

1908 **Chapter 9**
1909 **Weak Focusing Synchrotron**

1910 **Abstract** This Chapter introduces to the weak focusing synchrotron, and to the the-
1911 oretical material needed for the simulation exercises. It begins with a brief reminder
1912 of the historical context, and continues with beam optics and acceleration techniques
1913 which the weak synchrotron principle and methods lean on. Regarding the latter, it
1914 relies on basic charged particle optics and acceleration concepts introduced in the
1915 previous Chapters, and further addresses the following aspects:

- 1916 - fixed closed orbit,
- 1917 - periodic structure,
- 1918 - periodic motion stability,
- 1919 - optical functions,
- 1920 - synchrotron motion,
- 1921 - depolarizing resonances.

1922 The simulation of weak synchrotrons only require a very limited number of optical
1923 elements; actually two are enough: DIPOLE or BEND to simulate combined function
1924 dipoles, and DRIFT to simulate straight section. A third one CAVITE, is required
1925 for acceleration. Particle monitoring requires keywords introduced in the previous
1926 Chapters, including FAISCEAU, FAISTORE, possibly PICKUPS, and some others.
1927 Spin motion computation and monitoring resort to SPNTRK, SPNPRT, FAISTORE.
1928 Optics matching and optimization use FIT[2]. SYSTEM again is used to shorten the
1929 input data files.

1930 **Notations used in the Text**

| | |
|---------------------------------------|---|
| $B; \mathbf{B}, B_{x,y,s}$ | field value; field vector, its components in the moving frame |
| $B\rho = p/q; B\rho_0$ | particle rigidity; reference rigidity |
| $C; C_0$ | orbit length, $C = 2\pi R + \left[\begin{array}{l} \text{straight} \\ \text{sections} \end{array} \right]$; reference, $C_0 = C(p = p_0)$ |
| E | particle energy |
| EFB | Effective Field Boundary |
| $f_{\text{rev}}, f_{\text{rf}}$ | revolution and accelerating voltage frequencies |
| h | RF harmonic number, $h = f_{\text{rf}}/f_{\text{rev}}$ |
| $m; m_0; M$ | mass, $m = \gamma m_0$; rest mass; in units of MeV/c ² |
| $n = \frac{\rho}{B} \frac{dB}{d\rho}$ | focusing index |
| $\mathbf{p}; p; p_0$ | momentum vector; its modulus; reference |
| P_i, P_f | polarization, initial, final |
| q | particle charge |
| r, R | orbital radius ; average radius, $R = C/2\pi$ |
| s | path variable |
| v | particle velocity |
| $V(t); \hat{V}$ | oscillating voltage; its peak value |
| x, x', y, y' | horizontal and vertical coordinates in the moving frame |
| α | momentum compaction |
| α | trajectory angle |
| $\beta = v/c; \beta_0; \beta_s$ | normalized particle velocity; reference; synchronous |
| β_u | betatron functions ($u : x, y, Y, Z$) |
| $\gamma = E/m_0$ | Lorentz relativistic factor |
| δp | momentum offset or Dirac distribution |
| Δp | momentum offset |
| ε | wedge angle |
| ε_u | Courant-Snyder invariant ($u : x, r, y, l, Y, Z, s$, etc.) |
| ε_R | strength of a depolarizing resonance |
| μ_u | betatron phase advance, $\mu_u = \int_{\text{period}} ds/\beta_u(s)$ ($u : x, y, Y, Z$) |
| ν_u | wave number or “tune”, radial, vertical, synchrotron ($u : x, y, Y, Z, l$) |
| ρ, ρ_0 | curvature radius; reference |
| σ | beam matrix |
| $\phi; \phi_s$ | particle phase at voltage gap; synchronous phase |
| ϕ_u | betatron phase advance, $\phi_u = \int ds/\beta_u$ ($u : x, y, Y, or Z$) |
| φ | spin angle to the vertical axis |

1932 **Introduction**

1933 The synchrotron is an outcome of the mid-1940s longitudinal phase focusing syn-
 1934 chronous acceleration concept [1, 2]. In its early version, transverse beam stability

1935 in the synchrotron during the thousands of turns that the acceleration lasts was based
 1936 on the technique known at the time: weak focusing, as in the cyclotron and in the be-
 1937 tatron. An existing betatron was used to first demonstrate phase-stable synchronous
 1938 acceleration with slow variation of the magnetic field, on a fixed orbit, in 1946 [3],
 1939 - closely following the demonstration of the principle of phase focusing using a
 1940 fixed-field cyclotron [4].

1941 Phase focusing states that stability of the longitudinal motion, longitudinal focus-
 1942 ing, is obtained if particles in a bunch, which have a natural energy spread, arrive
 1943 at the accelerating gap in the vicinity of a proper phase of the oscillating voltage,
 1944 the synchronous phase; if this condition is fulfilled the bunch stays together, in the
 1945 vicinity of the latter, during acceleration. Synchrotrons operate in general in a non-
 1946 isochronous regime: the revolution period changes with energy; as a consequence,
 1947 in order to maintain an accelerated bunch on the synchronous phase, the RF voltage
 1948 frequency, which satisfies $f_{rf} = hf_{rev}$, has to change continuously from injection to
 1949 top energy. The reference orbit in a synchrotron is maintained at constant radius by
 1950 ramping the guiding field in the main dipoles in synchronism with the acceleration,
 1951 as in the betatron [5].

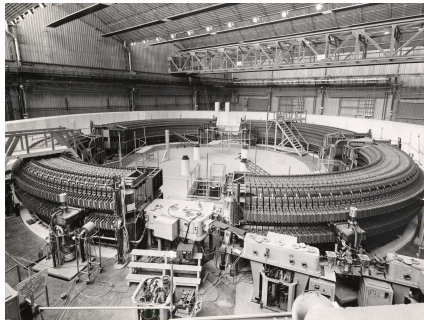


Fig. 9.1 SATURNE I at Saclay [6], a 3 GeV, 4-period, 68.9 m circumference, weak focusing synchrotron, constructed in 1956-58. The injection line can be seen in the foreground, injection is from a 3.6 MeV Van de Graaff (not visible)

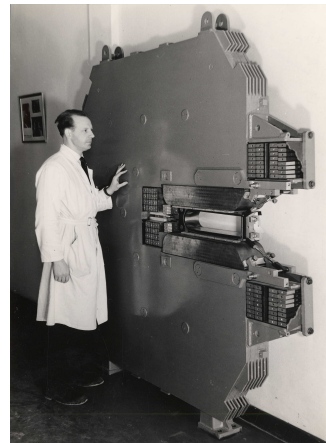


Fig. 9.2 A slice of SATURNE I dipole [6]. The slight gap tapering is hardly visible (increasing outward), it determines the weak index condition $0 < n < 1$

1952 The synchrotron concept allowed the highest energy reach by particle accelerators
 1953 at the time, it led to the construction of a series of proton rings with increasing
 1954 energy [7]: 1 GeV at Birmingham (1953), 3.3 GeV at the Cosmotron (Brookhaven
 1955 National Laboratory, 1953-1969), 6.2 GeV at the Bevatron (Berkeley, 1954-1993),
 1956 10 GeV at the Synchro-Phasotron (JINR, Dubna, 1957-2003), and a few additional
 1957 ones in the late 1950s well into the era of the concept which would essentially
 1958 dethrone the weak focusing method and its quite bulky rings of magnets which were

1959 a practical limit to further increase in energy¹: the strong focusing synchrotron (the
 1960 object of Chapter 10). The general layout of these first weak focusing synchrotrons
 1961 included straight sections (often 4, Fig. 9.1), which allowed insertion of injection
 1962 (Fig. 9.1) and extraction systems, accelerating cavities, orbit correction and beam
 1963 monitoring equipment.

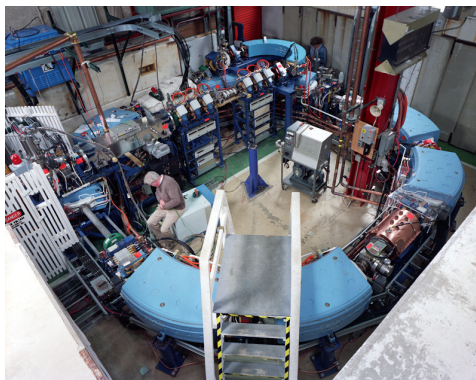


Fig. 9.3 Left: Loma Linda University medical synchrotron [8], during commissioning in 1989 at the Fermilab National Laboratory where it was designed

1964 The next decades following the invention of the synchrotron saw applications in
 1965 many fields of science including fixed-target nuclear physics for particle discovery,
 1966 material science, medicine, industry. Its technological simplicity still makes it an
 1967 appropriate technology today in low energy beam application when relatively low
 1968 current is not a concern, as in the hadrontherapy application (Fig. 9.3) [9, 10]: it
 1969 essentially requires a single type of a simple dipole magnet, an accelerating gap, some
 1970 command-control instrumentation, whereas it procures greater beam manipulation
 1971 flexibilities compared to (synchro-)cyclotrons.

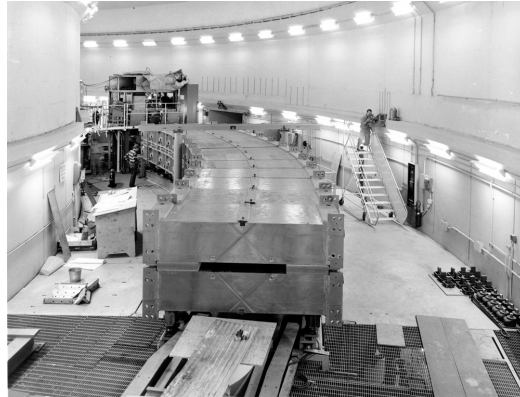
1972 *Polarized beams*

1973 The availability of polarized proton sources allowed the acceleration of polarized
 1974 beams to high energy. The possibility was considered from the early times at Argonne
 1975 ZGS (Zero-Gradient Synchrotron), a 12 GeV weak focusing synchrotron operated
 1976 over 1964-1979 [11] (Fig. 9.4). Up to 70% polarization transmission through the syn-
 1977 chrotron was achieved, for the first time in a synchrotron² and reaching multi-GeV
 1978 energy in 1973, up to 17.5 GeV/c with *appreciable polarizations* [12]. Polariza-
 1979 tion preservation techniques included harmonic orbit correction and fast betatron
 1980 tune jump at strongest depolarizing resonances [13] (Fig. 9.16). Experiments were
 1981 performed to assess the possibility of polarization transmission in strong focusing

¹ The story has it that it is possible to ride a bicycle in the vacuum chamber of Dubna's Synchro-Phasotron.

² Polarized beam had been accelerated in cyclotrons, at earlier times.

Fig. 9.4 The ZGS at Argonne during construction. A 12 GeV, 8-dipole, 4-period, 172 m circumference, wedge focusing synchrotron. The two persons inside and outside the ring, in the background, give an idea of the size of the magnets



1982 synchrotrons, and polarization lifetime in colliders [14]. Acceleration of polarized
1983 deuteron was achieved in the late 1970s, when sources were made available [15].

1984 9.1 Basic Concepts and Formulae

1985 The synchrotron is based on two key principles. On the one hand, a slowly varying
1986 magnetic field to maintain a constant orbit during acceleration,

$$B(t) \times \rho = p(t)/q, \quad \rho = \text{constant}, \quad (9.1)$$

1987 with $p(t)$ the particle momentum and ρ the bending radius in the dipoles. On the other
1988 hand, on synchronous acceleration for longitudinal phase stability. In a regime where
1989 the velocity change with energy cannot be ignored (non-ultrarelativistic particles),
1990 the latter requires a modulation of the accelerating voltage frequency so to satisfy

$$f_{\text{rf}}(t) = h f_{\text{rev}}(t) \quad (9.2)$$

1991 Synchronism between accelerating voltage oscillation and the revolution motion
1992 keeps the bunch on the synchronous phase at traversal of the accelerating gaps.
1993 Synchronous acceleration is technologically simpler in the case of electrons, as
1994 frequency modulation is unnecessary beyond a few MeV; for instance, from $v/c =$
1995 0.9987 at 10 MeV to $v/c \rightarrow 1$ the relative change in revolution frequency amounts
1996 to $\delta f_{\text{rev}}/f_{\text{rev}} = \delta\beta/\beta < 0.0013$.

1997 These are two major evolutions compared to the cyclotron, where, instead, the
1998 magnetic field is fixed - the reference orbit spirals out, and, by virtue of the isochro-
1999 nism of the orbits, the oscillating voltage frequency is fixed as well.

2000 A fixed orbit reduces the radial extent of individual guiding magnets, allowing a
2001 ring structure comprised of a circular string of dipoles. For the sake of comparison:
2002 a synchrocyclotron instead uses a single, massive dipole; increased energy requires

2003 increased radial extent of the magnet to allow for the greater bending field integral
 2004 (*i.e.*, $\oint B dl = 2\pi R_{max} \hat{B} = p_{max}/q$), thus a volume of iron increasing more than
 2005 quadratically with bunch rigidity.

2006 One or the other of the weak index ($-1 < k < 0$, Sect. 4.2.2) and/or wedge
 2007 focusing (Sect. 15.3.1) are used in weak focusing synchrotrons. Transverse stability
 2008 was based on the latter at Argonne ZGS (Zero-Gradient Synchrotron: the main
 2009 magnet had no field index); ZGS accelerated polarized proton beams, weak focusing
 2010 resulted in weak depolarizing resonances, an advantage in that matter [14].

2011 Due to the necessary ramping of the field, and of the RF frequency to follow,
 2012 in order to maintain a constant orbit, the synchrotron is a pulsed accelerator, the
 2013 acceleration is cycled, from injection to top energy, repeatedly. The repetition rate
 2014 of the acceleration cycle depends on the type of power supply. If the ramping uses a
 2015 constant electromotive force ($E=V+ZI$ is constant), then

$$B(t) \propto (1 - e^{-\frac{t}{\tau}}) = 1 - \left[1 - \left(\frac{t}{\tau}\right) + \left(\frac{t}{\tau}\right)^2 - \dots \right] \approx \frac{t}{\tau} \quad (9.3)$$

2016 essentially linear; $\dot{B} = dB/dt$ does not exceed a few Tesla/second: the repetition rate
 2017 of the acceleration cycle if of the order of a Hertz. If instead the magnet winding
 2018 is part of a resonant circuit then the field oscillates from an injection threshold to a
 2019 maximum value, $B(t) : B_0 \rightarrow B_0 + \hat{B}$, as in the betatron; the repetition rate is up to
 2020 a few tens of Hertz. In both cases anyway B imposes its law and the other quantities
 2021 comprising the acceleration cycle (RF frequency in particular) will follow B(t).

2022 For the sake of comparison: in a synchrocyclotron the field is constant, thus
 2023 acceleration can be cycled as fast as the swing of the voltage frequency allows
 2024 (hundreds of Hz are common practice); assume a conservative 10 kVolts per turn,
 2025 thus of the order of 10,000 turns to 100 MeV, with velocity $0.046 < v/c < 0.43$
 2026 from 1 to 100 MeV, proton. Take $v \approx 0.5c$ to make it simple, an orbit circumference
 2027 below 30 meter, thus the acceleration takes of the order of $10^4 \times C/0.5c \approx \text{ms}$ range,
 2028 potentially a repetition rate in kHz range, more than an order of magnitude beyond
 2029 the reach of a rapid-cycling pulsed synchrotron.

2030 9.1.1 Periodic Stability

2031 This section introduces the various components of the transverse focusing and the
 2032 conditions for periodic stability in a weak focusing synchrotron. It builds on material
 2033 introduced in Chap. 4, Classical Cyclotron.

2034 9.1.1.1 Closed orbit

2035 The concept is found in the betatron, which accelerates particles on a constant orbit
 2036 (Chap. 7). The closed orbit is fixed, and maintained during acceleration by ensuring

2037 that the relationship Eq. 9.1 is satisfied. In a perfect ring, the closed orbit is along an
 2038 arc in the bending magnets and straight along the drifts, Fig. 9.5.

2039 Particle motion is defined in a moving frame (O;s,x,y) whose origin coincides
 2040 with the location of an ideal particle following the reference orbit. The moving frame
 2041 s axis is tangent to the reference orbit, its transverse horizontal axis x is normal to
 2042 the s axis, its vertical axis y is normal to the (s, x) plane (Fig. 4.8, Sect. 4.2.2).

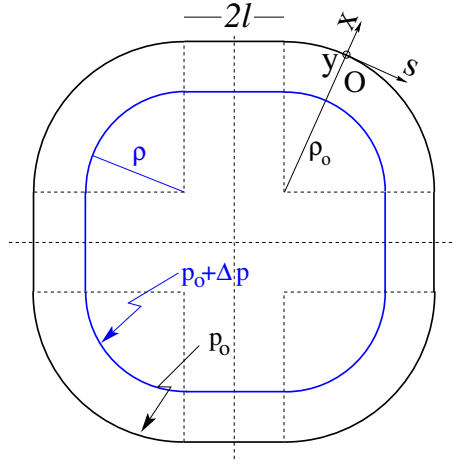


Fig. 9.5 A $2\pi/4$ axially symmetric structure with four drift spaces. Orbit length on reference momentum p_0 is $C = 2\pi\rho_0 + 8l$. (O;s,x,y) is the moving frame, along the reference orbit. The orbit for momentum $p = p_0 + \Delta p$ ($\Delta p < 0$, here) is at constant distance $\Delta x = \frac{\rho_0}{1-n} \frac{\Delta p}{p_0} = \frac{R}{(1+k)(1-n)} \frac{\Delta p}{p_0}$ from the reference orbit

2043 **9.1.1.2 Transverse Focusing**

2044 Radial motion stability around a reference closed orbit in an axially symmetric dipole
 2045 field requires a field index (Sect. 4.2.2),

$$n = -\frac{\rho_0}{B_0} \left. \frac{\partial B_y}{\partial x} \right|_{x=0, y=0} \quad (9.4)$$

2046 a quantity evaluated on the reference arc in the dipoles, satisfying the weak focusing
 2047 condition (Eq. 4.11 with $n = -k$)

$$0 < n < 1 \quad (9.5)$$

2048 This condition can be obtained with a tapered gap (as in SATURNE dipoles, Fig. 9.2)
 2049 causing the magnetic field to decrease slowly with radius, so resulting in both axial
 2050 and radial focusing (Figs. 9.6, 9.7). Note the sign convention here, the cyclotron uses
 2051 the opposite sign (Eq. 4.10). This condition holds regardless of the presence of drifts
 2052 or not. Adding drift spaces between the dipoles, the reference orbit is comprised of
 2053 arcs of radius ρ_0 in the magnets, and straight segments along the drift spaces that
 2054 connect these arcs. This requires defining two radii, namely,

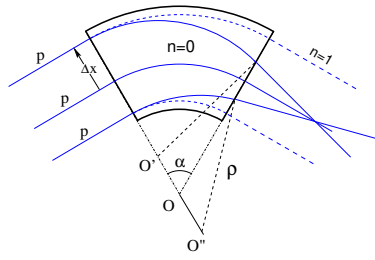


Fig. 9.6 Geometrical focusing: in a sector dipole with focusing index $n = 0$, parallel incoming rays of equal momenta experience the same curvature radius ρ , their trajectories converge as outer trajectories have a longer path in the field, inner ones shorter. An index value $n=1$ cancels that effect: parallel incoming rays exit parallel

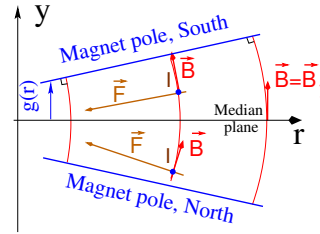


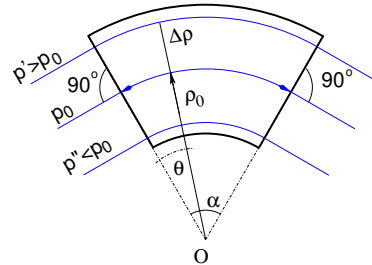
Fig. 9.7 Axial motion stability requires proper shaping of field lines: B_y has to decrease with radius. The Laplace force pulls a positive charge with velocity pointing out of the page, at I, toward the median plane. Increasing the field gradient (n closer to 1, gap opening up faster) increases the focusing

- 2055 (i) the magnet curvature radius ρ_0 ,
 2056 (ii) an average radius $R = C/2\pi = \rho_0 + Nl/\pi$ (with C the length of the reference
 2057 closed orbit and $2l$ the drift length) (Fig. 9.5) which also writes

$$R = \rho_0(1 + k), \quad k = \frac{Nl}{\pi\rho_0} \quad (9.6)$$

2058 Adding drift spaces decreases the average focusing around the ring.

Fig. 9.8 In a sector dipole with radial index $n \neq 0$, closed orbits follow arcs of constant field. A closed orbit at $p_0 + \Delta p$ follows an arc of radius $\rho_0 + \Delta\rho$, $\Delta\rho = \Delta p / (1 + n)qB_0$



2059 Geometrical focusing

2060 The limit $n \rightarrow 1$ of the transverse motion stability domain corresponds to a cancel-
 2061 lation of the geometrical focusing (Fig. 9.6): in a constant field dipole (radial field
 2062 index $n=0$) the longer (respectively shorter) path in the magnetic field for parallel
 2063 trajectories entering the magnet at greater (respectively smaller) radius result in
 2064 convergence. This effect is cancelled, *i.e.*, trajectory angle is the same whatever the

2065 entrance radius, if the curvature center is made independent of the entrance radius:
 2066 $OO' = 0, O''O = 0$. This occurs if trajectories at an outer (inner) radius experience a
 2067 smaller (greater) field such as to satisfy $BL = B\rho\alpha = C^{st}$. Differentiating $B\rho = C^{st}$
 2068 gives $\frac{\Delta B}{B} + \frac{\Delta\rho}{\rho} = 0$, with $\Delta\rho = \Delta x$, so yielding $n = -\frac{\rho_0}{B_0} \frac{\Delta B}{\Delta x} = 1$. The focal distance
 2069 associated with the curvature is (Eq. 4.12 with $R = \rho_0$) $f = \frac{\rho_0^2}{L}$. Optical drawbacks
 2070 of the weak focusing method include the weakness of the focusing and the absence
 2071 of independent radial and axial focusing.

2072 *Wedge Focusing*

2073 Entrance and exit wedge angles may be used to ensure transverse focusing, Fig. 9.9:
 2074 opening the magnetic sector increases the horizontal focusing (and decreases the
 2075 vertical focusing); closing the magnetic sector has the reverse effect (see Sect. 15.3.1).

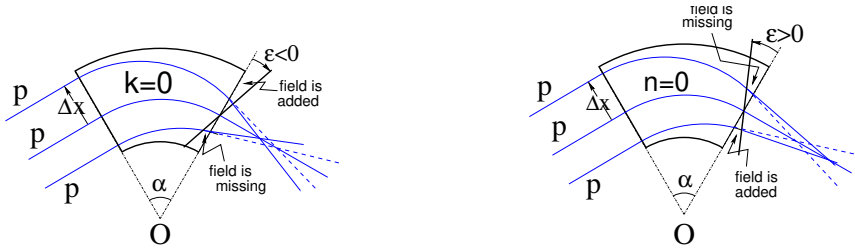


Fig. 9.9 Left: a focusing wedge ($\varepsilon < 0$); opening the sector increases horizontal focusing and decreases vertical focusing. Right: a defocusing wedge ($\varepsilon > 0$), closing the sector, has the reverse effect. This is the origin of the focusing in the ZGS zero-gradient dipoles

2076 In a point transform approximation, at the wedge the trajectory undergoes a local
 2077 deviation proportional to the distance to the optical axis, amounting to

$$\Delta x' = \frac{\tan \varepsilon}{\rho_0} \Delta x, \quad \Delta y' = -\frac{\tan(\varepsilon - \psi)}{\rho_0} \Delta y \quad (9.7)$$

2078 The ψ angle component is a correction for the fringe field extent (Eq. 15.21); the
 2079 effect of the latter, of the first order on the vertical focusing, is of second order
 2080 horizontally.

2081 Profiling the magnet gap in order to adjust the focal distance complicates the
 2082 magnet; a parallel gap, $n = 0$, makes it simpler, for that reason edge focusing may
 2083 be preferred. Wedge vertical focusing in the ZGS ($\varepsilon > 0$) was at the expense of
 2084 horizontal geometrical focusing (Fig. 9.6). This was an advantage though, for the
 2085 acceleration of polarized beams, as radial field components (which are responsible for
 2086 depolarization) were only met at the EFBs of the eight main dipoles, and weak [12].
 2087 Preserving beam polarization at high energy required tight control of the tunes, this

2088 was achieved by pole face windings added at the ends of the dipoles [17, 18], pulsed
2089 to control the amplitude detuning, resulting in a control of the tunes at 0.01 level.

2090 9.1.1.3 Betatron motion

2091 The first order differential equations of motion in the moving frame (Fig. 9.5) derive
2092 from the Lorentz equation

$$\frac{d\mathbf{mv}}{dt} = q\mathbf{v} \times \mathbf{B} \Rightarrow m \frac{d}{dt} \begin{pmatrix} \frac{ds}{dt} \mathbf{s} \\ \frac{dx}{dt} \mathbf{x} \\ \frac{dy}{dt} \mathbf{y} \end{pmatrix} = q \begin{pmatrix} \left(\frac{dx}{dt} B_y - \frac{dy}{dt} B_x \right) \mathbf{s} \\ -\frac{ds}{dt} B_y \mathbf{x} \\ \frac{ds}{dt} B_x \mathbf{y} \end{pmatrix} \quad (9.8)$$

2093 Motion in a weak index dipole field is solved in Sect. 4.2.2, Classical Cyclotron: in
2094 the latter substitute ρ to R , $n = -\frac{\rho_0}{B_0} \frac{\partial B_y}{\partial x}$ to $-k$, evaluated on the reference orbit.
2095 Taylor expansions of the transverse field components in the moving frame (Eq. 4.6)
2096 lead to

$$\begin{aligned} B_y(\rho)|_{y=0} &= B_0(1 - n \frac{x}{\rho_0}) + \mathcal{O}(x^2) \\ B_x(0+y) &= -n \frac{B_0}{\rho_0} y + \mathcal{O}(y^3) \end{aligned} \quad (9.9)$$

2097 Assume transverse stability: $0 < n < 1$; in the approximation $ds \approx vdt$ (Eq. 4.13)
2098 Eqs. 9.8, 9.9 lead to the differential equations of motion

$$\frac{d^2x}{ds^2} + \frac{1-n}{\rho_0^2} x = 0, \quad \frac{d^2y}{ds^2} + \frac{n}{\rho_0^2} y = 0 \quad (9.10)$$

2099 It results that, in an S-periodic structure comprised of gradient dipoles, wedges
2100 and drift spaces, the differential equation of motion takes the general form of Hill's
2101 equation, a second order differential equation with periodic coefficient, namely (with
2102 u standing for x or y),

$$\begin{cases} \frac{d^2u}{ds^2} + K_u(s)u = 0 \\ K_u(s+S) = K_u(s) \end{cases} \quad \text{with} \quad \begin{cases} \text{in dipoles : } \begin{cases} K_x = \frac{1-n}{\rho_0^2} \\ K_y = \frac{n}{\rho_0^2} \end{cases} \\ \text{at a wedge at } s = s_0 : K_x = \frac{\pm \tan \varepsilon}{\rho_0} \delta(s - s_0) \\ \text{in drift spaces : } \frac{1}{\rho_0} = 0, K_x = K_y = 0 \end{cases} \quad (9.11)$$

2103 $K_u(s)$ is S-periodic, $S = 2\pi R/N$ ($S = C/4$ for instance in a 4-periodic ring,
2104 Figs. 9.1, 9.5).

2105 The solution of Eqs. 9.11 is not as straightforward as in the cyclotron where K_u is
2106 constant around the ring (Eq. 4.14), which results in a sinusoidal motion (Eq. 4.16)
2107 - the latter is on the other hand a reasonable approximation, see below, *Weak focusing*
2108 *approximation*. G. Floquet has established [19] that the two independent solutions
2109 of Hill's second order differential equation have the form [16]

$$\left\{ \begin{array}{l} u_1(s) = \sqrt{\beta_u(s)} e^{i \int_0^s \frac{ds}{\beta_u(s)}} \\ du_1(s)/ds = \frac{i - \alpha_u(s)}{\beta_u(s)} u_1(s) \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} u_2(s) = u_1^*(s) \\ du_2(s)/ds = du_1^*(s)/ds \end{array} \right. \quad (9.12)$$

2110 wherein $\beta_u(s)$ and $\alpha_u(s) = -\beta'_u(s)/2$ are S-periodic functions, from what it results
2111 that

$$u_{\frac{1}{2}}(s+S) = u_{\frac{1}{2}}(s) e^{\pm i \mu_u} \quad (9.13)$$

2112 wherein

$$\mu_u = \int_{s_0}^s \frac{ds}{\beta_u(s)} \quad (9.14)$$

2113 is the betatron phase advance at s , from the origin s_0 . A real solution of Hill's
2114 equation is the linear combination $A u_1(s) + A^* u_2^*(s)$. With $A = \frac{1}{2} \sqrt{\varepsilon_u/\pi} e^{i\phi}$
2115 following conventional notations, ϕ the phase of the motion at the origin $s = s_0$, the
2116 general solution of Eq. 9.11 writes

$$\left\{ \begin{array}{l} u(s) = \sqrt{\beta_u(s) \varepsilon_u/\pi} \cos\left(\int_{s_0}^s \frac{ds}{\beta_u} + \phi\right) \\ u'(s) = -\sqrt{\frac{\varepsilon_u/\pi}{\beta_u(s)}} \sin\left(\int_{s_0}^s \frac{ds}{\beta_u} + \phi\right) + \alpha_u(s) \cos\left(\int_{s_0}^s \frac{ds}{\beta_u} + \phi\right) \end{array} \right. \quad (9.15)$$

2117 An invariant of the motion, known as the Courant-Snyder invariant, is

$$\frac{1}{\beta_u(s)} \left[u^2 + (\alpha_u(s)u + \beta_u(s)u')^2 \right] = \frac{\varepsilon_u}{\pi} \quad (9.16)$$

2118 At a given azimuth s of the periodic structure the observed turn-by-turn motion
2119 lies on that ellipse (Fig. 9.10). The form and inclination of the ellipse depend on
2120 the observation azimuth s via the respective local values of $\alpha_u(s)$ and $\beta_u(s)$, but
2121 its surface ε_u is invariant. Motion along the ellipse is clockwise, as can be figured
2122 from Eq. 9.15 considering an observation azimuth s where the ellipse is upright,
2123 $\alpha_u(s) = 0$. The phase advance over a turn (from one position to the next on the
2124 ellipse, Fig. 9.10) in an N-periodic ring yields the wave number

$$\nu_u = N \mu_u = \int_{s_0}^{s_0+NS} \frac{ds}{\beta_u(s)} = N \int_{\text{period}} \frac{ds}{\beta_u(s)} \quad (9.17)$$

2125 *Weak focusing approximation*

2126 In a cylindrically symmetric structure a sinusoidal motion is the exact solution of the
2127 first order differential equations of motion (Eqs. 4.15, 4.16, Classical Cyclotron Chap-
2128 ter), the coefficients $K_x = (1-n)/R_0^2$ and $K_y = n/R_0^2$ are constant (s-independent).
2129 Adding drift spaces results in Hill's differential equation with periodic coefficient
2130 $K(s+S) = K(s)$ (Eq. 9.11), and in a pseudo harmonic solution (Eq. 9.15). Due to the

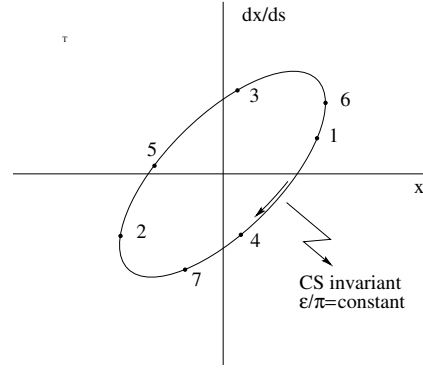


Fig. 9.10 Courant-Snyder invariant and turn-by-turn harmonic motion along the invariant, observed at some azimuth s . The form and tilt-angle of the ellipse depend on the observation azimuth s but its surface ε_u is invariant

2131 weak focusing the beam envelope is only weakly modulated (see below), thus so is
 2132 $\beta_u(s)$. In a practical manner, the modulation of $\beta_u(s)$ does not exceed a few percent,
 2133 this justifies introducing the average value $\overline{\beta_u}$ to approximate the phase advance by

$$\int_0^s \frac{ds}{\beta_u(s)} \approx \frac{s}{\overline{\beta_u}} = \nu_u \frac{s}{R} \quad (9.18)$$

2134 The right equality is obtained by applying this approximation to the phase advance
 2135 per period, namely (Eq. 9.14) $\mu_u = \int_{s_0}^{s_0+S} \frac{ds}{\beta_u(s)} \approx S/\overline{\beta_u}$, and introducing the wave
 2136 number of the N-period optical structure $\nu_u = \frac{N\mu_u}{2\pi} = \frac{\text{phase advance over a turn}}{2\pi}$ so that

$$\overline{\beta_u} = \frac{R}{\nu_u} \quad (9.19)$$

2137 the wavelength of the betatron oscillation around the ring. With $k \ll 1$ and using
 2138 Eq. 9.23,

$$\overline{\beta_x} = \frac{\rho_0(1+k/2)}{\sqrt{1-n}}, \quad \overline{\beta_y} = \frac{\rho_0(1+k/2)}{\sqrt{n}} \quad (9.20)$$

2139 Substituting $\nu_u \frac{s}{R}$ to $\int \frac{ds}{\beta_u(s)}$ in Eq. 9.15 yields the approximate solution

$$\begin{cases} u(s) \approx \sqrt{\beta_u(s)\varepsilon_u/\pi} \cos\left(\nu_u \frac{s}{R} + \phi\right) \\ u'(s) \approx -\sqrt{\frac{\varepsilon_u/\pi}{\beta_u(s)}} \sin\left(\nu_u \frac{s}{R} + \phi\right) + \alpha_u(s) \cos\left(\nu_u \frac{s}{R} + \phi\right) \end{cases} \quad (9.21)$$

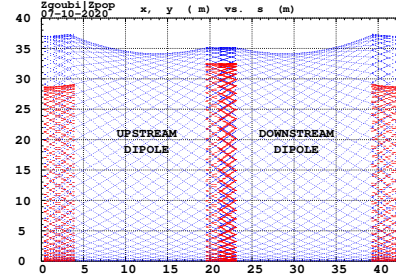
2140 *Beam envelopes*

2141 The beam envelope $\hat{u}(s)$ (with u standing for x or y) is determined by the particle of
 2142 maximum invariant ε_u/π , it is given at all s by

$$\hat{u}_{\text{env}}(s) = \pm \sqrt{\beta_u(s) \frac{\mathcal{E}_u}{\pi}} \quad (9.22)$$

As $\beta_u(s)$ is S -periodic, so is the envelope, $\hat{u}(s+S) = \hat{u}(s)$. In a cell with symmetries,

Fig. 9.11 Multi-turn particle excursion along the ZGS 2-dipole 43 m cell. The motion extrema (Eq. 9.22) tangent the envelopes, respectively horizontal (red), and vertical (blue). Envelopes have the symmetry of the cell



2143 beam envelopes feature the same symmetries, as in Fig. 9.11 for instance: a symmetry
 2144 with respect to the center of the cell; envelop extrema are at azimuth s of $\beta_u(s)$
 2145 extrema, *i.e.* where $d\hat{u}(s)/ds \propto \beta'_u(s) = 0$ or $\alpha_u = 0$ as $\beta'_u = -2\alpha_u$.
 2146

2147 Working point

2148 The “working point” of the synchrotron is the wave number couple (ν_x, ν_y) at which
 2149 the accelerator is operated, it fully characterizes the focusing. In a structure with
 2150 cylindrical symmetry (such as the Classical Cyclotron) $\nu_x = \sqrt{1-n}$ and $\nu_y = \sqrt{n}$
 2151 (Eq. 4.17) so that $\nu_x^2 + \nu_y^2 = 1$: when the radial field index n is changed the working
 2152 point stays on a circle of radius 1 in the stability diagram (or “tune diagram”,
 2153 Fig. 9.12). If drift spaces are added, from Eqs. 9.19, 9.20, with $1 + \frac{k}{2} \approx \sqrt{R/\rho_0}$
 2154 (Eq. 9.6), it comes

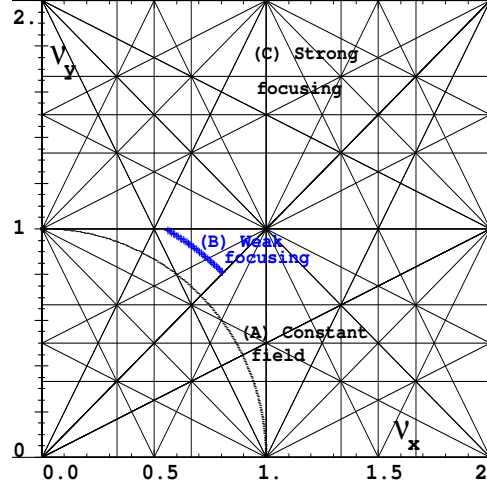
$$\nu_x \approx \sqrt{(1-n) \frac{R}{\rho_0}}, \quad \nu_y \approx \sqrt{n \frac{R}{\rho_0}}, \quad \nu_x^2 + \nu_y^2 \approx \frac{R}{\rho_0} \quad (9.23)$$

thus the working point is located on the circle of radius $\sqrt{R/\rho_0} > 1$ (Fig. 9.12).
 Tunes can not exceed the limits

$$0 < \nu_{x,y} \lesssim \sqrt{R/\rho_0}$$

2155 Horizontal and vertical focusing are not independent (Eq. 9.11): if ν_x increases then
 2156 ν_y decreases and reciprocally. This is a lack of flexibility which the advent of strong
 2157 focusing will overcome by providing two knobs allowing separate adjustment of the
 2158 tunes.

Fig. 9.12 Location of the working point in the tune diagram. (A) field with revolution symmetry: (ν_x, ν_y) is on a circle of radius 1; (B) sector field with field index $0 < n < 1$ and drift spaces: (ν_x, ν_y) is on a circle of radius $(\sqrt{R/\rho_0})$; (C) strong focusing, field index $|n| \gg 1$, ν_x and ν_y are independent and large



2159 Off-momentum orbits; periodic dispersion

2160 In the linear approximation in $\Delta p/p_0$, a momentum offset $\Delta p = p - p_0$ changes
 2161 mv to $mv(1 + \Delta p/p_0)$ in Eq. 9.8; this changes the horizontal equation of motion
 2162 (Eq. 9.10) to

$$\frac{d^2x}{ds^2} + K_x x = \frac{1}{\rho_0} \frac{\Delta p}{p_0}, \quad \text{or} \quad \frac{d^2x}{ds^2} + K_x \left(x - \frac{1}{\rho_0 K_x} \frac{\Delta p}{p_0} \right) = 0 \quad (9.24)$$

2163 A change of variable $x - \frac{1}{K_x \rho_0} \frac{\Delta p}{p_0} \rightarrow x$ (with $1/\rho_0 K_x = \rho_0/(1-n)$) restores the
 2164 unperturbed equation of motion; thus orbits of different momenta $p = p_0 + \Delta p$ are
 2165 distant

$$\Delta x = \frac{\rho_0}{1-n} \frac{\Delta p}{p_0} \quad (9.25)$$

2166 from the reference orbit (Fig. 9.8). Introduce the geometrical radius $R = (1+k)\rho_0$
 2167 (Eq. 9.6) to account for the added drifts; this yields the dispersion function

$$D_x = \frac{\Delta x}{\Delta p/p_0} \equiv \frac{\Delta R}{\Delta p/p_0} = \frac{R}{(1-n)(1+k)} = \frac{\rho_0}{1-n}, \quad \text{constant, positive} \quad (9.26)$$

2168 D_x is the chromatic dispersion of the orbits, an s -independent quantity: in a structure
 2169 with axial symmetry, comprising drift sections (Fig. 9.5) or not (classical and AVF
 2170 cyclotrons for instance), the ratio $\frac{\Delta x}{\rho_0 \Delta p/p_0}$ is independent of the azimuth s , the
 2171 distance of a chromatic orbit to the reference orbit is constant around the ring.

2172 Given that $n < 1$,

- 2173 - higher momentum orbits, $p > p_0$, have a greater radius,
- 2174 - lower momentum orbits, $p < p_0$, have a smaller radius.

2175 The horizontal motion of an off-momentum particle is a superposition of the
 2176 betatron motion (solution of Hill's Eq. 9.21 with $\delta p/p = 0$) and of a particular
 2177 solution of the inhomogeneous equation ($\delta p/p \neq 0$), namely

$$x(s) = \sqrt{\beta_u(s)\varepsilon_u/\pi} \cos\left(\nu_u \frac{s}{R} + \phi\right) + \frac{\rho_0}{1-n} \frac{\Delta p}{p_0} \quad (9.27)$$

2178 whereas the vertical motion is unchanged.

2179 *Chromatic orbit length*

2180 In an axially symmetric structure the difference in closed orbit length $\Delta C = 2\pi\Delta R$
 2181 resulting from the difference in momentum arises in the dipoles, as all orbits are
 2182 parallel in the drifts (Fig. 9.5). Hence, from Eq. 9.26, the relative closed orbit
 2183 lengthening factor, or momentum compaction

$$\alpha = \frac{\Delta C}{C} \Big/ \frac{\Delta p}{p_0} \equiv \frac{\Delta R}{R} \Big/ \frac{\Delta p}{p_0} = \frac{1}{(1-n)(1+k)} \approx \frac{1}{\nu_x^2} \quad (9.28)$$

2184 with $k = Nl/\pi\rho_0$ (Eq. 9.6). Note that the relationship $\alpha \approx 1/\nu_x^2$ between momentum
 2185 compaction and horizontal wave number established for a revolution symmetry
 2186 structure (Eq. 4.21) still holds when adding drifts.

2187 **9.1.2 Acceleration**

In a synchrotron, the field B is varied during acceleration (a function performed by the magnet power supply) concurrently with the variation of the bunch momentum p (a function performed by the accelerating cavity) in such a way that the beam is maintained on the design orbit. Given the energies involved, the magnet supply imposes its law $B(t)$ (Fig. 9.13) and the cavity follows, the best it can. The accelerating voltage $\hat{V}(t) = \sin \omega_{rf}t$ is maintained in synchronism with the revolution motion, its angular frequency satisfying

$$\omega_{rf} = h\omega_{rev} = h \frac{c}{R} \frac{B(t)}{\sqrt{\left(\frac{m_0}{q\rho}\right)^2 + B^2(t)}}$$

2188 *Energy gain*

2189 The variation of the particle energy over a turn amounts to the work of the force
 2190 $F = dp/dt = q\rho dB/dt$ on the charge at the cavity, namely

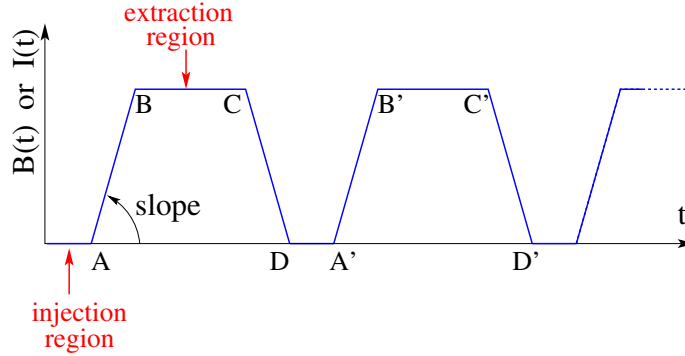


Fig. 9.13 Cycling $B(t)$ in a pulsed synchrotron. Ignoring saturation, $B(t)$ is proportional to the magnet power supply current $I(t)$. Beam injection occurs at low field, in the region of A, extraction occurs at top energy, on the high field plateau. (AB): field ramp up (acceleration); (BC): flat top; (CD): field ramp down; (DA'): thermal relaxation. (AA'): repetition period; $(1/AA')$: repetition rate; *slope*: ramp velocity $\dot{B} = dB/dt$ (Tesla/s).

$$\Delta W = F \times 2\pi R = 2\pi R q \rho \dot{B} \quad (9.29)$$

In a slow-cycling synchrotron \dot{B} is usually constant over most of the acceleration cycle (Eq. 9.3), thus so is ΔW . At SATURNE I for instance (the object of Exercise 9.1, parameters in Tab. 9.1)

$$\frac{\Delta W}{q} = 2\pi R \rho \dot{B} = 68.9 \times 8.42 \times 1.8 = 1044 \text{ volts}$$

The field ramp lasts

$$\Delta t = (B_{\max} - B_{\min})/\dot{B} \approx B_{\max}/\dot{B} = 0.8 \text{ s}$$

The number of turns to the top energy ($W_{\max} \approx 3 \text{ GeV}$) is

$$N = \frac{W_{\max}}{\Delta W} = \frac{3 \cdot 10^9 \text{ eV}}{1044 \text{ eV/turn}} \approx 3 \cdot 10^6 \text{ turns}$$

The dependence of particle mass on field writes

$$m(t) = \gamma(t)m_0 = \frac{q\rho}{c} \sqrt{\left(\frac{m_0}{qc\rho}\right)^2 + B(t)^2}$$

2191 *Adiabatic damping of the betatron oscillations*

The focusing index (Eq. 9.4) does not change during acceleration, thus the tunes ν_x and ν_y do not change either. As a result of the longitudinal acceleration at the cavity

though, the longitudinal energy of the particles is modified. This results in a decrease of the amplitude of betatron oscillations (an increase if the cavity is decelerating). The mechanism is sketched in Fig. 9.14: the slope, respectively before and after (index 2) the cavity is

$$\frac{dx}{ds} = \frac{m \frac{dx}{dt}}{m \frac{ds}{dt}} = \frac{p_x}{p_s}, \quad \frac{dx}{ds} \Big|_2 = \frac{m \frac{dx}{dt}}{m \frac{ds}{dt}} \Big|_2 = \frac{p_{x,2}}{p_{s,2}}$$

Particle mass and velocity are modified at the traversal of the cavity but, as the

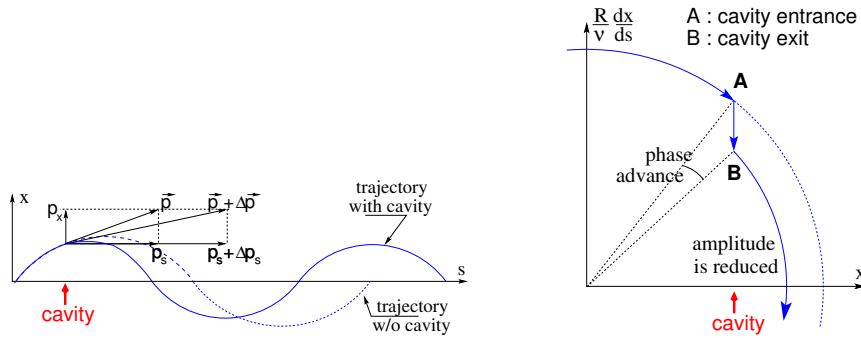


Fig. 9.14 Adiabatic damping of betatron oscillations, here from $x' = p_x/p_s$ before the cavity, to $x'_2 = p_x/(p_s + \Delta p_s)$ after the cavity. In the horizontal phase space, to the right, decrease of $\Delta \left(\frac{dx}{ds} \right)$ if $\frac{dx}{ds} > 0$, increase of $\Delta \left(\frac{dx}{ds} \right)$ if $\frac{dx}{ds} < 0$

force is longitudinal, $dp_x/dt = 0$ thus $p'_x = p_x$, the increase in momentum is purely longitudinal, $p'_s = p_s + \Delta p$. Thus

$$\frac{dx}{ds} \Big|_2 = \frac{p_x}{p_s + \Delta p} \approx \frac{p_x}{p_s} \left(1 - \frac{\Delta p}{p_s} \right)$$

and as a consequence the slope dx/ds varies across the cavity,

$$\Delta \left(\frac{dx}{ds} \right) = \frac{dx}{ds} \Big|_2 - \frac{dx}{ds} = - \frac{dx}{ds} \frac{\Delta p_s}{p_s}$$

- 2192 The variation of the slope is proportional to the slope, with opposite sign if $\Delta p/p > 0$
 2193 (acceleration) thus a decrease of the slope. This variation has two consequences on
 2194 the betatron oscillation (Fig. 9.14):
 2195 - a change of the betatron phase,
 2196 - a modification of the betatron amplitude.

2197 *Coordinate transport*

2198 at the cavity writes $\begin{cases} x_2 = x \\ x'_2 \approx \frac{p_x}{p_s} (1 - \frac{dp}{p}) = x'(1 - \frac{dp}{p}) \end{cases}$. In matrix form, $\begin{pmatrix} x_2 \\ x'_2 \end{pmatrix} =$
2199 $[C] \begin{pmatrix} x \\ x' \end{pmatrix}$ with

$$[C] = \begin{bmatrix} 1 & 0 \\ 0 & 1 - \frac{dp}{p} \end{bmatrix} \quad (9.30)$$

2200 and $\det[C] = 1 - \frac{dp}{p} \neq 1$: the system is non-conservative, the surface of the beam
2201 ellipse in phase space is not conserved. Assume one cavity in the ring and note
2202 $[T] \times [C]$ the one-turn coordinate transport matrix with origin at entrance of the
2203 cavity. Its determinant is $\det[T] \times \det[C] = \det[C] = 1 - \frac{dp}{p}$; the variation of
2204 the transverse ellipse surface satisfies $\varepsilon_u = (1 - \frac{dp}{p_0})\varepsilon_0$ or, with $d\varepsilon_u = \varepsilon_u - \varepsilon_0$,
2205 $\frac{d\varepsilon_u}{\varepsilon_u} = -\frac{dp}{p_0}$, the solution of which is

$$p \varepsilon_u = \text{constant}, \text{ or } \beta\gamma\varepsilon_u = \text{constant} \quad (9.31)$$

2206 Over N turns the coordinate transport matrix is $[T_N] = ([T][C])^N$, its determinant
2207 is $(1 - \frac{dp}{p})^N \approx 1 - N\frac{dp}{p}$: the ellipse surface changes by that factor.

2208 *Synchrotron motion; phase stability*

2209 “Synchrotron motion” designates the mechanism of phase stability, or longitudinal
2210 focusing (Fig. 9.15), that stabilizes the longitudinal motion of a particle in the vicinity
2211 of a synchronous phase, ϕ_s , in virtue of

2212 (i) the presence of an accelerating cavity with its frequency indexed on the
2213 revolution time,

2214 (ii) with the bunch centroid positioned either on the rising slope of the oscillating
2215 voltage (low energy regime), or on the falling slope (high energy regime).

The synchronous (or “ideal”) particle follows the equilibrium trajectory around
the ring (the reference closed orbit, about which all other particles will undergo a
betatron oscillation), its velocity satisfies $v(t) = \frac{qB\rho(t)}{m}$; at each turn it reaches the
accelerating gap when the oscillating voltage is at the synchronous phase ϕ_s , and
undergoes an energy gain

$$\Delta W = q\hat{V} \sin \phi_s$$

The condition $|\sin \phi_s| < 1$ imposes a lower limit to the cavity voltage for acceleration
to happen, namely, after Eq. 9.29,

$$\hat{V} > 2\pi R\rho\dot{B}$$

2216 Referring to Fig. 9.15, the synchronous phase can be placed on the left (A A' A"...
2217 series in the Figure, or on the right (B B' B"... series) of the oscillating voltage crest.

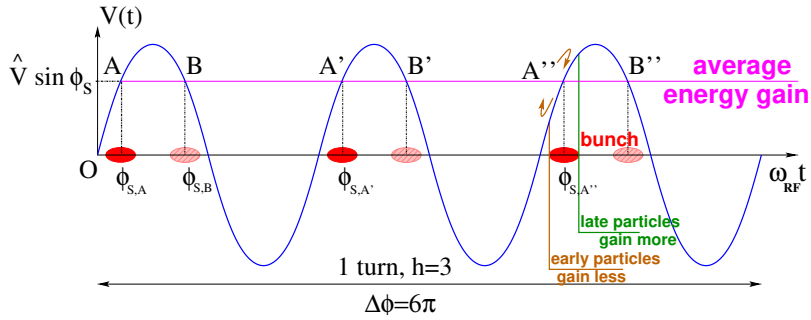


Fig. 9.15 A sketch of the mechanism of phase stability, $h = 3$ in this example. Below transition phase stability occurs for a synchronous phase taken at either one of A, A', A'' arrival times at the gap: a particle with higher energy goes around the ring more rapidly than the synchronous particle; if both are launched together, the former arrives earlier at the voltage gap (at $\phi < \phi_{s,A}$) so experiencing weaker acceleration; at lower energy the particle is slower, it arrives at the gap later, $\phi > \phi_{s,A}$, so experiencing a greater voltage; this results in an overall stable oscillatory motion around the synchronous phase. Beyond transition the stable phase is at either one of B, B', B'' locations: a particle which is less energetic than the synchronous particle arrives earlier, $\phi < \phi_{s,B}$, so experiencing a greater voltage, and inversely, resulting in overall stable synchrotron motion.

2218 One and only one of these two possibilities, and which one depending upon the optical
 2219 lattice and on particle energy, ensures that particles in a bunch remain grouped in
 2220 the vicinity of the synchronous particle. The transition is between two time-of-flight
 2221 regimes: a particle which gains momentum compared to the synchronous particle
 2222 has a greater velocity, while

2223 - in the high bunch energy regime the increase in path length around the ring
 2224 is faster than the increase in velocity (velocity essentially does not even change
 2225 in ultrarelativistic regime), a revolution around the ring takes more time (this is the
 2226 classical cyclotron and synchrocyclotron regime, and as well the high energy electron
 2227 synchrotron regime); consider such a particle, arriving at the accelerating gap late
 2228 ($\phi(t) > \phi_s$), in order for it to be pulled toward bunch center (*i.e.*, take less time
 2229 around the ring) it has to undergo deceleration; this is the B series, above transition;

2230 - in the low bunch energy regime velocity increase is faster than path length
 2231 increase, thus a revolution around the ring is faster; consider such a particle, arriving
 2232 at the accelerating gap early ($\phi(t) < \phi_s$), in order for it to be pulled toward bunch
 2233 center (*i.e.*, take more time around the ring) it has to be slowed down, it has to
 2234 undergo deceleration; this is the A series, below transition.

2235 *Transition energy*

2236 The transition between the two time-of-flight regimes occurs at $\frac{dT_{\text{rev}}}{T_{\text{rev}}} = 0$. With

2237 $T = 2\pi/\omega = C/v$, this can be written $\frac{d\omega_{\text{rev}}}{\omega_{\text{rev}}} = -\frac{dT_{\text{rev}}}{T_{\text{rev}}} = \frac{dv}{v} - \frac{dC}{C}$. With $\frac{dv}{v} = \frac{1}{\gamma^2} \frac{dp}{p}$

2238 and momentum compaction $\alpha = \frac{dC}{C} / \frac{dp}{p}$, (Eq. 9.28), this can be written

$$\frac{d\omega_{\text{rev}}}{\omega_{\text{rev}}} = -\frac{dT_{\text{rev}}}{T_{\text{rev}}} = \left(\frac{1}{\gamma^2} - \alpha \right) \frac{dp}{p} = \eta \frac{dp}{p} \quad (9.32)$$

2239 wherein the phase-slip factor has been introduced,

$$\eta = \overbrace{\frac{1}{\gamma^2}}^{\text{kinematics}} - \underbrace{\alpha}_{\text{lattice}} = \frac{1}{\gamma^2} - \frac{1}{\gamma_{\text{tr}}^2} \quad (9.33)$$

2240 The transition γ appears to be a property of the lattice.

2241 In a weak focusing lattice $\gamma_{\text{tr}} = 1/\sqrt{\alpha} \approx v_x$ (Eqs. 4.21, 9.28), thus the phase
2242 stability regime is

$$\begin{aligned} &\text{below transition, i.e. } \phi_s < \pi/2, \quad \text{if } \gamma < v_x \\ &\text{above transition, i.e. } \phi_s > \pi/2, \quad \text{if } \gamma > v_x \end{aligned} \quad (9.34)$$

2243 In a weak focusing synchrotron the horizontal tune $\nu_x = \sqrt{(1-n)R/\rho_0}$ (Eq. 9.23)
2244 may be ≥ 1 , and subsequently $\gamma_{\text{tr}} > 1$ is a possibility. There is no transition-gamma if
2245 $\nu_x < 1$. Acceleration to 3 GeV in SATURNE I for instance, from 50 MeV at injection,
2246 and with $\nu_x \approx 0.7$ (Tab. 9.1) did not require transition-gamma crossing³.

2247 9.1.3 Depolarizing Resonances

2248 The field index is essentially zero in the ZGS, transverse focusing is ensured by
2249 wedge angles at the ends of the height dipoles, the only location where non-zero
2250 radial field components are found. The latter are weak anyway, as a consequence so
2251 are depolarizing resonances: “As we can see from the table, the transition probability
2252 [from spin state $\psi_{1/2}$ to spin state $\psi_{-1/2}$] is reasonably small up to $\gamma = 7.1$ ” [12], i.e.
2253 $G\gamma = 12.73$, $p = 6.6$ GeV/c; the table referred to stipulates a transition probability
2254 $P_{\frac{1}{2}, -\frac{1}{2}} < 0.042$, whereas resonances beyond that energy range feature $P_{\frac{1}{2}, -\frac{1}{2}} > 0.36$.
2255 Beam depolarization up to 6 GeV/c, under the effect of these resonances, is illustrated
2256 in Fig. 9.16.

2257 In a synchrotron using gradient dipoles, particles experience radial fields all along
2258 the latter as they undergo vertical betatron oscillations, as an effect of the radial field
2259 index [12, 20, 21]. However these radial field components are weak, and so is there

³ Transition-gamma crossing, or “gamma jump”, is a common beam manipulation during acceleration in strong focusing synchrotrons, it requires an RF phase jump, the technique is addressed in Chapter 10.

2260 effect on spin motion as long as the particle energy is low enough (an effect of the γ
 2261 factor in the spin precession Eq. 4.29, Chap. 4).

Assuming a defect-free ring, the vertical betatron motion excites “intrinsic” spin resonances, located at

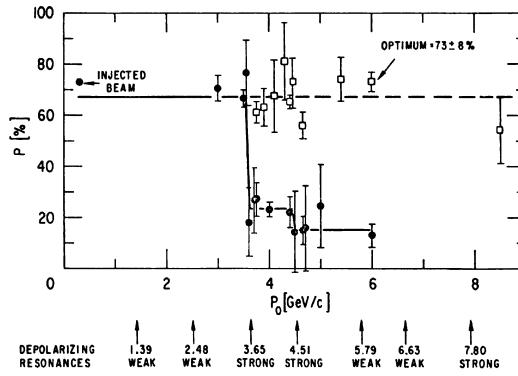
$$G\gamma_R = k P \pm \nu_y$$

with k an integer and P the period of the ring. In the ZGS for instance, $\nu_y \approx 0.8$ (Tab. 9.2), the ring is $P=4$ -periodic, thus $G\gamma_R = 4k \pm 0.8$. Strongest resonances are located at

$$G\gamma_R = mk P \pm \nu_y$$

2262 with m the number of cells per superperiod [22, Sec. 3.II]. In the ZGS, $m=2$ thus
 2263 strongest resonances occur at $G\gamma_R = 2 \times 4k \pm 0.8 = 7.2$ ($p = 3.65$ GeV/c), 8.8 (4.51 GeV/c), 15.2 (7.9 GeV/c), ... (Fig. 9.16).

Fig. 9.16 Depolarizing intrinsic resonance landscape up to 9 GeV/c at the ZGS (solid circles) [23]. Systematic resonances are located at $G\gamma_R = 4 \times \text{integer} \pm \nu_y$, stronger ones at $G\gamma_R = 8 \times \text{integer} \pm \nu_y$. A tune jump was applied to preserve polarization when crossing strong resonances (empty circles)



2264 In the presence of vertical orbit defects, non-zero periodic transverse fields are experienced along the closed orbit, they excite “imperfection” depolarizing resonances, located at

$$G\gamma_R = k$$

with k an integer. In the case of systematic defects the periodicity of the orbit is that of the lattice, P , imperfection resonances are located at $G\gamma_R = kP$. Strongest imperfection resonances are located at [22, Sec. 3.II]

$$G\gamma_R = mk P$$

2265 Crossing a depolarizing resonance of strength ϵ_R causes a loss of polarization
 2266 given by (Froissart-Stora formula [24])

$$\frac{P_f}{P_i} = 2e^{-\frac{\pi}{2} \frac{|\epsilon_R|^2}{\alpha}} - 1 \quad (9.35)$$

2267 from a value P_i upstream to an asymptotic value P_f downstream of the resonance.
 2268 This assumes an isolated resonance, crossed at an energy gain ΔE per turn, with a
 2269 crossing speed

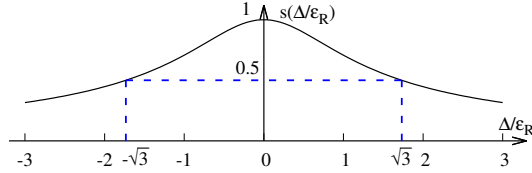
$$\alpha = G \frac{d\gamma}{d\theta} = \frac{1}{2\pi} \frac{\Delta E}{M} \quad (9.36)$$

2270 *Spin precession axis. Resonance width*

2271 Consider the spin vector $\mathbf{S}(\theta) = (S_\eta, S_\xi, S_y)$ of a particle in the laboratory frame,
 2272 with θ the orbital angle around the accelerator. Introduce the projection $s(\theta)$ of \mathbf{S}
 2273 in the median plane

$$s(\theta) = S_\eta(\theta) + jS_\xi(\theta) \quad (\text{and } S_y^2 = 1 - s^2) \quad (9.37)$$

2274 **Fig. 9.17** Modulus of the
 2275 horizontal spin component.
 2276 $s = 1/2$ at distance $\Delta =$
 $\pm\sqrt{3}\epsilon_R$ from $G\gamma_R$



2274 It can be shown that in the case of a stationary solution of the spin motion, viz.
 2275 the spin precession axis, s satisfies [21] (Fig. 9.17)
 2276

$$s^2 = \frac{1}{1 + \frac{\Delta^2}{|\epsilon_R|^2}} \quad (9.38)$$

2277 with $\Delta = G\gamma - G\gamma_R$ the distance to the resonance. The resonance width is a measure
 2278 of its strength (Fig. 9.18). The quantity of interest is the angle, ϕ , of the spin
 2279 precession direction to the vertical axis, given by (Fig. 9.18)

$$\cos \phi(\Delta) \equiv S_y(\Delta) = \sqrt{1 - s^2} = \frac{\Delta/|\epsilon_R|}{\sqrt{1 + \Delta^2/|\epsilon_R|^2}} \quad (9.39)$$

2280 On the resonance, $\Delta = 0$, the spin precession axis lies in the bend plane: $\phi = \pm\pi/2$.
 2281 $S_y = 0.99$ (1% depolarization) corresponds to a distance to the resonance $\Delta = 7|\epsilon_R|$,
 2282 spin precession axis at an angle $\phi = \arccos(0.99) = 8^\circ$ from the vertical.

2283 Conversely, given S_y ,

$$\frac{\Delta^2}{|\epsilon_R|^2} = \frac{S_y^2}{1 - S_y^2} \quad (9.40)$$

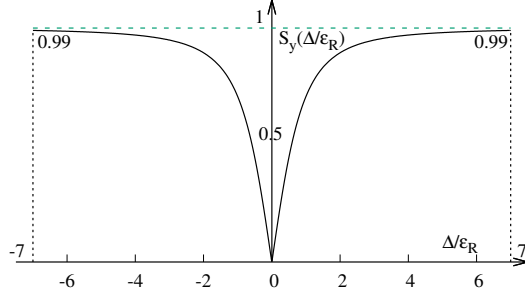


Fig. 9.18 Dependence of polarization on the distance to the resonance. For instance $S_y = 0.99$, 1% depolarization, corresponds to $\Delta = 7|\epsilon_R|$. On the resonance, $\Delta = 0$, the precession axis lies in the median plane, $S_y = 0$

The precession axis is common to all spins, S_y is a measure of the polarization along the vertical axis,

$$S_y = \frac{N^+ - N^-}{N^+ + N^-}$$

2284 wherein N^+ and N^- denote the number of particles in spin states $\frac{1}{2}$ and $-\frac{1}{2}$ respectively.
2285

2286 *Spin motion through weak resonances*

Depolarizing resonances are weak up to several GeV in a weak focusing synchrotron, as the radial and/or longitudinal fields, which stem from a small radial field index and from dipole fringe fields, are weak. Spin motion $S_y(\theta)$ through a resonance in that case can be assumed to satisfy $S_{y,f} \approx S_{y,i}$, with $S_{y,f}$ and $S_{y,i}$ the asymptotic vertical spin component values respectively upstream and downstream of the resonance). As a consequence it can be calculated in terms of the Fresnel integrals [20, 21]

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right) dt, \quad S(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right) dt$$

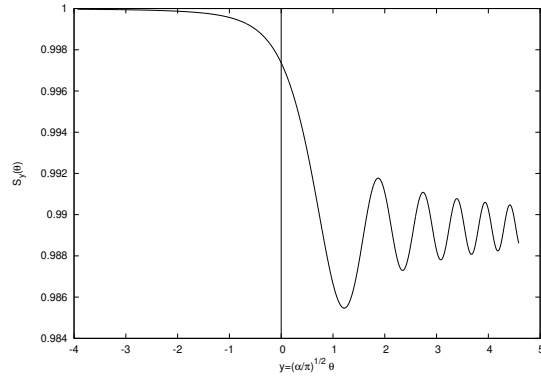
2287 namely, with the origin of the orbital angle is taken at the resonance (Fig. 9.19),

$$\begin{aligned} \text{if } \theta < 0 : \left(\frac{S_y(\theta)}{S_{y,i}}\right)^2 &= 1 - \frac{\pi}{\alpha}|\epsilon_R|^2 \left\{ \left[0.5 - C\left(-\theta\sqrt{\frac{\alpha}{\pi}}\right)\right]^2 + \left[0.5 - S\left(-\theta\sqrt{\frac{\alpha}{\pi}}\right)\right]^2 \right\} \\ \text{if } \theta > 0 : \left(\frac{S_y(\theta)}{S_{y,i}}\right)^2 &= 1 - \frac{\pi}{\alpha}|\epsilon_R|^2 \left\{ \left[0.5 + C\left(\theta\sqrt{\frac{\alpha}{\pi}}\right)\right]^2 + \left[0.5 + S\left(\theta\sqrt{\frac{\alpha}{\pi}}\right)\right]^2 \right\} \end{aligned} \quad (9.41)$$

2288 In the asymptotic limit,

$$\frac{S_y(\theta)}{S_{y,i}} \xrightarrow{\theta \rightarrow \infty} 1 - \frac{\pi}{\alpha}|\epsilon_R|^2 \quad (9.42)$$

Fig. 9.19 Vertical component of spin motion $S_y(\theta)$ through a weak depolarizing resonance (after Eq. 9.41). The vertical bar is at the location of the resonance, which coincides with the origin of the orbital angle



2289 which identifies with the development of Froissart-Stora formula $P_f/P_i =$
 2290 $2 \exp(-\frac{\pi}{2} \frac{|\epsilon_R|^2}{\alpha}) - 1$ to the first order in $|\epsilon_R|^2/\alpha$. This approximation holds in the
 2291 limit that higher order terms can be neglected: $|\epsilon_R|^2/\alpha \ll 1$.

2292 9.2 Exercises

2293 9.1 Construct SATURNE I (weak index) synchrotron. Spin Resonances

2294 Solution: page 304

2295 In this exercise, SATURNE I weak focusing 3 GeV synchrotron is modeled. Spin
 2296 resonances in a weak dipole gradient lattice are studied.

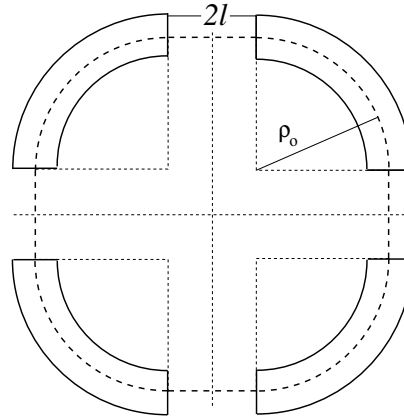


Fig. 9.20 A schematic layout of SATURNE I, a $2\pi/4$ axial symmetry structure, comprised of 4 radial field index 90 deg dipoles and 4 drift spaces. The cell in the simulation exercises is taken as a $\pi/4$ quadrant: 1-drift/90 $^\circ$ -dipole/1-drift

Table 9.1 Parameters of SATURNEI weak focusing synchrotron [25]. ρ_0 denotes the reference bending radius in the dipole; the reference orbit, field index, wave numbers, etc., are taken along that radius

| | | |
|---------------------------------|----------|------------------------------------|
| Orbit length, C | cm | 6890 |
| Average radius, $R = C/2\pi$ | cm | 1096.58 |
| Straight section length, $2l$ | cm | 400 |
| Magnetic radius, ρ_0 | cm | 841.93 |
| R/ρ_0 | | 1.30246 |
| Field index n , nominal value | | 0.6 |
| Wave numbers, $\nu_x; \nu_y$ | | 0.724; 0.889 **** verif wrt. simul |
| Stability limit | | $0.5 < n < 0.757$ |
| Injection energy | MeV | 3.6 |
| Field at injection | kG | 0.326 |
| Top energy | GeV | 2.94 |
| \dot{B} | kG/s | 18 |
| Field at top energy, B_{\max} | kG | 14.9 |
| $B_{\max}\rho$ | T m | 13 |
| Field ramp at injection | kG/s | 20 |
| Synchronous energy gain | keV/turn | 1.160 |
| RF harmonic | | 2 |

2297 (a) Construct a model of SATURNEI 90° cell dipole in the hard-edge model,
 2298 using DIPOLE. Use the parameters given in Tab. 9.1, and Fig. 10.7 as a guidance. In
 2299 order to allow beam monitoring, split the dipole in two 45° deg halves. It is judicious
 2300 to take $RM=841.93$ cm in DIPOLE, as this is the reference radius for the definition
 2301 of the radial index. Take an integration step size in centimeter range - small enough
 2302 to ensure numerical convergence, as large as doable for fast multiturn raytracing.

2303 Validate the model by producing the 6×6 transport matrix of the cell dipole
 2304 (MATRIX[IFOC=0] can be used for that, with OBJET[KOBJ=5] to define a proper
 2305 set of paraxial initial coordinates) and checking against theory (Sect. 15.2, Eq. 15.6).

2306 (b) Construct a model of SATURNEI cell, with origin at the center of the drift.
 2307 Find the closed orbit, that particular trajectory which has all its coordinates zero in
 2308 the drifts: use DIPOLE[KPOS] to cancel the closed orbit coordinates at DIPOLE
 2309 ends. While there, check the expected value of the dispersion (Eq. 9.26) and of
 2310 the momentum compaction (Eq. 9.28), from the raytracing of a chromatic closed
 2311 orbit - *i.e.*, the orbit of an off-momentum particle. Plot these two orbits (on- and
 2312 off-momentum), over a complete turn around the ring, on a common graph.

2313 Compute the cell periodic optical functions and tunes, using either MA-
 2314 TRIX[IFOC=11] or TWISS; check their values against theory. Check consistency
 2315 with previous dispersion function and momentum compaction outcomes.

2316 Move the origin of the lattice at a different azimuth s along the cell: verify that,
 2317 while the transport matrix depends on the origin, its trace does not.

2318 Produce a graph of the optical functions (betatron functions and dispersion) along
 2319 the cell. Check the expected average values of the betatron functions (Eq. 9.20).

2320 Produce a scan of the tunes over the field index range $0.5 \leq n \leq 0.757$. RE-
 2321 BELOTE can be used to repeatedly change n over that range. Superimpose the
 2322 theoretical curves $\nu_x(n)$, $\nu_y(n)$.

(c) Justify considering the betatron oscillation as sinusoidal, namely,

$$y(\theta) = A \cos(\nu_y \theta + \phi)$$

2323 wherein $\theta = s/R$, $R = \oint ds/2\pi$.

2324 (d) Launch a few particles evenly distributed on a common paraxial horizontal
2325 Courant-Snyder invariant, vertical motion taken null (OBJET[KOBJ=8] can be used),
2326 for a single pass through the cell. Store particle data along the cell in zgoubi.plt,
2327 using DIPOLE[IL=2] and DRIFT[split,N=20,IL=2]. Use these to generate a graph
2328 of the beam envelopes.

2329 Using Eq. 9.22 compare with the results obtained in (b). Find the minimum
2330 and maximum values of the betatron functions, and their azimuth $s(\min[\beta_x])$,
2331 $s(\max[\beta_x])$. Check the latter against theory.

2332 Repeat for the vertical motion, taking $\varepsilon_x = 0$, ε_y paraxial.

2333 Repeat, using, instead of several particles on a common invariant, a single particle
2334 traced over a few tens of turns.

2335 (e) Produce an acceleration cycle from 3.6 MeV to 3 GeV, for a few particles
2336 launched on a common $10^{-4} \pi m$ initial invariant in each plane. Ignore synchrotron
2337 motion (CAVITE[IOPT=3] can be used in that case). Take a peak voltage $\hat{V} = 200$ kV
2338 (unrealistic though, as it would result in a nonphysical \hat{B} (Eq. 9.29)) and synchronous
2339 phase $\phi_s = 150$ deg (justify $\phi_s > \pi/2$).

2340 Check the betatron damping over the acceleration range: compare with theory
2341 (Eq. 9.31).

2342 How close to symplectic the numerical integration is (it is by definition *not*
2343 symplectic, being a truncated Taylor series method [26, Eq. 1.2.4]), depends on the
2344 integration step size, and on the size of the flying mesh in the DIPOLE method [26,
2345 Fig. 20]; check a possible departure of the betatron damping from theory as a function
2346 of these parameters.

2347 Produce a graph of the horizontal and vertical wave number values over the
2348 acceleration cycle.

2349 (f) Some spin motion, now. Adding SPNTRK at the beginning of the sequence
2350 will ensure spin tracking.

2351 Based on the file worked out for question (d), simulate the acceleration of a single
2352 particle, through the intrinsic resonance $G\gamma_R = 4 - \nu_Z$, from a few thousand turns
2353 upstream to a few thousand turns downstream. On a common graph, plot $S_y(\text{turn})$
2354 for a few different values of the vertical betatron invariant (the horizontal invariant
2355 value does not matter - explain that statement, it can be taken zero).

2356 (g) Produce a graph of the average value of S_Z over a 200 particle set, as a function
2357 of $G\gamma$, across the $G\gamma_R = 4 - \nu_Z$ resonance. Indicate on that graph the location of
2358 the resonant $G\gamma_R$ values.

2359 Perform this resonance crossing for five different values of the particle invariant:
2360 $\varepsilon_Z/\pi = 2, 10, 20, 40, 200 \mu m$. Compute P_f/P_i in each case, check the dependence
2361 on ε_Z against theory.

2362 Compute the resonance strength, ε_Z , from these trackings.

2363 Re-do this crossing simulation for a different crossing speed (take for instance
2364 $\hat{V} = 10\text{kV}$) and a couple of vertical invariant values, compute P_f/P_i so obtained.
2365 Check the crossing speed dependence of P_f/P_i against theory.

2366 (h) Show that the previous weak resonance crossings ($P_f/P_i \approx 1$) satisfies
2367 Eq. 9.41. Match the tracking data to the latter to get the vertical betatron tune ν_y , the
2368 location of the resonance $G\gamma_R$, and its strength.

2369 (i) Track a few particles at fixed energy, at distances from the resonance $G\gamma_R =$
2370 $4 - \nu_y$ of up to a $7 \times \epsilon_R$ (this distance corresponds to 1% depolarization).

2371 Produce on a common graph the spin motion $S_Z(\text{turn})$ for all these particles, as
2372 observed at some azimuth along the ring.

2373 Produce a graph of $\langle S_y \rangle|_{\text{turn}}(\Delta)$ (as in Fig. 9.18).

Produce the vertical betatron tune ν_y , the location of the resonance $G\gamma_R$, and its
strength, obtained from a match of these tracking trials to (Eq. 9.39)

$$\langle S_y \rangle (\Delta) = \frac{\Delta}{\sqrt{|\epsilon_R|^2 + \Delta^2}}$$

2374 9.2 Construct the ZGS (zero-gradient) synchrotron. Spin Resonances

2375 Solution: page 328

2376 In this exercise, the ZGS 12 GeV synchrotron is modeled. Spin resonances in a
2377 zero-gradient, wedge focusing synchrotron are studied.

2378 A photo taken in the ZGS tunnel is given in Fig. 9.4; a schematic layout of the ring
2379 is shown in Fig. 9.21, and a sketch of the double dipole cell in Fig. 9.22. Table 9.2
2380 details the parameters of the synchrotron resorted to in these simulations.

2381 (a) Construct a model of ZGS 45° cell dipole in the hard-edge model, using
2382 DIPOLE. Use the parameters given in Tab. 9.2, and Figs. 9.21, 9.22 as a guidance.
2383 In order to allow beam monitoring, split the dipole in two 22.5° deg halves. Take the
2384 closed orbit radius as the reference $RM=2076$ cm in DIPOLE: it will be assumed
2385 that the orbit is the same at all energies⁴. Take an integration step size in centimeter
2386 range - small enough to ensure numerical convergence, as large as doable for fast
2387 multiturn raytracing.

2388 Validate the model by producing the 6×6 transport matrices of both dipole
2389 (MATRIX[IFOC=0] can be used for that, with OBJET[KOBJ=5] to define a proper
2390 set of paraxial initial coordinates) and checking against theory (Sect. 15.2, Eq. 15.6).

2391 Add fringe fields in DIPOLE[$\lambda, C_0 - C_5$], the rest if the exercise will use that
2392 model. Take fringe field extent and coefficient values

$$\lambda = 60 \text{ cm } C_0 = 0.1455, C_1 = 2.2670, C_2 = -0.6395, C_3 = 1.1558, C_4 = C_5 = 0 \quad (9.43)$$

2393 ($C_0 - C_5$ determine the shape of the field fall-off, they have been computed from a
2394 typical measured field profile $B(s)$).

⁴ Note that in reality the reference orbit in ZGS moved outward during acceleration [27].

2395 (b) Construct a model of ZGS cell accounting for dipole fringe fields, with origin
 2396 at the center of the long drift. In doing so, use DIPOLE[KPOS] to cancel the closed
 2397 orbit coordinates at DIPOLE ends.

2398 Compute the periodic optical functions at cell ends, and cell tunes, using MA-
 2399 TRIX[IFOC=11]; check their values against theory.

2400 Move the origin at the location (azimuth s along the cell) of the betatron functions
 2401 extrema: verify that, while the transport matrix depends on the origin, its trace does
 2402 not. Verify that the local betatron function extrema, and the dispersion function, have
 2403 the expected values.

2404 Produce a graph of the optical functions (betatron functions and dispersion) along
 2405 the cell.

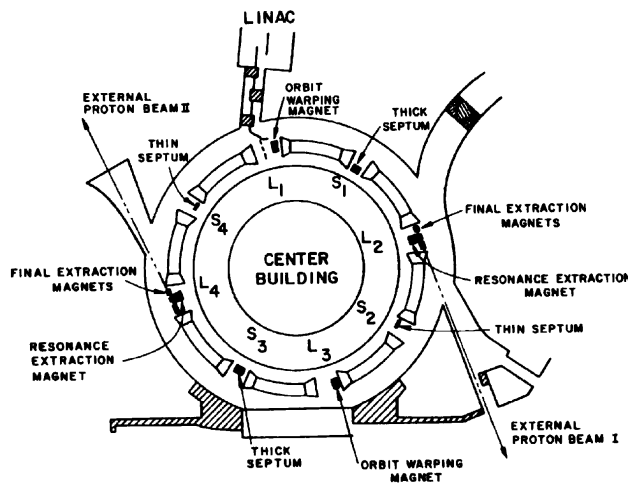


Fig. 9.21 A schematic layout of the ZGS [23], a $\pi/2$ -periodic structure, comprised of 8 zero-index dipoles, 4 long and 4 short straight sections

2406 (c) Additional verifications regarding the model.

2407 Produce a graph of the field $B(s)$

2408 - along the on-momentum closed orbit, and along off-momentum chromatic closed
 2409 orbits, across a cell;

2410 - along orbits at large horizontal excursion;

2411 - along orbits at large vertical excursion.

2412 For all these cases, verify qualitatively, from the graphs, that $B(s)$ appears as
 2413 expected.

(d) Justify considering the betatron oscillation as sinusoidal, namely,

$$y(\theta) = A \cos(\nu_y \theta + \phi)$$

2414 wherein $\theta = s/R$, $R = \oint ds/2\pi$.

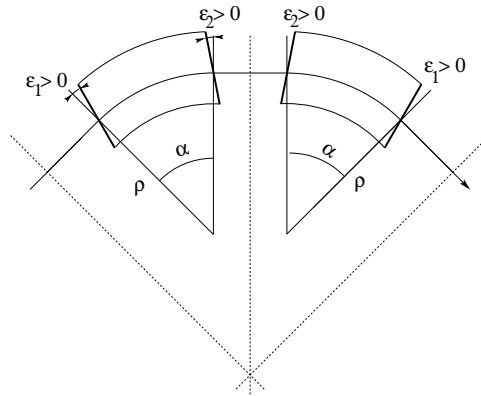


Fig. 9.22 A sketch of ZGS cell layout. In defining the entrance and exit faces (EFBs) of the magnet, beam goes from left to right. Wedge angles at the long straight sections (ε_1) and at the short straight sections (ε_2) are different

Table 9.2 Parameters of the ZGS weak focusing synchrotron after Refs. [27, 28] [23, pp.288-294,p. 716] (2nd column, when they are known) and in the present simplified model and numerical simulations (3rd column). Note that the actual orbit moves during ZGS acceleration cycle, tunes change as well - this is not taken into account in the present modeling, for simplicity

| | | From Refs. [27, 28] | Simplified model |
|--|-------------------|------------------------|--|
| Injection energy | MeV | | 50 |
| Top energy | GeV | | 12.5 |
| $G\gamma$ span | | | 1.888387 - 25.67781 |
| Length of central orbit | m | 171.8 | 170.90457 |
| Length of straight sections, total | m | 41.45 | 40.44 |
| <i>Lattice</i> | | | |
| Wave numbers ν_x ; ν_y | | 0.82; 0.79 | 0.849; 0.771 |
| Max. β_x ; β_y | m | | 32.5; 37.1 |
| <i>Magnet</i> | | | |
| Length | m | 16.3 | 16.30486 (magnetic) |
| Magnetic radius | m | 21.716 | 20.76 |
| Field min.; max. | kG | 0.482; 21.5 | 0.4986; 21.54 |
| Field index | | | 0 |
| Yoke angular extent | deg | 43.02590 | 45 |
| Wedge angle | deg | ≈ 10 | 13 and 8 |
| <i>RF</i> | | | |
| Rev. frequency | MHz | 0.55 - 1.75 | 0.551 - 1.751 |
| RF harmonic $h = \omega_{rf} / \omega_{rev}$ | | | 8 |
| Peak voltage | kV | 20 | 200 |
| B-dot, nominal/max. | T/s | 2.15/2.6 | |
| Energy gain, nominal/max. | keV/turn | 8.3/10 | 100 |
| Synchronous phase, nominal | deg | | 150 |
| <i>Beam</i> | | | |
| ε_x ; ε_y (at injection) | $\pi \mu\text{m}$ | | 25; 150 |
| Momentum spread, rms | | | 3×10^{-4} |
| Polarization at injection | % | > 75 | 100 |
| Radial width of beam (90%), at inj. | inch | 2.5 | $\sqrt{\beta_x \varepsilon_x / \pi} = 1.1$ |

2415 (e) Produce an acceleration cycle from 50 MeV to 17 GeV about, for a few particles
2416 launched on a common $10^{-5} \pi\text{m}$ vertical initial invariant, with small horizontal
2417 invariant. Ignore synchrotron motion (CAVITE[IOPT=3] can be used in that case).
2418 Take a peak voltage $\hat{V} = 200\text{ kV}$ (this is unrealistic but yields 10 times faster
2419 computing than the actual $\hat{V} = 20\text{ kV}$, Tab. 9.2) and synchronous phase $\phi_s = 150\text{ deg}$
2420 (justify $\phi_s > \pi/2$). Add spin, using SPNTRK, in view of the next question, (f).

2421 Check the accuracy of the betatron damping over the acceleration range, compared
2422 to theory. How close to symplectic the numerical integration is (it is by definition
2423 *not* symplectic), depends on the integration step size, and on the size of the flying
2424 mesh in the DIPOLE method [26, Fig. 20]; check a possible departure of the betatron
2425 damping from theory as a function of these parameters.

2426 Produce a graph of the evolution of the horizontal and vertical wave numbers
2427 during the acceleration cycle.

2428 (f) Using the raytracing material developed in (e): produce a graph of the vertical
2429 spin component of the particles, and the average value over that 200 particle set, as
2430 a function of $G\gamma$. Indicate on that graph the location of the resonant $G\gamma_R$ values.

- 2431 (g) Based on the simulation file used in (f), simulate the acceleration of a single
 2432 particle, through one particular intrinsic resonance, from a few thousand turns
 2433 upstream to a few thousand turns downstream.
- 2434 Perform this resonance crossing for different values of the particle invariant.
 2435 Determine the dependence of final/initial vertical spin component value, on the
 2436 invariant value; check against theory.
- 2437 Re-do this crossing simulation for a different crossing speed. Check the crossing
 2438 speed dependence of final/initial vertical spin component so obtained, against theory.
- 2439 (h) Introduce a vertical orbit defect in the ZGS ring.
 2440 Find the closed orbit.
 2441 Accelerate a particle launched on that closed orbit, from 50 MeV to 17 GeV about,
 2442 produce a graph of the vertical spin component.
- 2443 Select one particular resonance, reproduce the two methods of (g) to check the
 2444 location of the resonance at $G\gamma_R = \text{integer}$, and to find its strength.

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