Homework 10. Due October 12

Problem 1. 4x5 points. Matrix of an ideal solenoid.
Consider particles with momentum p_o propagating along the axis of idealized solenoid with

$$B_s = \left\{ \begin{array}{c} 0, s < 0 \\ B_o, 0 \le s \le l \\ 0, s > 1 \end{array} \right\}$$

All other components of the field are zero, e.g. s=z, not curvature.

(a) Use Sylvester formula and calculate 4x4 transport matrix of the solenoid;

(b) Show that resulting matrix can be presented is form of focusing matrix in each direction and a rotation

$$M_s = \begin{bmatrix} I\cos\varphi & I\sin\varphi \\ -I\sin\varphi & I\cos\varphi \end{bmatrix} \cdot \begin{bmatrix} F & 0 \\ 0 & F \end{bmatrix}$$

where

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; F = \begin{bmatrix} a & b \\ c & a \end{bmatrix}; ab - cd = 1$$

are 2x2 matrices and F is focusing one. Write expressions for φ, F through $p_o, B_o, l...,$

(c) Finally use one tricks available for you since we can use torsion and decouple x and y motion:

$$\tilde{h}_n = \frac{\pi_1^2 + \pi_3^2}{2} + f\frac{x^2}{2} + g\frac{y^2}{2} + L(x\pi_3 - y\pi_1)$$

$$f = \left(\frac{eB_s}{2p_oc}\right)^2; g = \left(\frac{eB_s}{2p_oc}\right)^2; L = \kappa + \frac{e}{2p_oc}B_s;$$

by choosing $\kappa = -\frac{e}{2p_oc}B_s$. Show that matrix in this coordinates system is block diagonal (e.g. de-coupled)

$$M_s = \begin{bmatrix} F & 0 \\ 0 & F \end{bmatrix}$$

with F identical to that in the problem (b) above. Show also that rotation is angle aroin z-axis is $\kappa l = -\varphi$.

(d) Finally, explain why a simple trajectory x=const and y=const (which intuitively is trajectory parallel to the magnetic lines) is not a solution?

$$\mathrm{v}_{x,y} = 0; \rightarrow \vec{\mathrm{v}} = \hat{z}\mathrm{v}_o; \vec{f} = \frac{e}{c}\left[\hat{z}\mathrm{v}_o \times \hat{z}B_0\right] = 0$$

Hint: consider what is happening at the entrance and exit to the solenoid.