PHY 554. Homework 8.

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Electronic copies accepted at *vladimir.litvinenko@stonybrook.edu*

HW 1, 3 points. Using representation of transport matrices using β-functions from Lecture, and a weak quadrupole error:

$$M_o(s_1|s_2) = \begin{bmatrix} \sqrt{\frac{\beta_2}{\beta_1}}\left(\cos\Delta\psi_{12} + \alpha_1 \sin\Delta\psi_{12}\right) & \sqrt{\beta_1\beta_2}\sin\Delta\psi_{12} \\ -\frac{1+\alpha_1\alpha_2}{\sqrt{\beta_1\beta_2}}\sin\Delta\psi_{12} - \frac{\alpha_1-\alpha_2}{\sqrt{\beta_1\beta_2}}\cos\Delta\psi_{12} & \sqrt{\frac{\beta_1}{\beta_2}}\left(\cos\Delta\psi_{12} - \alpha_2\sin\Delta\psi_{12}\right) \end{bmatrix};$$

$$\Delta\psi_{12} = \psi_2 - \psi_1; M_\delta(s_1) = \begin{bmatrix} 1 & 0 \\ -k(s_1)ds & 1 \end{bmatrix};$$

$$M(s_2|s_2+C) = M_o(s_1|s_2+C)M_\delta(s_1)M_o(s_2|s_1); \beta_i \equiv \beta_o(s_i); \psi_i \equiv \psi_o(s_i) = \nu\phi_o(s_i);$$

$$\delta M_{12}(s_2|s_2+C) = M(s_2|s_2+C) - M_o(s_2|s_2+C);$$

prove the modification of the transport matrix element M_{12} is indeed what we used in Lecture 8:

$$\delta M_{12}(s_2|s_2+C) = -\beta_1\beta_2 k(s_1)ds\cdot\sin(\psi_1-\psi_2)\cdot\sin(\mu_o-\psi_1+\psi_2)$$

$$= \frac{1}{2}\beta_1\beta_2 k(s_1)ds\cdot\left[\cos\mu_o - \cos\left(\mu_o - 2(\psi_1-\psi_2)\right)\right]$$

HW2: 3 points. Prove that relative value of β-beat has the forced socillator equation with doubel betatron frequency:

$$f(s) = \frac{\delta\beta(s)}{\beta_o(s)} = -\frac{1}{2\sin\mu_o}\int_{\psi(s)}^{\psi(s)+\mu}\beta_o^2(z)k(z)\cdot\cos\left(\mu_o + 2(\psi-\varphi)\right)d\varphi; d\varphi = \frac{ds}{\beta_o};$$

$$\frac{d^2}{d\psi^2}f(s) + 4f(s) = -2\beta_o^2(s)k(s).$$

HW3, 4 points: Prove that it is impossible to compensate both horizontal and verticla chromaticity in a storage ring with uniform weak focusing.

Hints:

(a) Use the fact that β-functions are constants;
(b) Prove that both natural chromaticities are negative;
(c) Show hat dispersion fnction is constant and positive;
(d) Use this fact to show that sextupoles have equal opposite effect on tow chromaticities independently of locaion