Homework 1. PHY 564

Problem 1. 2 points. Lorentz transformations

Show explicitly that two successive Lorentz transformations in the same direction are equivalent to a single Lorentz transformation with $v = \dfrac{v_1 + v_2}{1 + (v_1 v_2 / c^2)}$.

Problem 2. 2 points. 4-invarints

Show that trace of a tensor is 4-invariant, i.e. $F_i^i \equiv \sum_{i=0}^{3} F_i^i = inv$.

Problem 3. Lorentz group (read additional material on the website)

a) **5 points.** For the Lorentz boost and rotation matrices **K** and **S** show that

$$\left(\vec{\varepsilon}\vec{\mathbf{S}}\right)^3 = -\vec{\varepsilon}\vec{\mathbf{S}};\ \left(\vec{\varepsilon}\vec{K}\right)^3 = \vec{\varepsilon}\vec{K};\forall\,\vec{\varepsilon} = \vec{\varepsilon}^*;|\vec{\varepsilon}| = 1;$$
$$\text{or } \left(\vec{a}\vec{\mathbf{S}}\right)^3 = -\vec{a}\vec{\mathbf{S}}\cdot\vec{a}^2;\ \left(\vec{a}\vec{\mathbf{K}}\right)^3 = \vec{a}\vec{\mathbf{K}}\cdot\vec{a}^2;\forall\,\vec{a} = \vec{a}.$$

b) **5 points.** Use these results to show that

$$e^{\vec{\omega}\vec{\mathbf{S}}} = I + \frac{\vec{\omega}\vec{\mathbf{S}}}{|\vec{\omega}|}\sin|\vec{\omega}| + \frac{\left(\vec{\omega}\vec{\mathbf{S}}\right)^2}{\vec{\omega}^2}(\cos|\vec{\omega}| - 1);$$
$$e^{\vec{\beta}\vec{K}} = I + \frac{\vec{\beta}\vec{\mathbf{K}}}{\left|\vec{\beta}\right|}\sinh\left|\vec{\beta}\right| + \frac{\left(\vec{\beta}\vec{\mathbf{K}}\right)^2}{\vec{\beta}^2}(\cosh\left|\vec{\beta}\right| - 1);$$

Draw connection to Lorentz transformations (e.g. boosts and rotations).