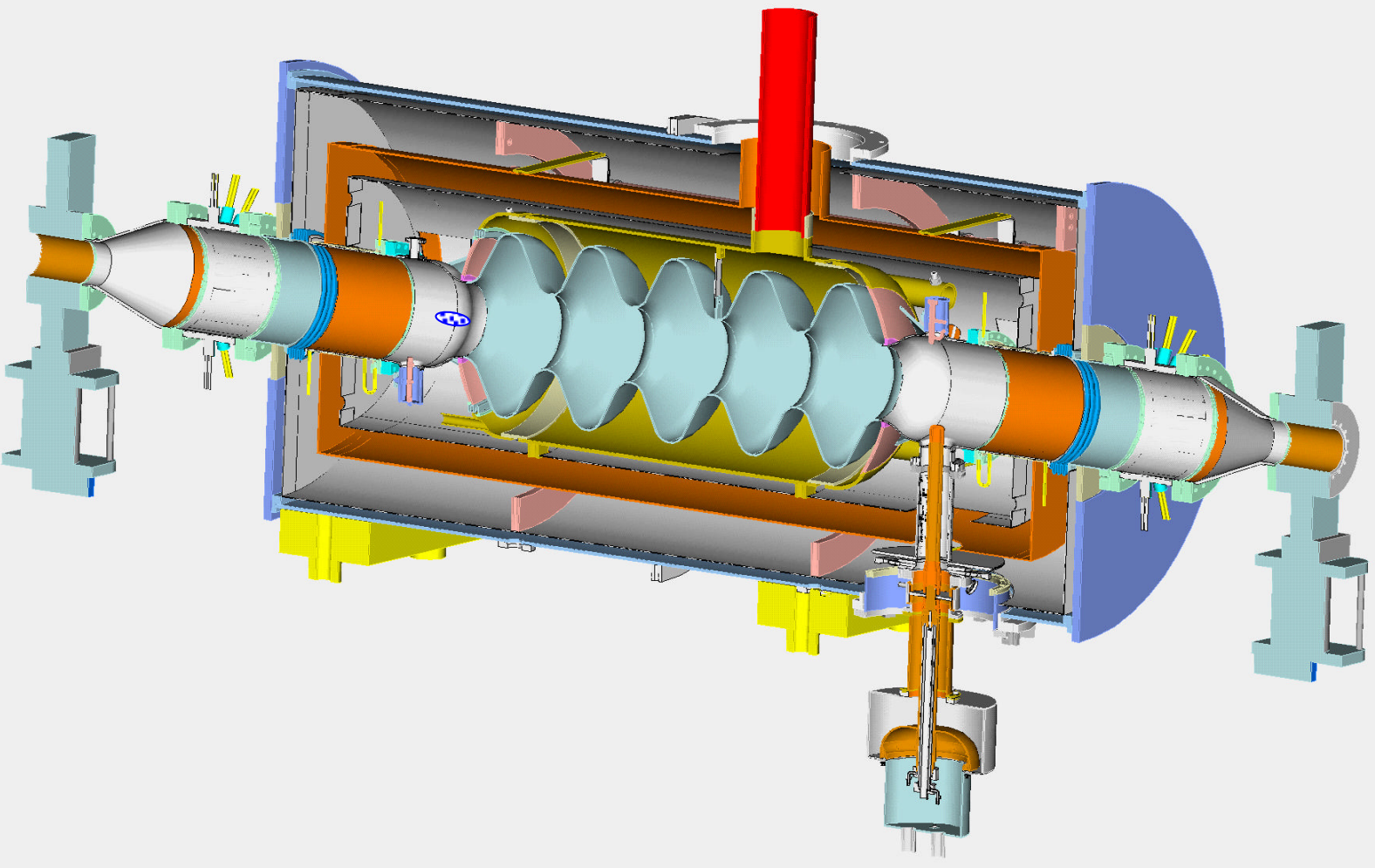
How $\beta=1$ RF accelerator works? In pictures



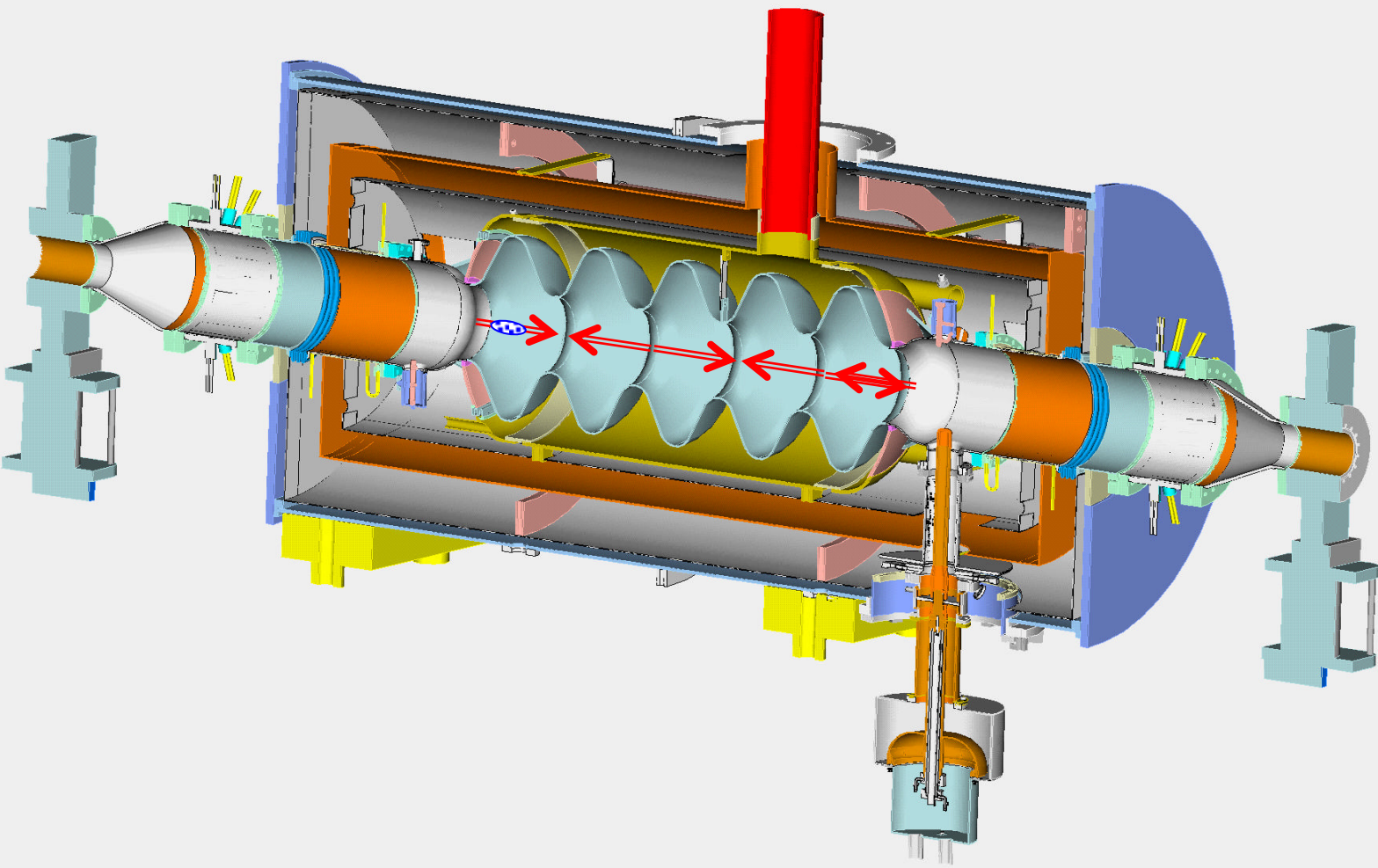
How $\beta=1$ RF accelerator works? In pictures



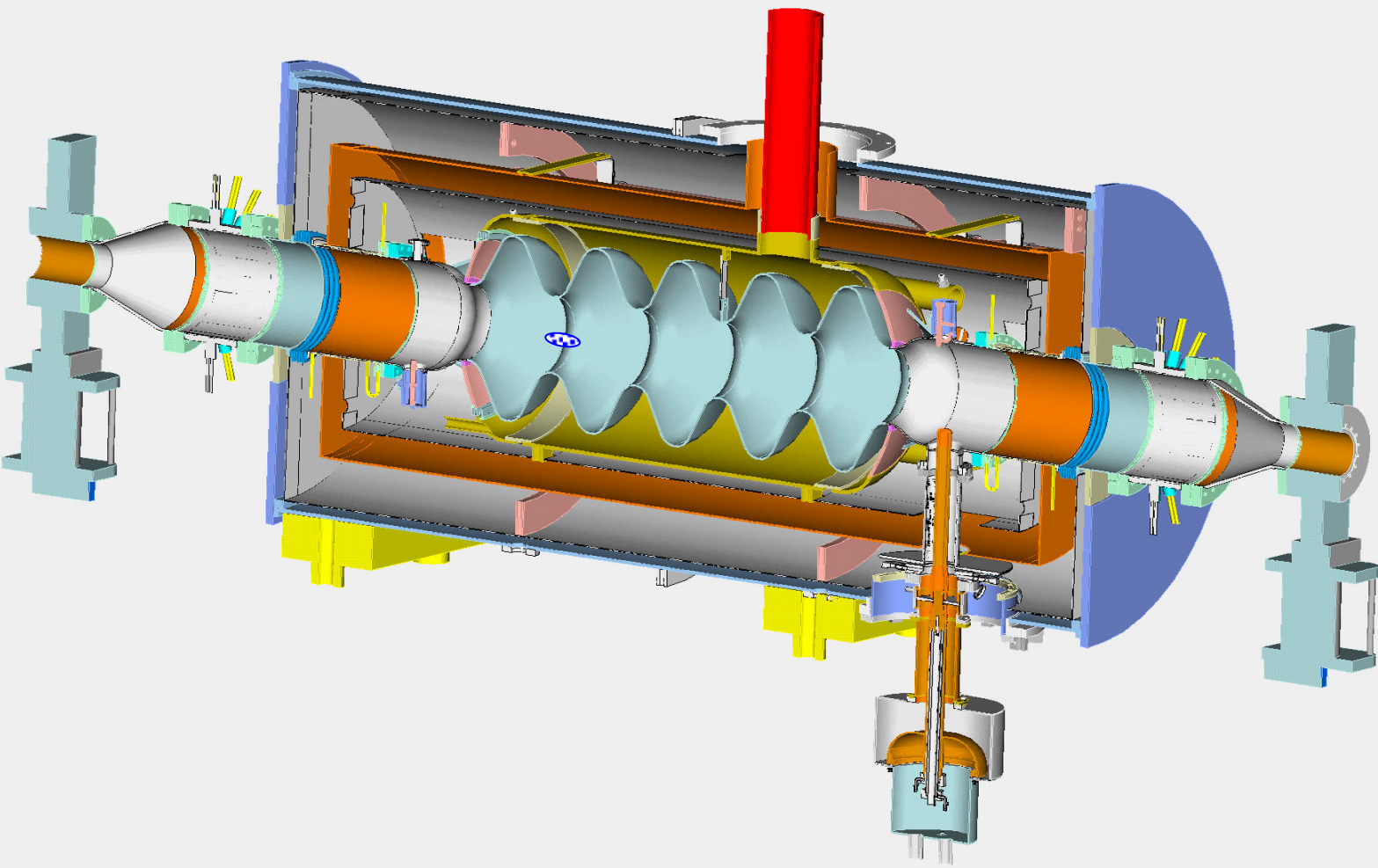
How $\beta=1$ RF accelerator works? In pictures



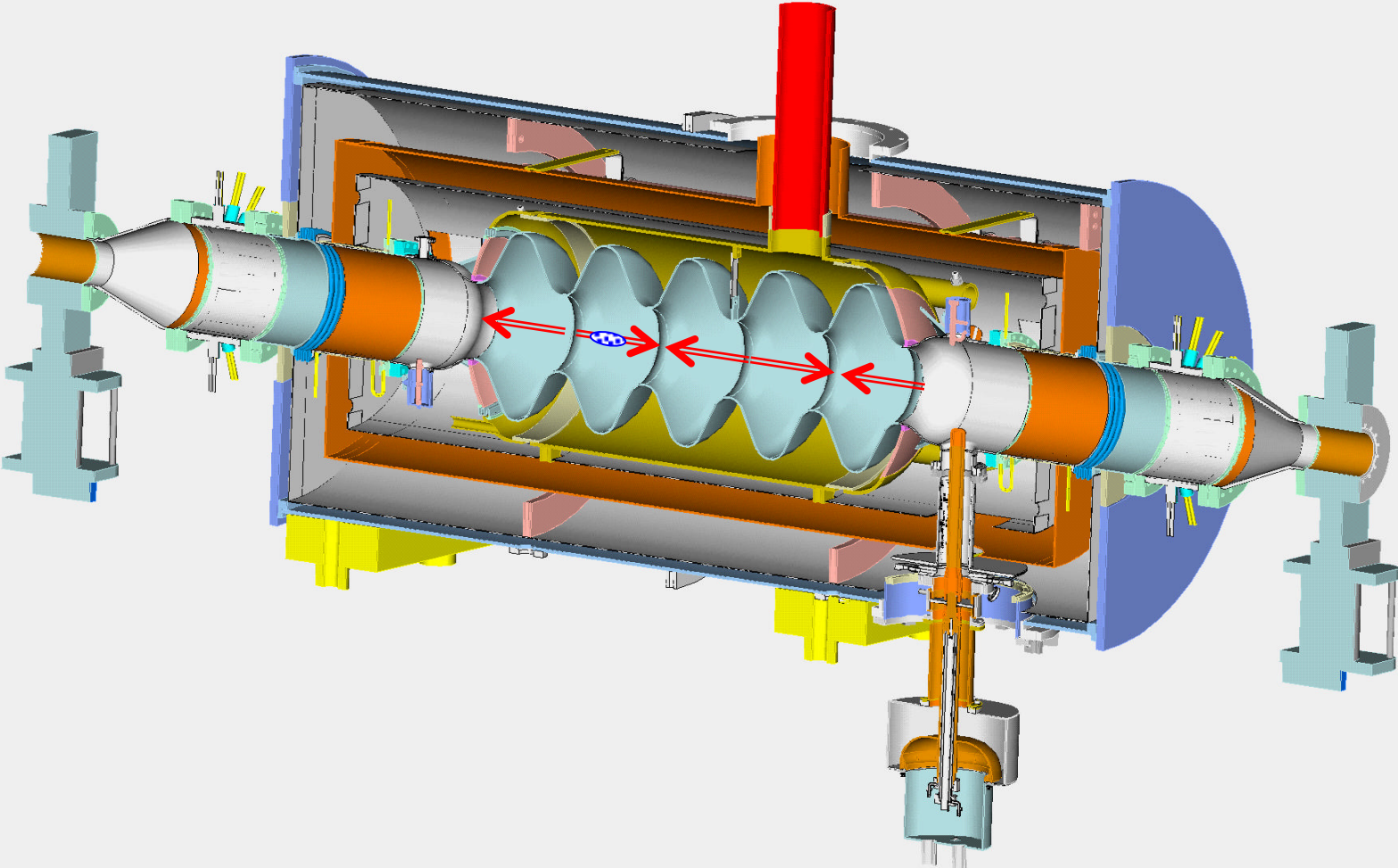
How $\beta=1$ RF accelerator works? In pictures



How $\beta=1$ RF accelerator works? In pictures

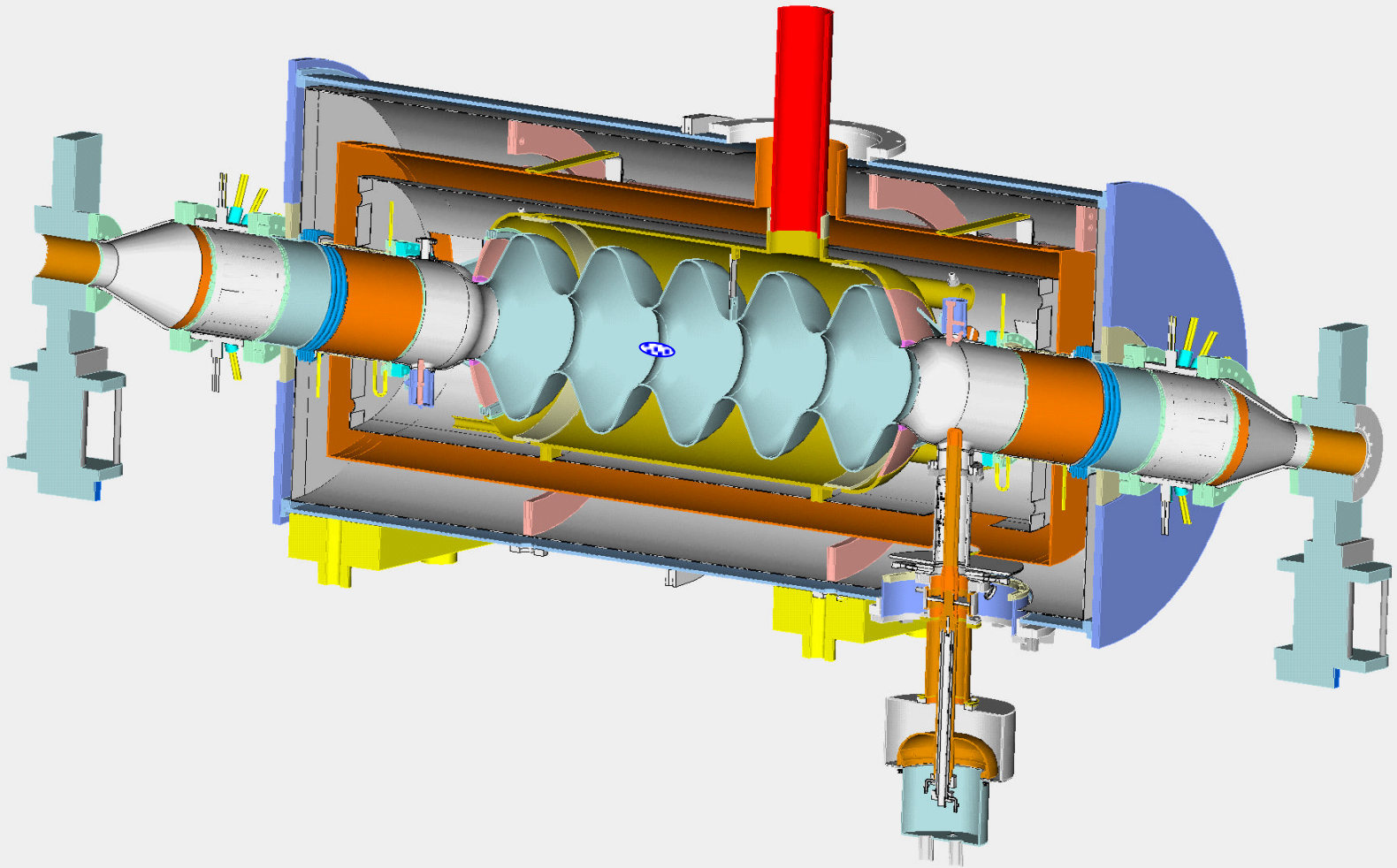


How $\beta=1$ RF accelerator works? In pictures

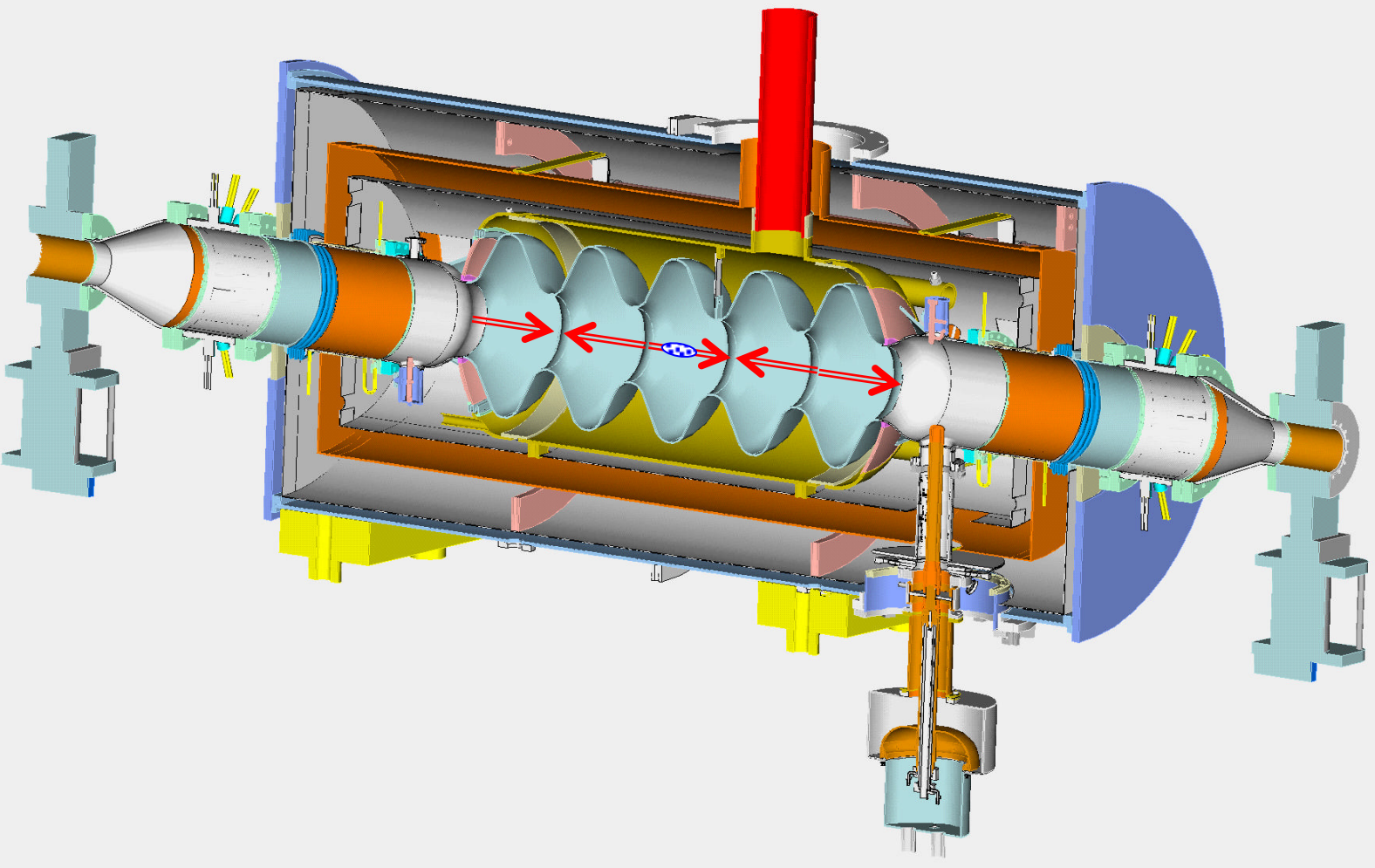


How $\beta=1$ RF accelerator works?

In pictures

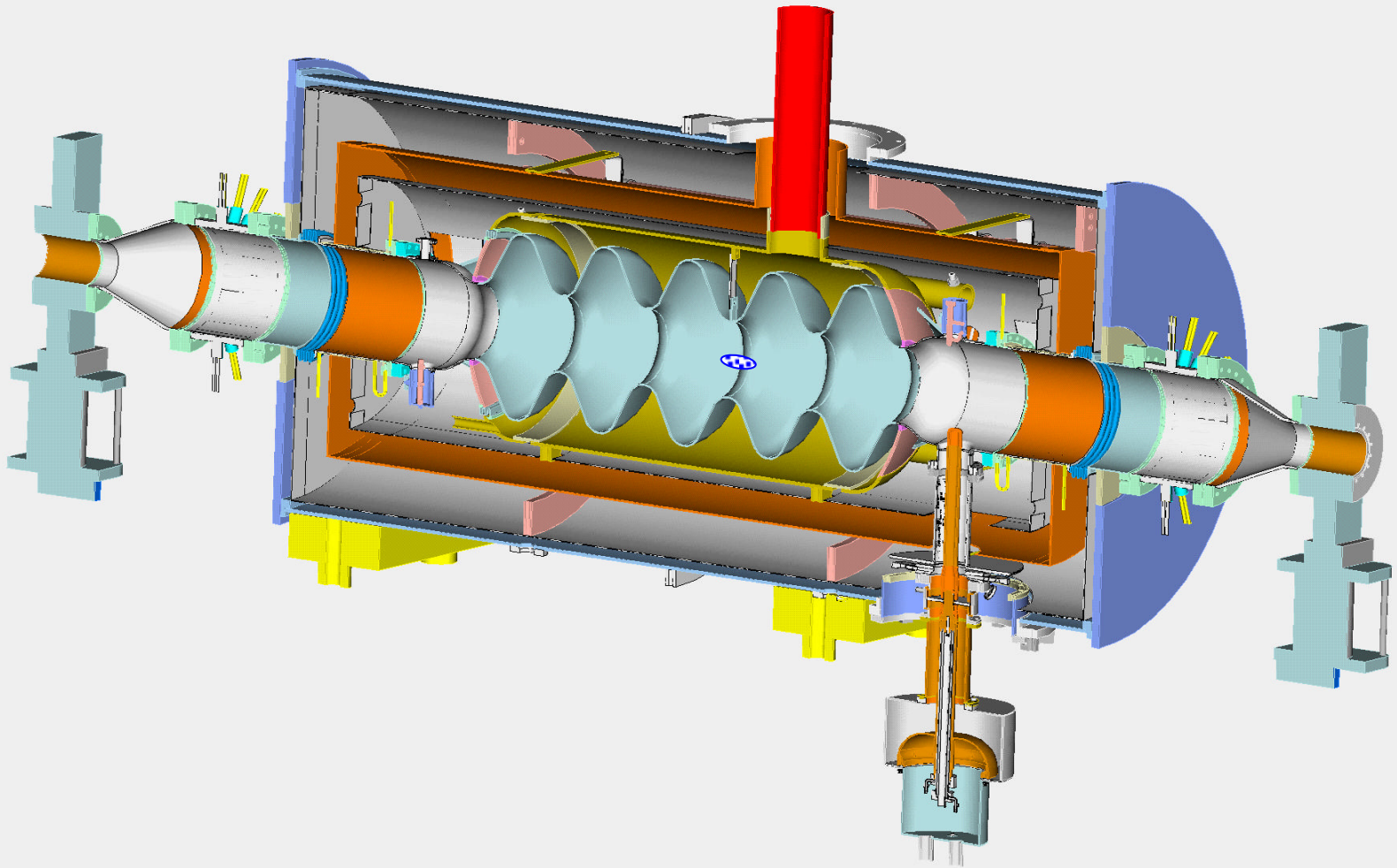


How $\beta=1$ RF accelerator works? In pictures

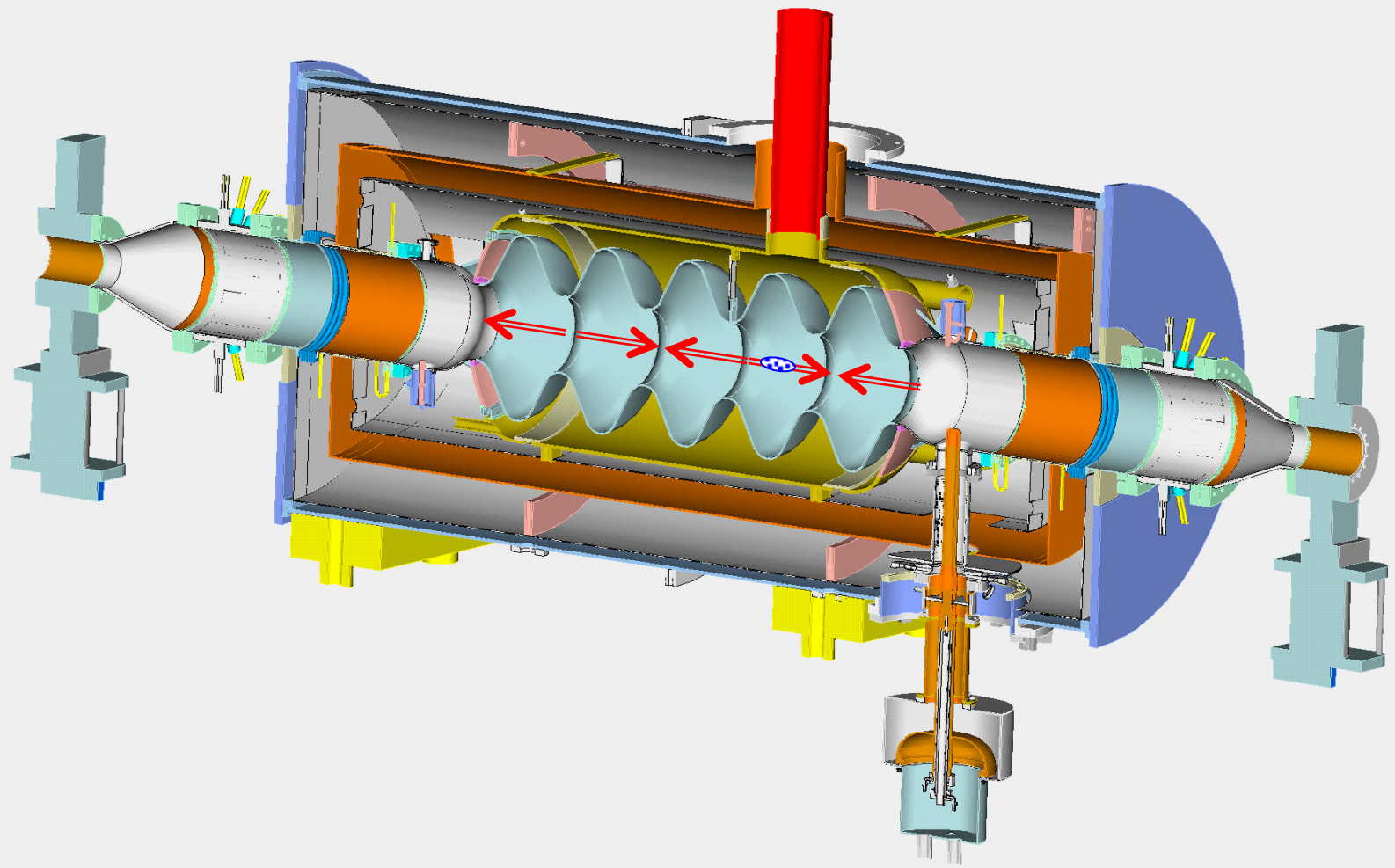


How $\beta=1$ RF accelerator works?

In pictures

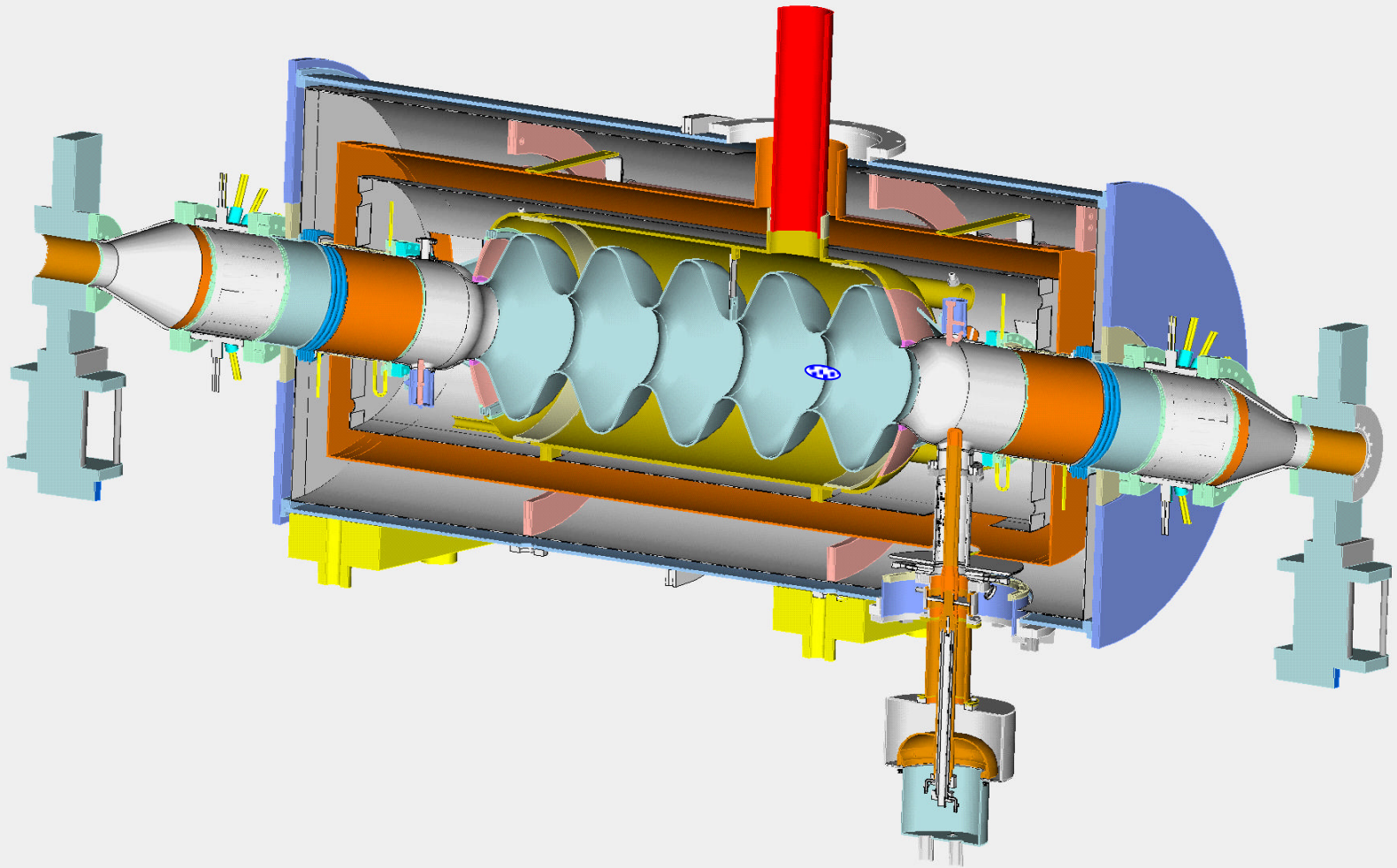


How $\beta=1$ RF accelerator works? In pictures

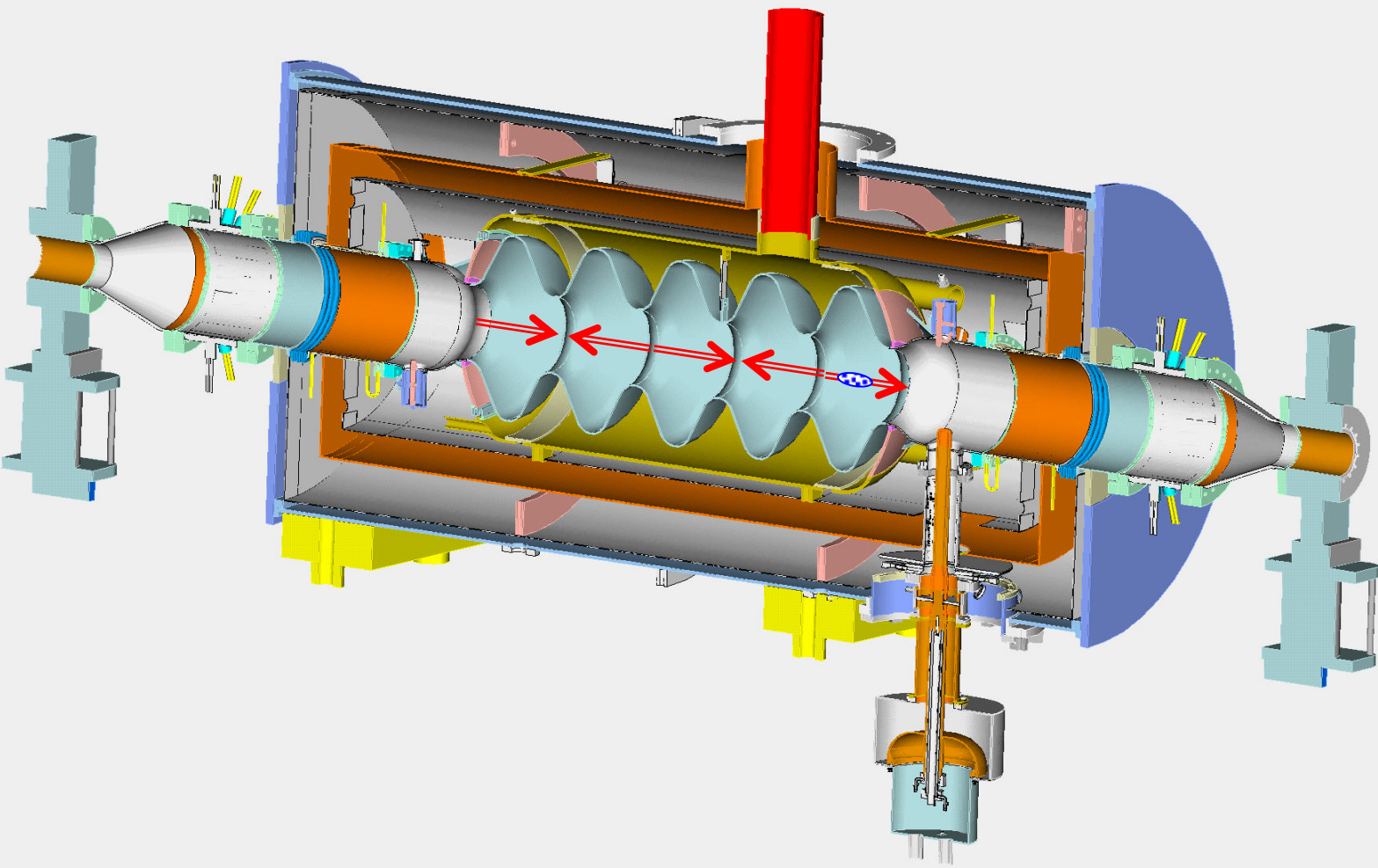


How $\beta=1$ RF accelerator works?

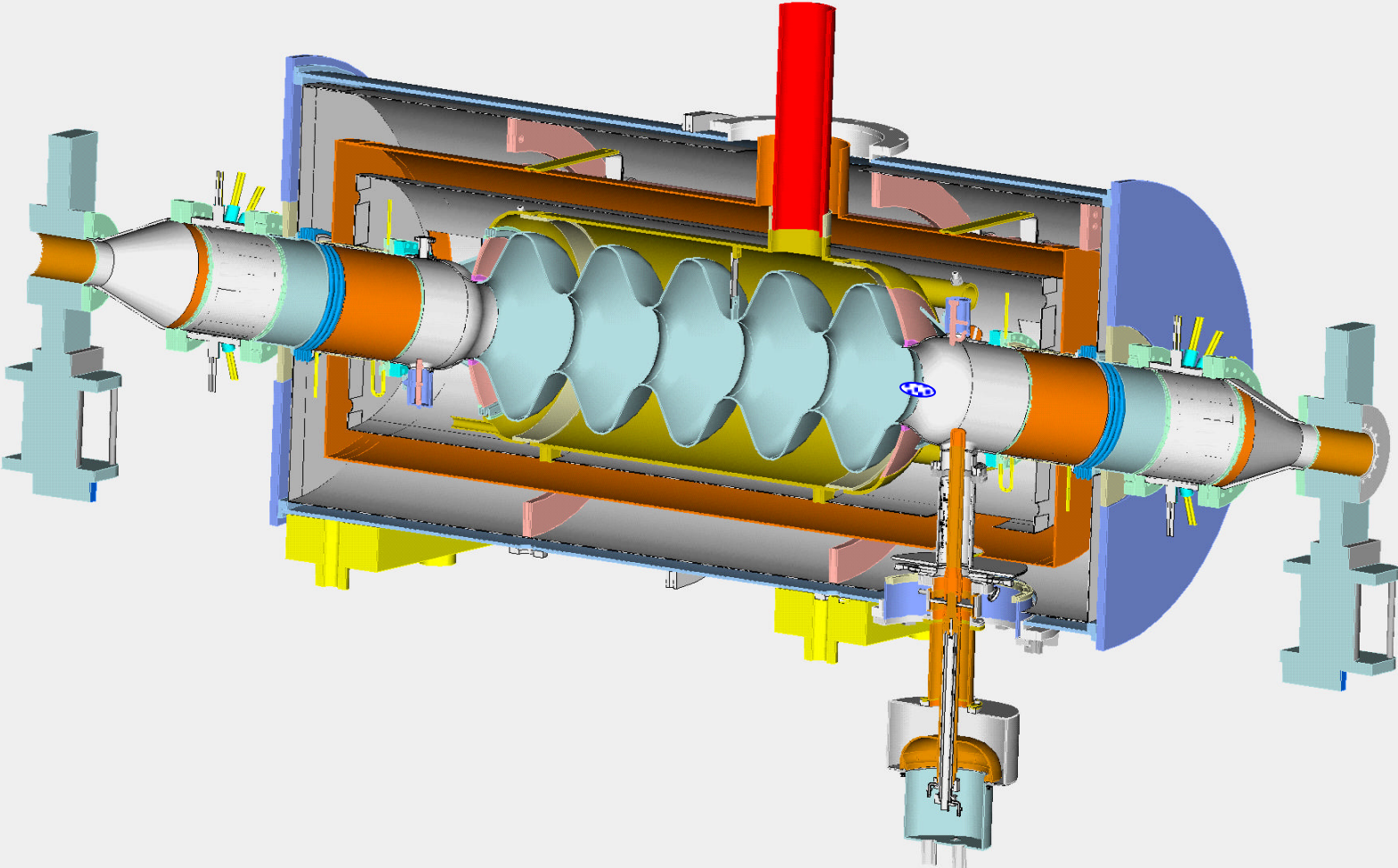
In pictures



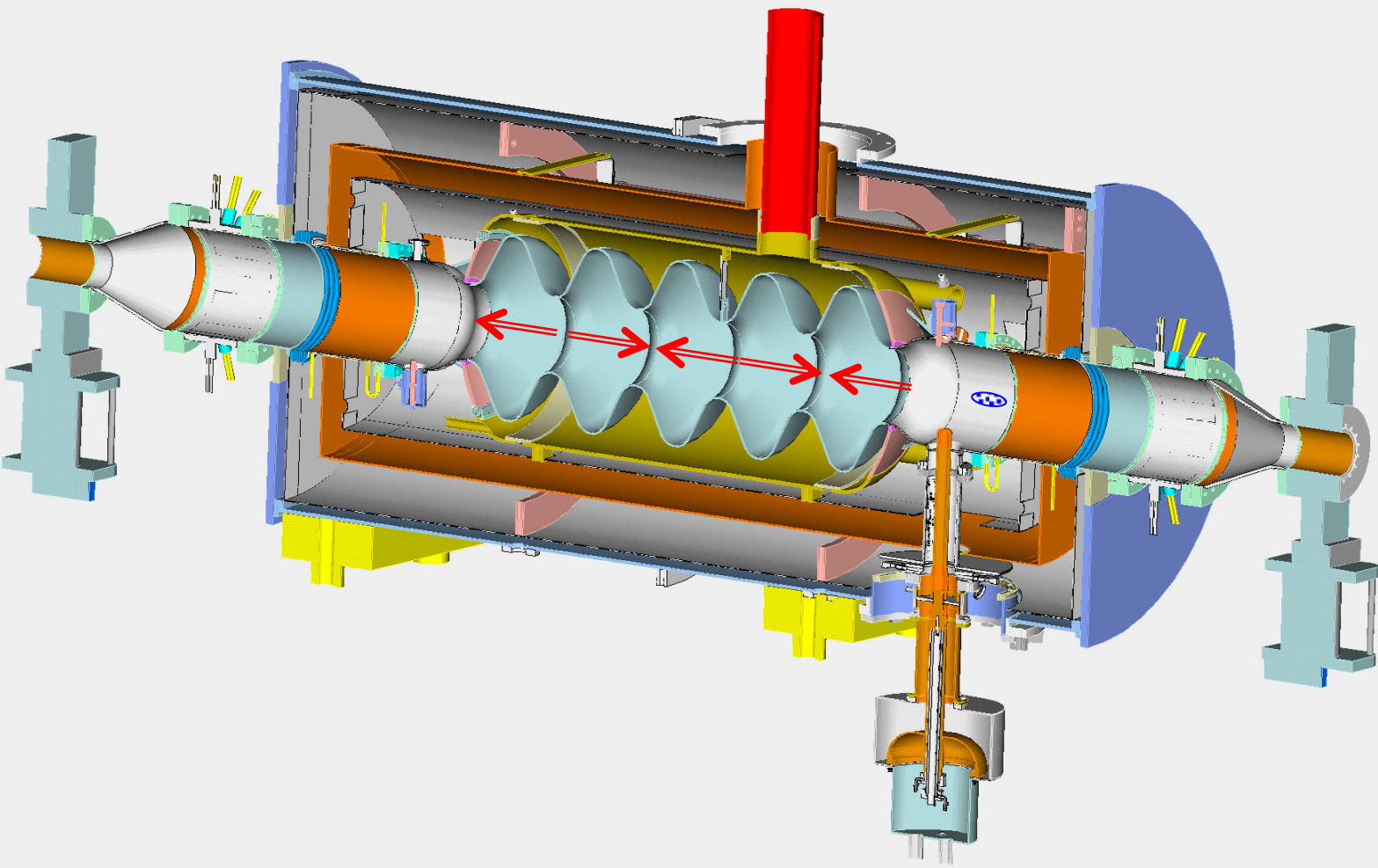
How $\beta=1$ RF accelerator works? In pictures



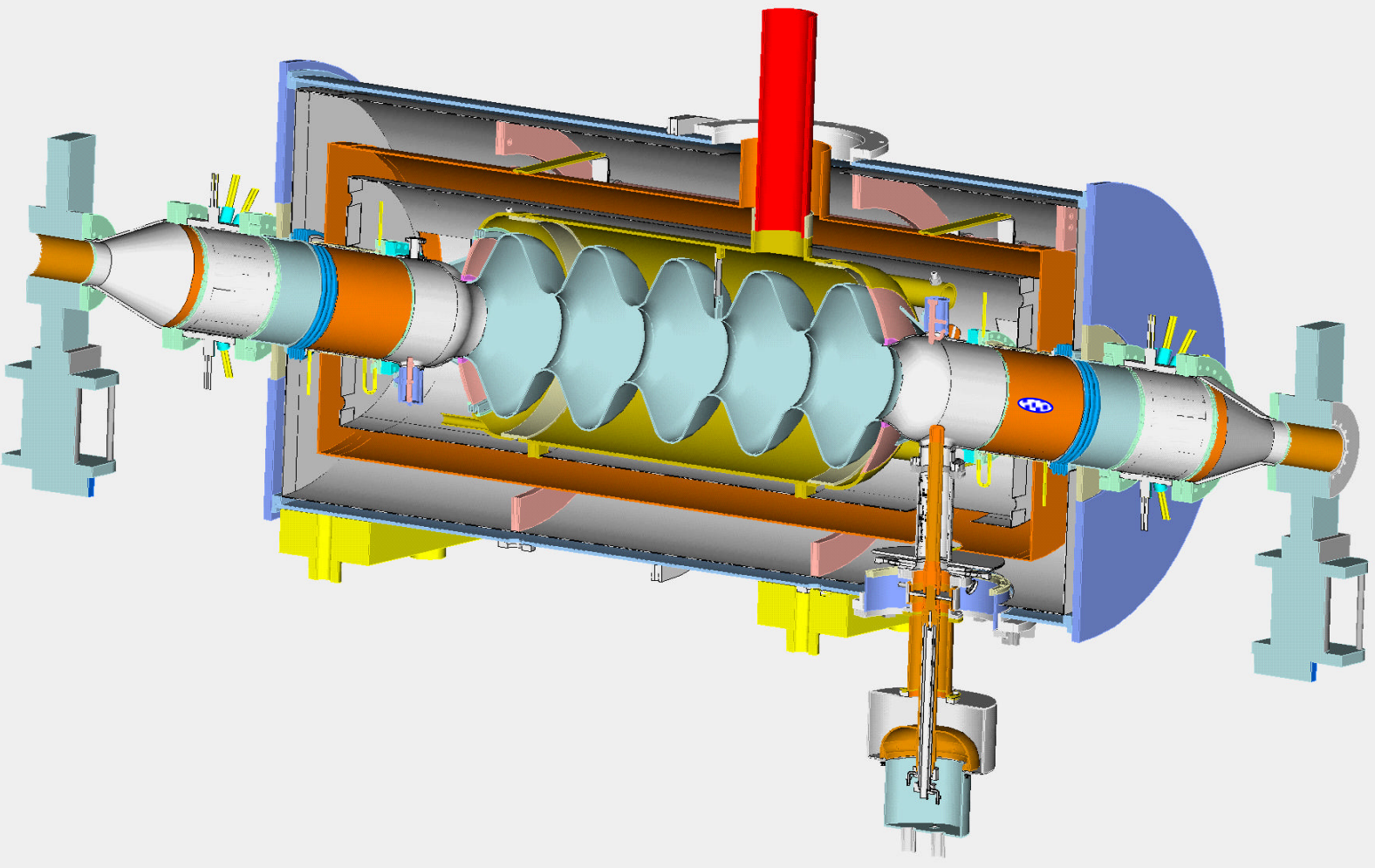
How $\beta=1$ RF accelerator works? In pictures



How $\beta=1$ RF accelerator works? In pictures



How $\beta=1$ RF accelerator works? In pictures



Multi-cell cavities

- We learned so far that single cell RF cavity has limited accelerating voltage

$$\text{Max}(V_{RF}) = \frac{E_0 \lambda_{RF}}{\pi}$$

- To gain more energy we can either use more individual cells or use multi-cell cavities
- The first path, while feasible, is expensive (each cavity would need individual transmitter, waveguide, controls, etc.) and less effective – the average accelerating gradient (energy gain per meter of real estate) would be low
- Thus, where the acceleration gradient is important, the accelerator community uses multi-cell cavities



9-cell Tesla design



7-cell



5-cell



What we learned

- Resonant modes in a cavity resonator belong to two families: TE and TM.
- There is an infinite number of resonant modes.
- The lowest frequency TM mode is usually used for acceleration.
- All other modes (HOMs) are considered parasitic as they can harm the beam.
- Several figures of merits are used to characterize accelerating cavities: main are accelerating voltage, transit time and Q-factor.
- In a multi-cell cavity every mode splits into a pass-band.
- The number of modes in each pass-band is equal to the number of cavity cells.
- The width of the pass-band is determined by the cell-to-cell coupling.
- Accelerating cavities operate at frequency below the cut-off frequency of vacuum pipes connected to them. The RF field decay exponentially along the pipes and reduces to a negligible level at length \sim few beam-pipe radii (assuming $R \ll \lambda_{\text{RF}}$)
- Coaxial lines and rectangular waveguides are commonly used in RF systems for power delivery to cavities
- Homework is posted on the website this evening: due in one week