

## Homework 12

### Problem 1. 10 points, 2D distribution function and RMS beam sizes

For the case of fully coupled transverse oscillations with eigen vectors

$$Y_1 = \begin{bmatrix} w_{1x} e^{i\varphi_{1x}} \\ \left( u_{1x} + i \frac{q}{w_{1x}} \right) e^{i\varphi_{1x}} \\ w_{1y} e^{i\varphi_{1y}} \\ \left( u_{1y} + i \frac{1-q}{w_{1y}} \right) e^{i\varphi_{1y}} \end{bmatrix}; Y_2 = \begin{bmatrix} w_{2x} e^{i\varphi_{2x}} \\ \left( u_{2x} + i \frac{1-q}{w_{2x}} \right) e^{i\varphi_{2x}} \\ w_{2y} e^{i\varphi_{2y}} \\ \left( u_{2y} + i \frac{q}{w_{2y}} \right) e^{i\varphi_{2y}} \end{bmatrix}$$

and known values of eigen emittances  $\varepsilon_{1,2} \equiv I_{1,2} = \frac{\langle a_{1,2}^2 \rangle}{2}$  of stationary Gaussian distribution

(solution of Fokker-Plank equation)

- (a) **6 points**; Write explicit expression for the distribution function in terms of  $x$ ,  $P_x$ ,  $y$  and  $P_y$ .
- (b) **4 points**; Write expression of the RMS beam sizes

$$\sigma_x = \sqrt{\langle x^2 \rangle}; \sigma_y = \sqrt{\langle y^2 \rangle}$$

using beam emittances and necessary components of eigen vectors.

### Problem 2. 10 points, 3D distribution function and RMS beam sizes

- (a) 5 points: For the case of fully coupled transverse oscillations with eigen vectors

$$Y_k(s) = \begin{bmatrix} w_{kx} e^{i\chi_{kx}} \\ \left( v_{kx} + i \frac{q_{kx}}{w_{kx}} \right) e^{i\chi_{kx}} \\ w_{ky} e^{i\chi_{ky}} \\ \left( v_{ky} + i \frac{q_{ky}}{w_{ky}} \right) e^{i\chi_{ky}} \\ w_{k\tau} e^{i\chi_{k\tau}} \\ \left( v_{k\tau} + i \frac{q_{k\tau}}{w_{k\tau}} \right) e^{i\chi_{k\tau}} \end{bmatrix}; k = 1, 2, 3$$

and known values of eigen emittances  $\varepsilon_k \equiv I_k = \frac{\langle a_k^2 \rangle}{2}; k = 1, 2, 3$  of stationary Gaussian distribution, write expression of the RMS beam sizes

$$\sigma_x = \sqrt{\langle x^2 \rangle}; \sigma_y = \sqrt{\langle y^2 \rangle}; \sigma_\tau = \sqrt{\langle \tau^2 \rangle}$$

using the beam emittances and necessary components of eigen vectors.

(b) 5 points: For the case of slow synchrotron oscillations and approximate expressions for the eigen vectors:

$$Y_k = \begin{bmatrix} Y_{k\beta} \\ y_{k\tau} \\ 0 \end{bmatrix} = \begin{bmatrix} w_{kx} e^{i\chi_{kx}} \\ \left( v_{kx} + \frac{iq_k}{w_{kx}} \right) e^{i\chi_{kx}} \\ w_{ky} e^{i\chi_{ky}} \\ \left( v_{ky} + \frac{i(1-q_k)}{w_{ky}} \right) e^{i\chi_{ky}} \\ y_{k\tau} = \eta^T S Y_{k\beta} \\ 0 \end{bmatrix}; k=1,2; Y_\delta = \begin{bmatrix} \eta \\ \chi_\tau \\ 1 \end{bmatrix} = \begin{bmatrix} \eta_x \\ \eta_{px} \\ \eta_y \\ \eta_{py} \\ \chi_\tau \\ 1 \end{bmatrix};$$

and known values of eigen emittances  $\varepsilon_k \equiv I_k = \frac{\langle a_k^2 \rangle}{2}; k=1,2$  and RMS values of the

relative energy spread  $\sigma_\delta = \sqrt{\langle \delta^2 \rangle}$  write expressions for transverse beam sizes:

$$\sigma_x = \sqrt{\langle x^2 \rangle}; \sigma_y = \sqrt{\langle y^2 \rangle}$$