# Options for Coherent electron Cooling


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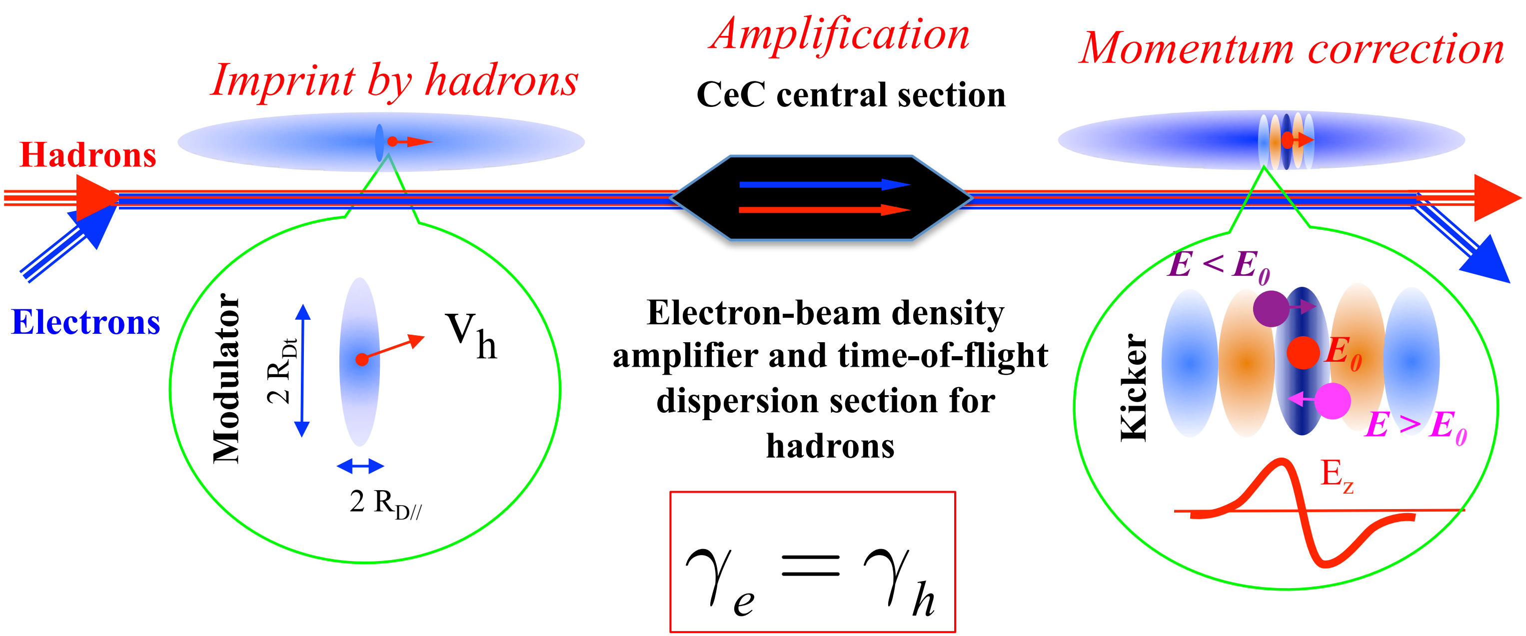
# Outline

- Options for Coherent electron Cooling
- What are advantages of various schemes
- What can be tested?
- How to evaluate cooling
- Conclusions

# What is Coherent electron Cooling

- Short answer – stochastic cooling of hadron beams with bandwidth at optical wave frequencies: 1 – 1000 THz
- Longer answer




$$\gamma_e = \gamma_h$$

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**Coherent Electron Cooling**

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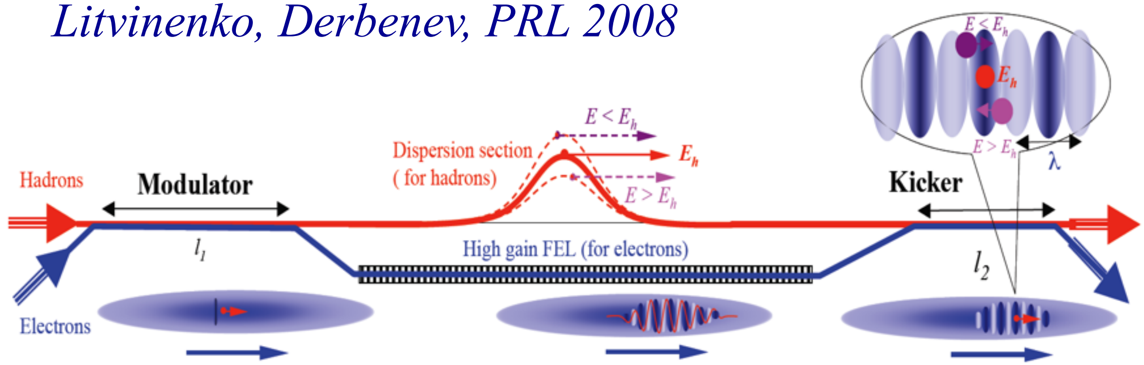
(Received 24 September 2008; published 16 March 2009)

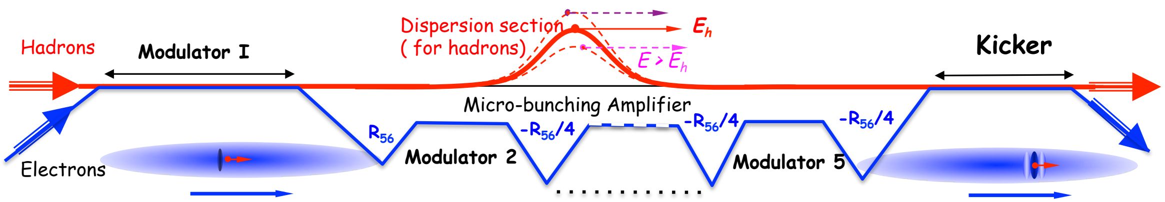
# CeC schemes

*Litvinenko, Derbenev, PRL 2008*



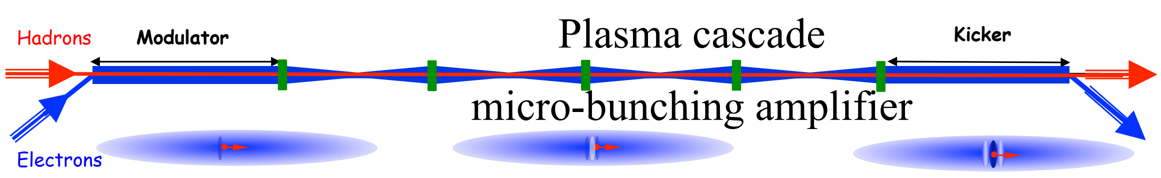

**High gain FEL amplifier**

*Ratner, PRL 2013*



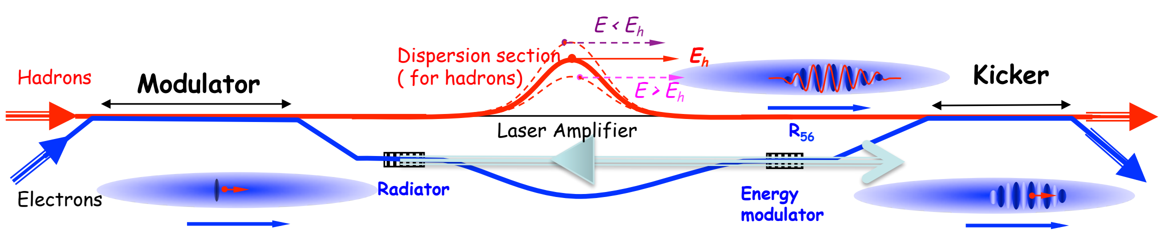

**Multi- Chicane Microbunching amplifier**

*Litvinenko, Wang, Kayran, Jing, Ma, 2017*




**Plasma-Cascade Microbunching amplifier**

*Litvinenko, Cool 13*



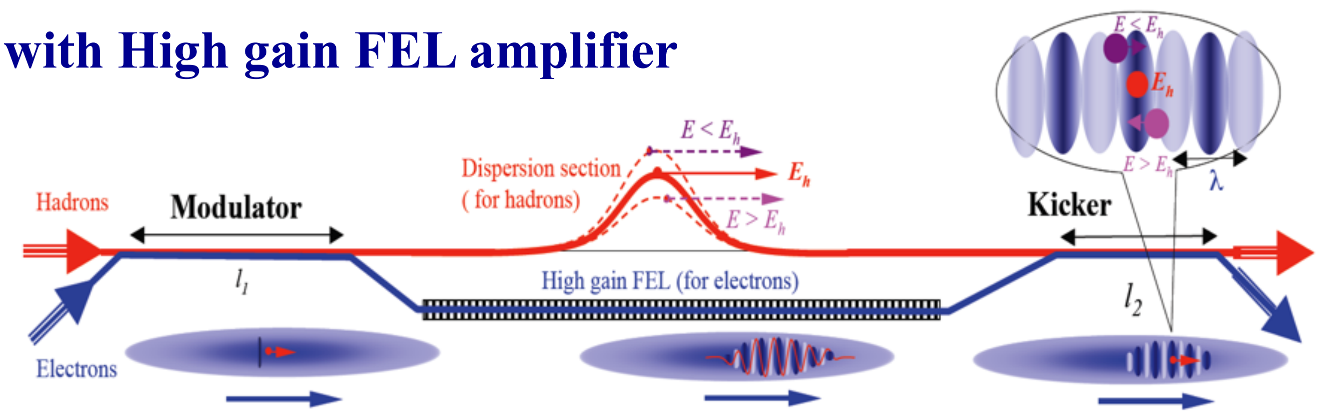

**Hybrid laser-beam amplifier**

# Advantages and Disadvantages

- The best studied and fully explored scheme
- Experimentally demonstrated both as instability and amplifier
- 3D FEL theory and simulation are very advanced
- Can operate at relatively low electron beam peak currents
- Allows – in principle – economic option without separating electron and hadron beams

- When compared with micro-bunching amplifier, it has relatively lower bandwidth ~ few % of the FEL frequency
- FEL saturates at lower gain than micro-bunching amplifier
- Semi-periodic structure of the modulation limits the range where cooling occurs

## CeC with High gain FEL amplifier

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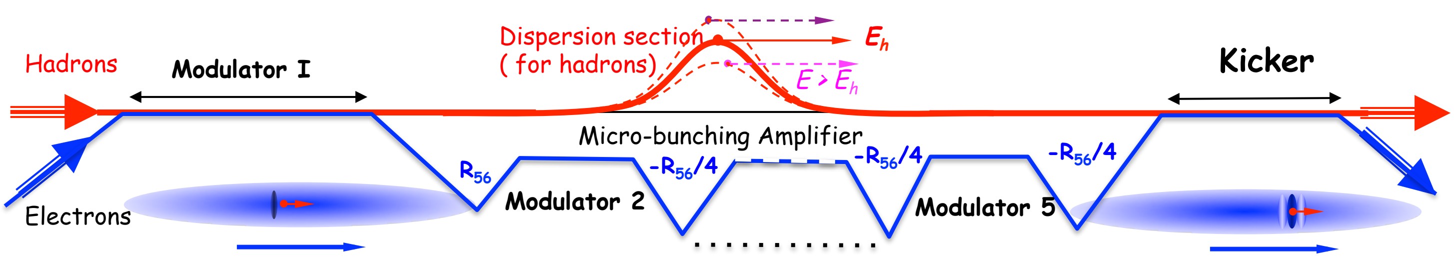
# Advantages and Disadvantages

- Very broad band amplifier
- Micro-bunching instability was experimentally demonstrated
- Can operate at significant gain without saturation and can be extended to LHC energies
- Ratner's original scheme is - in principle - insensitive to longitudinal space charge effects in the electron beam

- Micro-bunching amplifier was not demonstrated
- Less studied – especially numerically in 3D - than other CeC schemes
- Requires electron beam with low energy spread
- Definitely require separation of electron and hadron beams

## Multi-Chicane Microbunching amplifier

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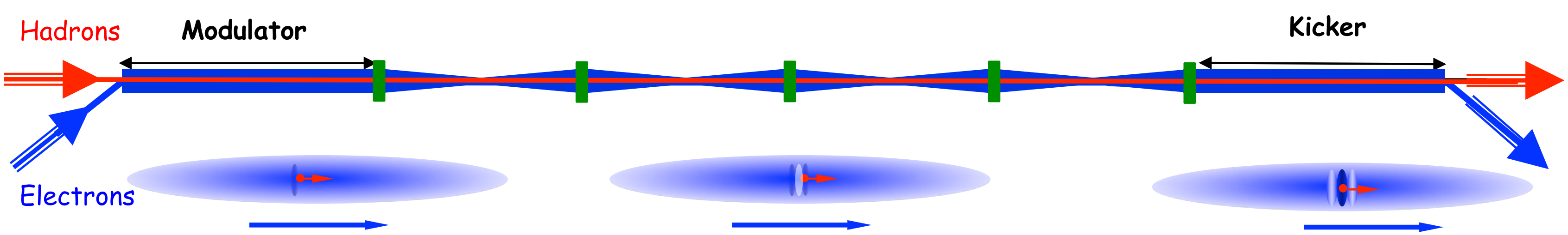
# Advantages and Disadvantages

- Very broad band amplifier, can operate at significant gain without saturation
- Plasma-cascade micro-bunching instability was experimentally demonstrated
- Has good theoretical model and is extensively studied in 3D numerical simulations
- Cool hadrons with all energy deviation (no anti-cooling)
- Does not require (full) separation of electron and hadron beams

- Micro-bunching amplifier was not demonstrated
- Requires better quality electron beam than FEL amplifier
- Can operate for medium hadron energies (up to hundreds of GeV, such as US EIC), but can not be extended to LHC energies
- Less studied than FEL-based CeC

## Plasma-Cascade Microbunching amplifier

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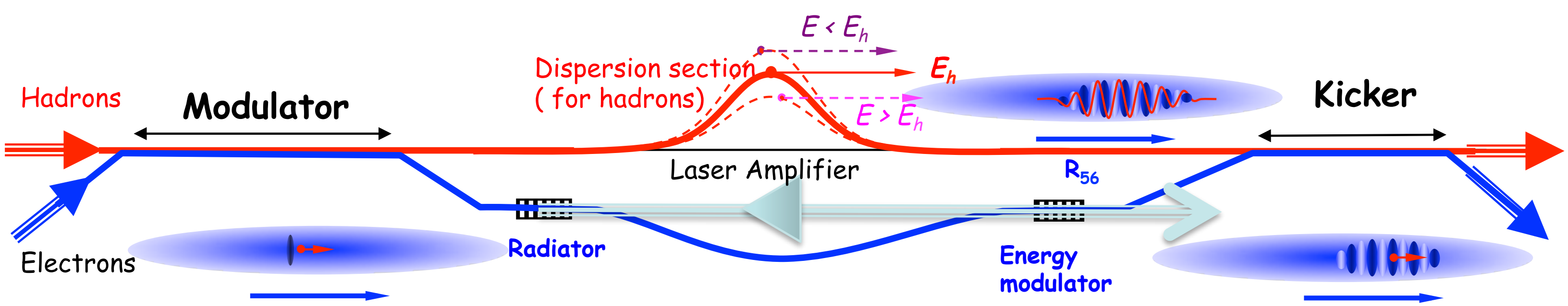
# Advantages and Disadvantages

- Relatively broad-band amplifier
- Take advantage of laser technology
- Does not require high peak current electron beam
- Take advantage of flexibility provided by high K-wigglers to adjust wavelength of radiation to that of the laser-amplifier
- Has some synergy with optical-stochastic cooling

- Not studied in details
- Definitely require separation of electron and hadron beams
- Would require rather significant delay of the hadron beam – may require $R_{56}$ reduction scheme for hadrons
- Semi-periodic structure of the modulation limits the energy range where cooling occurs

## Hybrid laser-beam amplifier

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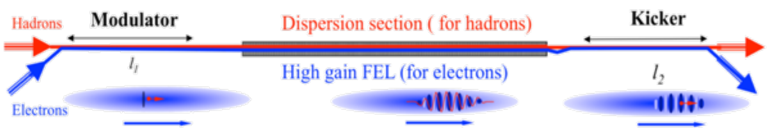
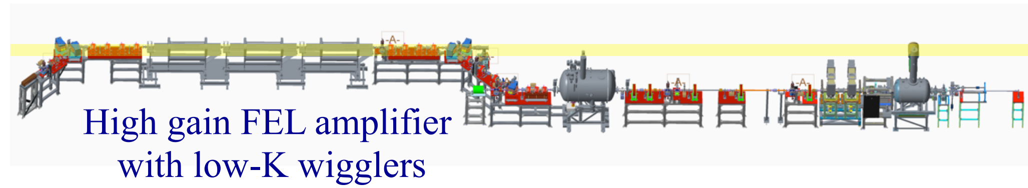
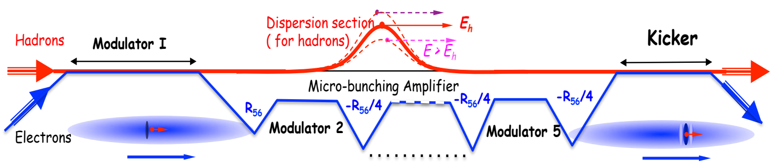
# What can be tested experimentally?



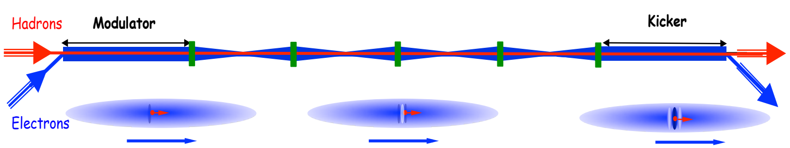



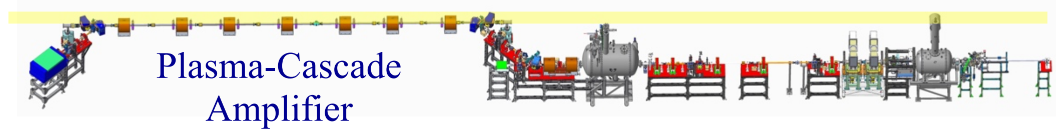

Cooling test requires serious modification of the RHIC lattice & superconducting magnets +$20-$30M

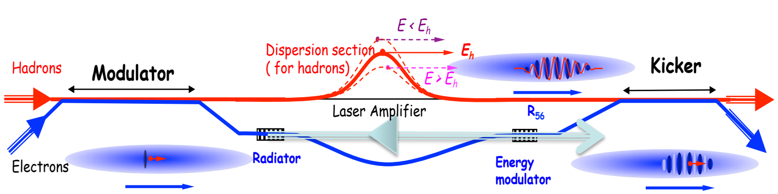
OR

Building new CeC system at another hadron storage ring




RHIC Runs 20-22




Cooling test requires serious modification of the RHIC lattice & superconducting magnets +$20-$30M

OR

Building new CeC system at another hadron storage ring

# Changing CeC amplifier from FEL to PCA

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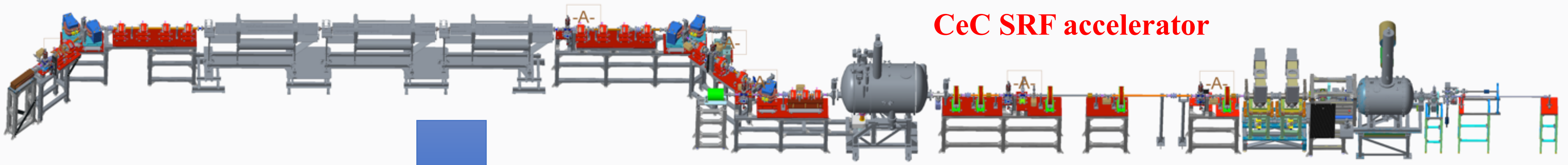
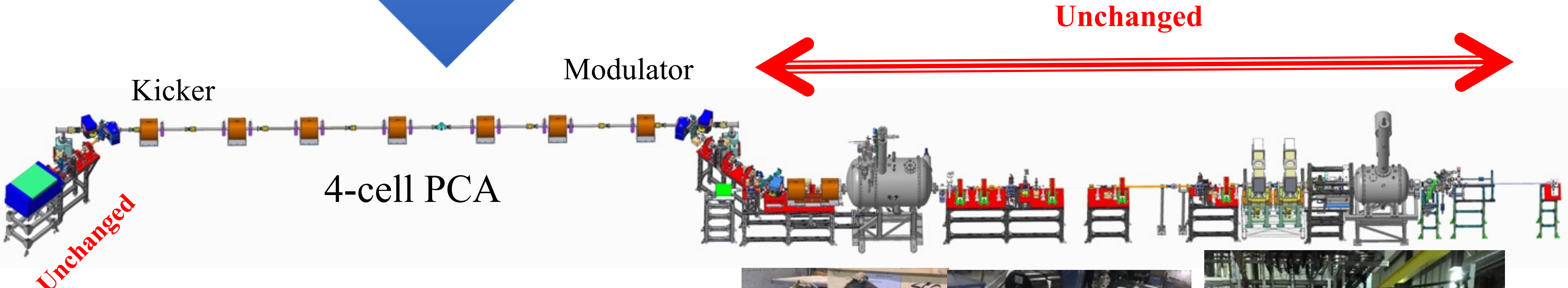
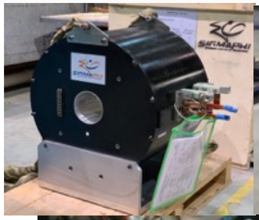
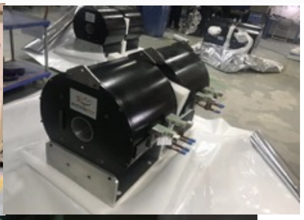
Small gap in FEL wigglers is not compatible with low energy RHIC operations of the Beam Energy Scan (BES-II) program

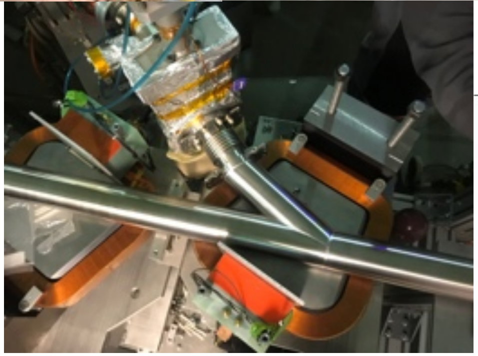


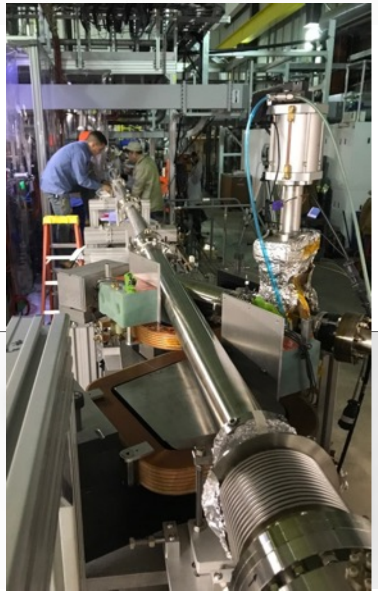




- Mechanical design new of the CeC system is completed. We used SBU NSF "Center for Accelerator Science and Education" grant to procure new hardware
- We procured and commissioned new laser system with controllable pulse structure
- All new vacuum chambers with beam diagnostics are built and installed
- All supports are built and installed
- All solenoids are designed, manufactured, delivered and undergo magnetic measurements
- Assembly of the plasma-cascade based CeC can be completed during this year's RHIC shut-down









# How to evaluate CeC: the original recipe

*Free Electron Lasers and High-energy Electron Cooling, Vladimir N. Litvinenko, Yaroslav S. Derbenev, Proceedings of 29th International Free Electron Laser Conference, Novosibirsk, Russia, August 27-31, 2007*

- Linear response of electron beam on perturbations – no saturation, superposition principle

$$\delta\vec{\mathbf{E}}_h = Ze\cdot\vec{\mathbf{G}}_{Eh}\left(\vec{r},\vec{r}_h,\gamma_h,t,t_h\right);\delta\vec{\mathbf{B}}_h = Ze\cdot\vec{\mathbf{G}}_{Bh}\left(\vec{r},\vec{r}_h,\gamma_h,t,t_h\right);$$

$$\delta\vec{\mathbf{E}}_e = -e\cdot\vec{\mathbf{G}}_{Ee}\left(\vec{r},\vec{r}_e,\gamma_e,t,t_e\right);\delta\vec{\mathbf{B}}_e = -e\cdot\vec{\mathbf{G}}_{Be}\left(\vec{r},\vec{r}_e,\gamma_e,t,t_e\right);$$

$$\vec{\mathbf{E}} = Ze\cdot\sum_h\vec{\mathbf{G}}_{Eh}\left(\vec{r},\vec{r}_h,\gamma_h,t,t_h\right) - e\cdot\sum_e\vec{\mathbf{G}}_{Ee}\left(\vec{r},\vec{r}_e,\gamma_e,t,t_e\right);$$

$$\vec{\mathbf{B}} = Ze\cdot\sum_h\vec{\mathbf{G}}_{Bh}\left(\vec{r},\vec{r}_h,\gamma_h,t,t_h\right) - e\cdot\sum_e\vec{\mathbf{G}}_{Be}\left(\vec{r},\vec{r}_e,\gamma_e,t,t_e\right)$$

- Evaluation of hadron distribution function using Fokker-Plank equation with both damping and diffusion terms
- Cooling transversely using coupling with longitudinal degrees of freedom

# How to evaluate CeC: the original recipe

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- Linear response of electron beam on perturbations – no saturation, superposition principle

$$\delta E_i = eZ\int \vec{\mathbf{E}}\cdot d\vec{r}_i;\ \ \delta \vec{p}_i = eZ\int\left(\vec{\mathbf{E}} + \frac{\left[\vec{p}_i \times \vec{\mathbf{B}}\right]}{\gamma_i m}\right)\cdot dt;\ \ \ X^T = (x,P_x,y,P_y,z,P_z);$$

$$\delta E_i = (eZ)^2 \cdot g_{Eh}\left(X_i,t_i\right) - Ze^2 \cdot g_{Ee}\left(X_i,t_i\right);$$

$$g_{Eh}\left(X_i,t_i\right) = \sum_h \int \vec{\mathbf{G}}_{Eh}\left(\vec{r}_i,\vec{r}_h,\gamma_h,t_i,t_h\right)\cdot d\vec{r}_i;\ \ g_{Ee} = \int\sum_e \vec{\mathbf{G}}_{Ee}\left(\vec{r}_i,\vec{r}_e,\gamma_e,t_i,t_e\right)\cdot d\vec{r}_i;$$

$$\delta \vec{p}_i = (eZ)^2 \cdot \vec{g}_{ph}\left(X_i,t_i\right) - Ze^2 \cdot \vec{g}_{pe}\left(X_i,t_i\right);$$

$$\vec{g}_{ph}\left(X_i,t_i\right) = \sum_h \int\left(\vec{\mathbf{G}}_{Eh} + \frac{\left[\vec{p}_i \times \vec{\mathbf{G}}_{Bh}\right]}{\gamma_i m}\right)\cdot dt_i;\ \vec{g}_{pe} = \sum_e \int\left(\vec{\mathbf{G}}_{Ee} + \frac{\left[\vec{p}_i \times \vec{\mathbf{G}}_{Be}\right]}{\gamma_i m}\right)\cdot dt_i$$

- Evaluation of hadron distribution function using Fokker-Plank equation with both damping and diffusion terms
- Cooling transversely using coupling with longitudinal degrees of freedom

# How to evaluate CeC: the original recipe

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- Linear response of electron beam on perturbations – no saturation, superposition principle
- Evaluation of hadron distribution function using Fokker-Plank equation with both damping and diffusion terms

$$\bar{f} = \langle \tilde{f} \rangle; \tilde{f} = \sum_h \delta\left(X - X_i(t)\right)$$

$$\frac{\partial \bar{f}(X,s)}{\partial t} + \frac{\partial}{\partial X_i}\left[\frac{dX_i(X,t)}{dt}\bar{f}(X,s)\right] - \frac{1}{2}\frac{\partial^2}{\partial X_i \partial X_k}\left[D_{ik}(X,t)\bar{f}(X,t)\right] = 0$$

$$\left\langle \frac{dX_i(X,t)}{dt} \right\rangle = \frac{1}{\tau}\int (X_i - Z_i)\cdot W(Z,X|\tau,t)\,dZ = \frac{1}{T_o}\langle \delta X_i \rangle$$

$$2D_{ik}(X,t) = \frac{1}{2\tau}\int (X_i - Z_i)(X_k - Z_k) W(Z,X|\tau,t)\,dZ = \frac{1}{T_o}\langle \delta X_i \cdot \delta X_i \rangle$$

..............................................

- Cooling transversely using coupling with longitudinal degrees of freedom

# Transverse cooling: the original recipe

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- Cooling transversely using coupling with longitudinal degrees of freedom by making energy kick depending on transverse motion (via $R_{51}$, $R_{52}$, $R_{53}$, $R_{54}$ or by displacing beam center in the kicker section)

$$X^T = [x, x', y, y', \tau, \delta]; \tau = -c(t - t_o); \delta = \frac{E - E_o}{\beta_o E_o};$$

$$\Delta E = F(X);\ \Delta\delta \equiv \Delta x_6 = \frac{F(X)}{\beta_o E_o} \approx const - \sum_{i=1}^{6} \zeta_i \cdot x_i;$$

$$\Delta X^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \Delta x_6 \end{bmatrix}; S = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix}; \sigma = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix};$$

$$X = \sum_{k=1}^{3} \left( a_k Y_k e^{i\psi_k} + a^*{}_k Y^*{}_k e^{-i\psi_k} \right);\ \ Y_j S Y_k^{*T} = -2i\delta_{kj};\ Y_k^T = \begin{bmatrix} y_{1k} & y_{2k} & y_{3k} & y_{4k} & y_{5k} & y_{6k} \end{bmatrix};$$

$$\Delta a_k = \frac{i}{2} \Delta X^T S Y_k^{*T} = \frac{i y^*{}_{5k}}{2} \Delta x_6 e^{-i\psi_k} = -\frac{i y^*{}_{5k}}{2} e^{-i\psi_k} \sum_{i=1}^{6} \zeta_i \cdot x_i;\ \ x_i = \sum_{j=1}^{3} \left( a_j y_{ij} e^{i\psi_j} + a^*{}_k Y^*{}_k e^{-i\psi_k} \right);$$

$$\Delta a_k = -\frac{i y^*{}_{5k}}{2} \sum_{i=1}^{6} \zeta_i \sum_{j=1}^{3} a_j y_{ij} e^{i(\psi_j - \psi_k)} \to \langle \Delta a_k \rangle = -\xi_k a_k \to a_k = a_{k0} e^{-n\xi_k};\ \sum_k \mathrm{Re}\,\xi_k = TrD = \zeta_6 \equiv \zeta_\delta$$

$$\left\langle e^{i(\psi_j - \psi_k)} \right\rangle = \delta_{kj}; \left\langle e^{-i(\psi_j + \psi_k)} \right\rangle = 0;\ \ \xi_k = \frac{i}{2} \sum_{i=1}^{6} \zeta_i y^*{}_{5k} y_{ik};\ \mathrm{Re}\,\xi_k = \mathrm{Im} \sum_{i=1}^{6} \zeta_i y^*{}_{5k} y_{ik}$$

# Transverse cooling: the original recipe

- Cooling transversely using coupling with longitudinal degrees of freedom by making energy kick depending on transverse motion (via $R_{51}$, $R_{52}$, $R_{53}$, $R_{54}$ or by displacing beam center in the kicker section) – we can only redistribute cooling decrements between three eigen modes

$$\langle \Delta a_k \rangle = -\xi_k a_k \rightarrow a_k = a_{k0} e^{-n\xi_k}; \quad \xi_k = \frac{i}{2}\sum_{i=1}^{6} \zeta_i y^*{}_{5k} y_{ik}; \quad \sum_k \xi_k = TrD = \zeta_6 \equiv \zeta_\delta$$

For slow synchrotron oscillations ($Q_s$<<1)

$$Q_s << Q_{1,2}$$

$$Y_{k=1,2} \cong \begin{pmatrix} y_{1k} \\ y_{2k} \\ y_{3k} \\ y_{4k} \\ y_{5k} \\ 0 \end{pmatrix} = \begin{pmatrix} Y_{k\beta} \\ -Y_{k\beta}^T SD \\ 0 \end{pmatrix}; Y_3 \cong \frac{1}{\sqrt{\Omega}} \begin{pmatrix} D_x \\ D'_x \\ D_y \\ D'_y \\ i\Omega \\ 1 \end{pmatrix};$$

$$y_{5k} = -Y_{k\beta}{}^T SD$$

$$\xi_k = \frac{i}{2}\left(Y_{k\beta}{}^T SD\right)\sum_{i=1}^{6} \zeta_i y_{ik};$$

Hence, introduction dependencies on the components of transverse motion and non-zero dispersion in the kicker section allows to re-distribute cooling decrements

# Instead of conclusion

- ✓ There is a variety of amplifiers suitable for CeC
  - ✓ In addition to what we discussed now, Yaroslav Derbenev is proposing using coherent synchrotron radiation instability as CeC amplifier – one need a specific schematic to understand how it fit into the CeC family
- ✓ Theoretical evaluation is typically limited to 1D, but 3D simulation are performed for two CeC schemes
- ✓ Two CeC options can be tested experimentally at RHIC – we currently pursuing CeC with plasma-cascade amplifier
- ✓ The evaluation scheme that I presented only looking simple – evil is always in details
- ✓ Following presentations will give a much deeper view into physics and realities of CeC

# Back-up

# Sum of decrements theorem



Let's consider an arbitrary linear s-dependent equation:

$$\frac{dX}{ds} = \mathbf{D}(s) \cdot X; \qquad \text{(SD-1)}$$

e.g. the overall motion is not necessary symplectic

$$X(s) = \mathbf{R}(s) X_o \rightarrow \frac{d\mathbf{R}}{ds} = \mathbf{DR} \rightarrow \frac{d}{ds}\det[\mathbf{R}(s)] = Trace[\mathbf{D}(s)] \cdot \det[\mathbf{R}(s)]$$

$$\det[\mathbf{R}(s)] = \int_o^s Trace[\mathbf{D}(\xi)] d\xi. \qquad \text{(SD-2)}$$

Prove of the later is rather trivial

$$\mathbf{R}(s+ds) = \left(\mathbf{I} + ds\mathbf{D}(s^*) + ds^2\mathbf{O}\right) \cdot \mathbf{R}(s);\ s^* \in \{s, s+ds\} \rightarrow \det \mathbf{R}(s+ds) = \det\left(\mathbf{I} + ds\mathbf{D}(s^*) + ds^2\mathbf{O}\right) \cdot \det \mathbf{R}(s);$$

$$\det A == \sum_{i,j,k} \varepsilon_{ijk...} a_{1i} a_{2j} a_{3k} \cdots; \det\left(\mathbf{I} + ds\mathbf{D}(s^*) + \varepsilon^2\mathbf{O}\right) = \sum_{i,j,k} \varepsilon_{ijk...} \left(\delta_{1i} + dsd_{1i}\right)\left(\delta_{2j} + dsd_{2j}\right)\left(\delta_{3k} + dsd_{3k}\right)... + O\left(\varepsilon^2\right)$$

$$\sum_{i,j,k} \varepsilon_{ijk...} \left(\delta_{1i} + dsd_{1i}\right)\left(\delta_{2j} + dsd_{2j}\right)\left(\delta_{3k} + dsd_{3k}\right)... = \prod_{i=1}^{2n} \left(\delta_{ii} + dsd_{ii}\right) + ds^2 \sum_{i \neq 1, j, k} \varepsilon_{ijk\cdot 1..} d_{1i} \sum_{i,j,k} \varepsilon_{ijk...} \left(\delta_{2j} + dsd_{2j}\right)\left(\delta_{3k} + dsd_{3k}\right)....d_{i1}... +$$

$$+ ds^2 \sum_{i, j \neq 2, k} \varepsilon_{ijk...} \sum_{i,j,k} \varepsilon_{ijk...} \left(\delta_{1i} + dsd_{1i}\right) d_{2j} \left(\delta_{3k} + dsd_{3k}\right)...d_{j2} + ... = \prod_{i=1}^{2n} \left(\delta_{ii} + dsd_{ii}\right) + O\left(ds^2\right);$$

$$\prod_{i=1}^{2n} \left(\delta_{ii} + dsd_{ii}\right) = 1 + ds \sum_{i=1}^{2n} d_{ii} + O\left(ds^2\right) = 1 + ds \cdot Tr\left[\mathbf{D}(s^*)\right];$$

$$\det \mathbf{R}(s+ds) \equiv \det \mathbf{R}(s) + d\left(\det \mathbf{R}(s)\right) + O\left(ds^2\right) = \left(1 + ds \cdot Tr\left[\mathbf{D}(s^*)\right] + O\left(ds^2\right)\right) \cdot \det \mathbf{R}(s);$$

$$ds \rightarrow 0 \Rightarrow d\left(\det \mathbf{R}(s)\right) = ds \cdot Tr[\mathbf{D}(s)] \cdot \det \mathbf{R}(s);$$

$$\frac{d}{ds} \det \mathbf{R}(s) = Tr[\mathbf{D}(s)] \cdot \det \mathbf{R}(s).$$

(SD-2)

# Status: *more details in my afternoon talk*

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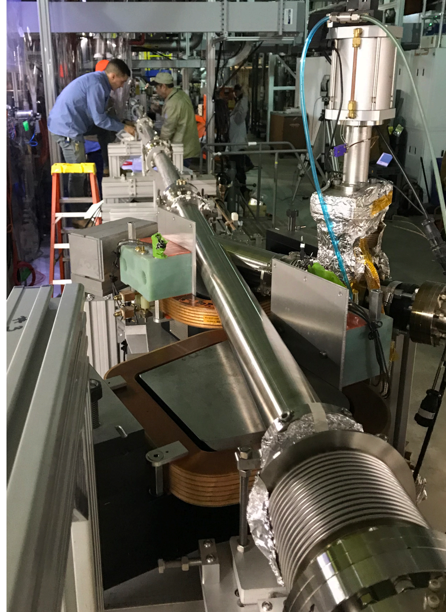
- Mechanical design new CeC system is completed
- We procured and commissioned new laser system with controllable pulse structure
- All new vacuum with beam diagnostics are built  chambers are installed
- All supports are built and installed
- All solenoids are designed, manufactured, delivered and undergo magnetic measurements
- Assembly of the plasma-cascade based CeC can be completed during this year's RHIC shut-down period

# Distribution of the decrements

$$X=\frac{1}{2}\sum_{k=1}^{3}\left(a_k Y_k(s)e^{i\psi_k}+c.c.\right);\;\; Y_j^{*T}SY_k=2i\delta_{jk};Y_j^{T}SY_k=0;\;\; S=\begin{bmatrix}\sigma & 0 & 0\\ 0 & \sigma & 0\\ 0 & 0 & \sigma\end{bmatrix};\sigma=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$$

$$\delta X=-\xi\begin{bmatrix}0\\0\\0\\0\\0\\ \delta+k_x x\end{bmatrix}=-\xi K\cdot X=-\xi\begin{bmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\k_x&0&0&0&0&1\end{bmatrix}\cdot\frac{1}{2}\sum_{k=1}^{3}\left(a_k Y_k(s)e^{i\psi_k}+c.c.\right);$$

$$\delta a_k=-\xi\frac{e^{-i\psi_k}}{2i}Y_k^{*T}SK\cdot\sum_{j=1}^{3}\left(a_j Y_j(s)e^{i\psi_j}+c.c.\right);$$

$$\xi_k=\frac{\langle\delta a_k\rangle}{a_k}=-\xi\frac{Y_k^{*T}SKY_k}{2i};\qquad 2\cdot\sum_{k=1}^{3}\xi_k=\xi\cdot Tr(K)=\xi;$$

$$\xi_k=\frac{\xi}{2i}\cdot Y_k^{5*}\left(k_x Y_k^1+Y_k^6\right)$$

$$X^T=\{x,x',y,y',-c\tau,\delta\}$$

$$k_x=\frac{R_{52e}}{D_{zh}}$$

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# Distribution of the decrements

$$Q_s << Q_{1,2}$$

$$Y_{k=1,2} \cong \begin{pmatrix} Y_{k1} \\ Y_{k2} \\ Y_{k3} \\ Y_{k4} \\ Y_{k5} \\ 0 \end{pmatrix} = \begin{pmatrix} Z_k \\ -Z_k^T SD \\ 0 \end{pmatrix}; Y_3 \cong \frac{1}{\sqrt{\Omega}} \begin{pmatrix} D_x \\ D'_x \\ D_y \\ D'_y \\ i\Omega \\ 1 \end{pmatrix};$$

$$\xi_k = \frac{\xi}{2i} \cdot Y_k^{*5} \left( k_x Y_k^1 + Y_k^6 \right)$$

$$\xi_s = \frac{\xi}{2}\left( k_x D_x + 1 \right);$$

$$\xi_{k=1,2} = -\frac{\xi}{2i} \cdot \left( Z_k^{*T} SD \right) \cdot k_x Z_k^1$$

$$\xi_1 + \xi_2 = -k_x D_x \frac{\xi}{2}$$

## Uncoupled case

$$\xi_y = 0; \quad \operatorname{Re} \xi_x = -\frac{\xi}{2} \cdot R_{52e} \frac{D_{xh}}{D_{zh}}; \quad \operatorname{Re} \xi_s = \frac{\xi}{2} \cdot \left( 1 - R_{52e} \frac{D_{xh}}{D_{zh}} \right)$$