Smearing of Wake due to Energy Deviation

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Will’s equation

$$w(z_k - z_m, \delta) \approx \int_{-L_m/2}^{L_m/2} \frac{ds_m}{L_m} \int_{-L_k/2}^{L_k/2} \frac{ds_k}{L_k} w(z_k - z_m + (s_k - s_m)\delta/\gamma^2)$$

$$w(z) = w_0 \exp\left(-\frac{z^2}{2\sigma_c^2}\right)\sin(kz)$$

$$w(\Delta z, \delta) = \frac{\sqrt{2\pi}\sigma_c w_0}{4R_{56}\delta} e^{-\frac{k^2\sigma_c^2}{2}} \int_{-1/2}^{1/2} \left\{ f_1\left(k\Delta z + kR_{56}\delta\left(\frac{1}{2} - \bar{s}_m\right)\right) - f_1\left(k\Delta z - kR_{56}\delta\left(\frac{1}{2} - \bar{s}_m\right)\right)\right\} d\bar{s}_m$$

$$f_1(x) = ierf\left(\frac{ik\sigma_c}{\sqrt{2}} + \frac{x}{\sqrt{2}k\sigma_c}\right) + ierf\left(\frac{ik\sigma_c}{\sqrt{2}} - \frac{x}{\sqrt{2}k\sigma_c}\right)$$

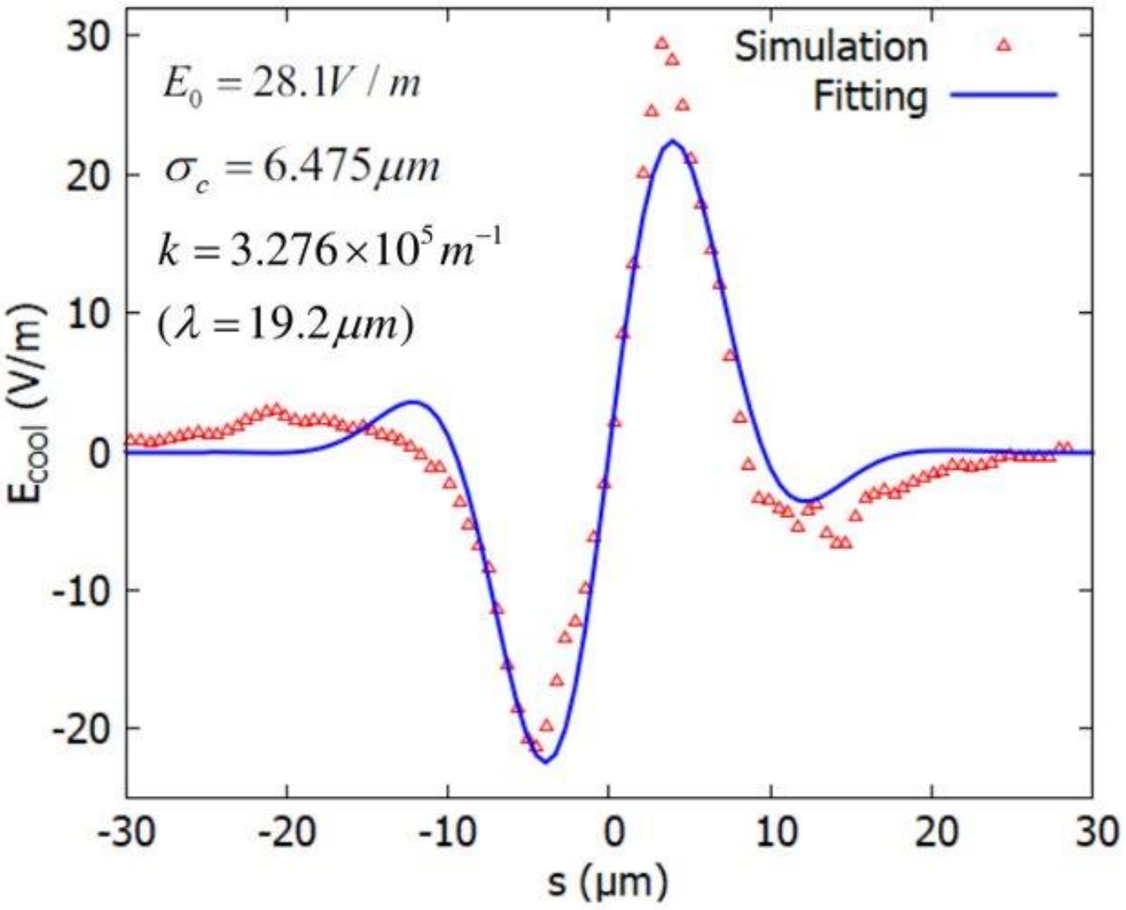
$$\Delta z = z_k - z_m$$

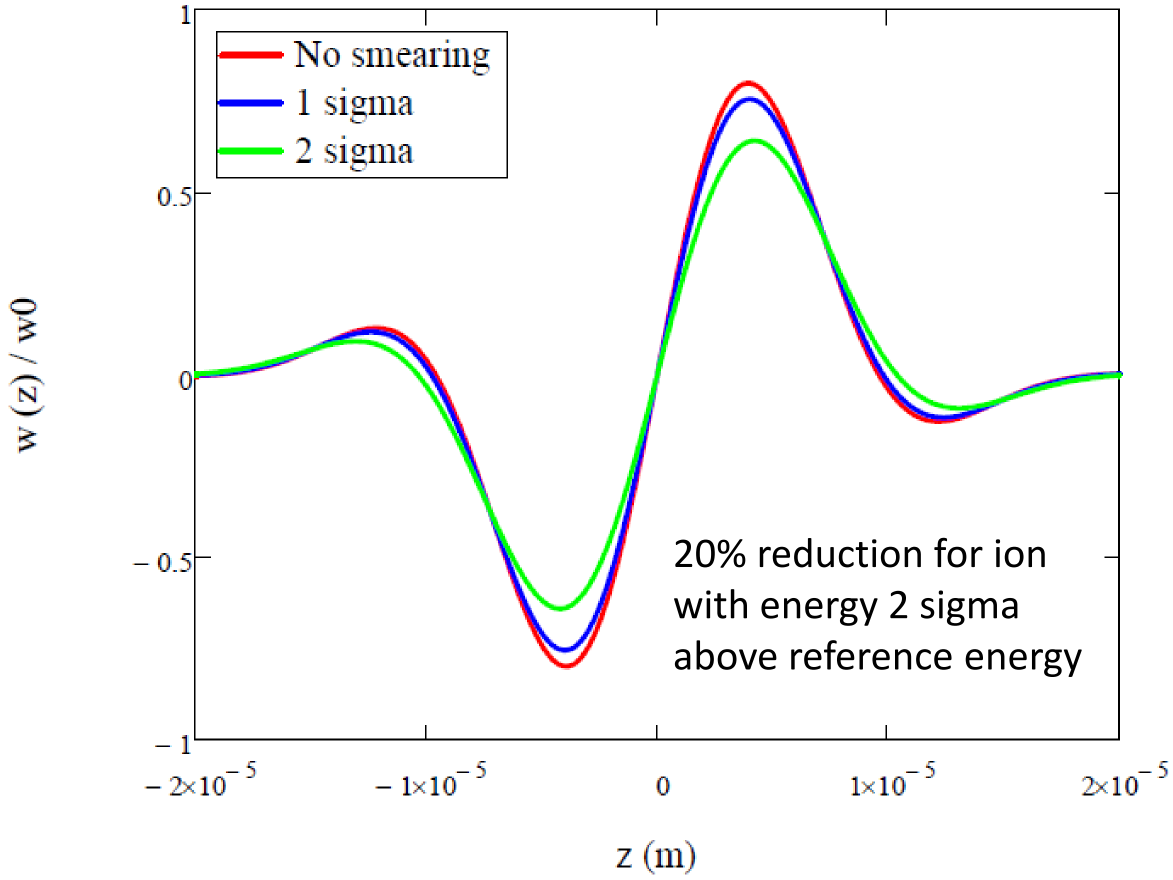
For the case of the CeC experiment

Notice that R56 is about 1.4 cm from modulator to kicker and the shift for 2 sigma is 16 um, i.e. we are not cooling these ions anyway. Our goal is to observe cooling of the ions with relatively small energy deviation and related profile change of the ion beam, not to reduce the RMS energy spread.

$$L_{k,m} = 3m$$

$$\sigma_p = 6\times10^{-4}$$





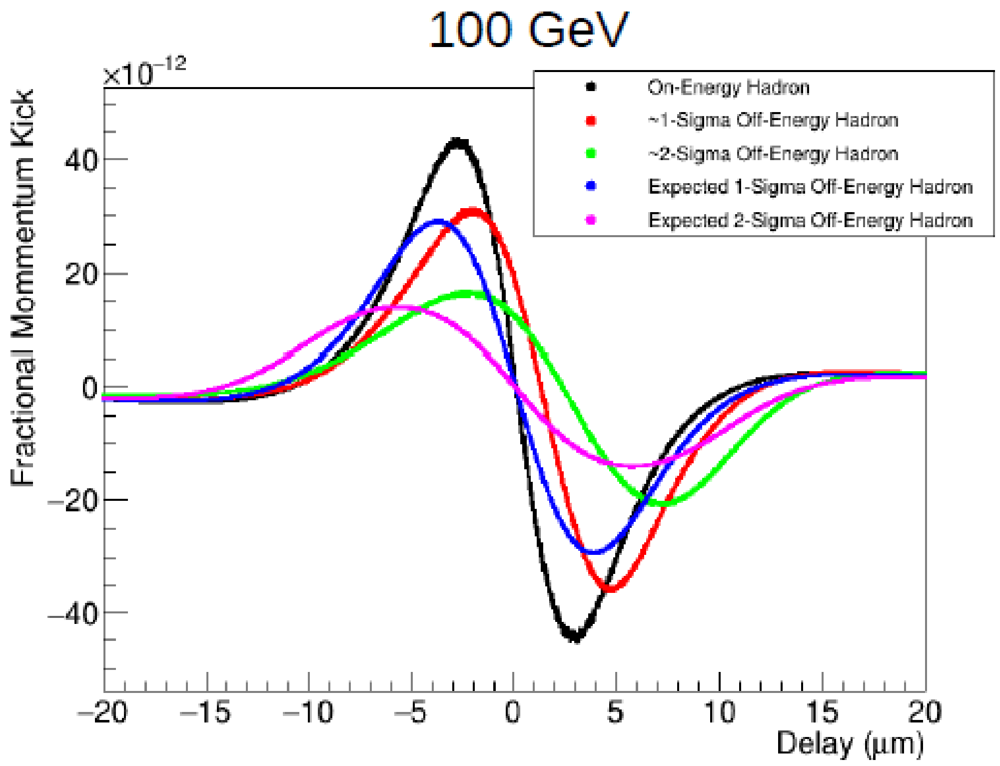
For the case of EIC with 60 m of kicker length and 100 GeV protons

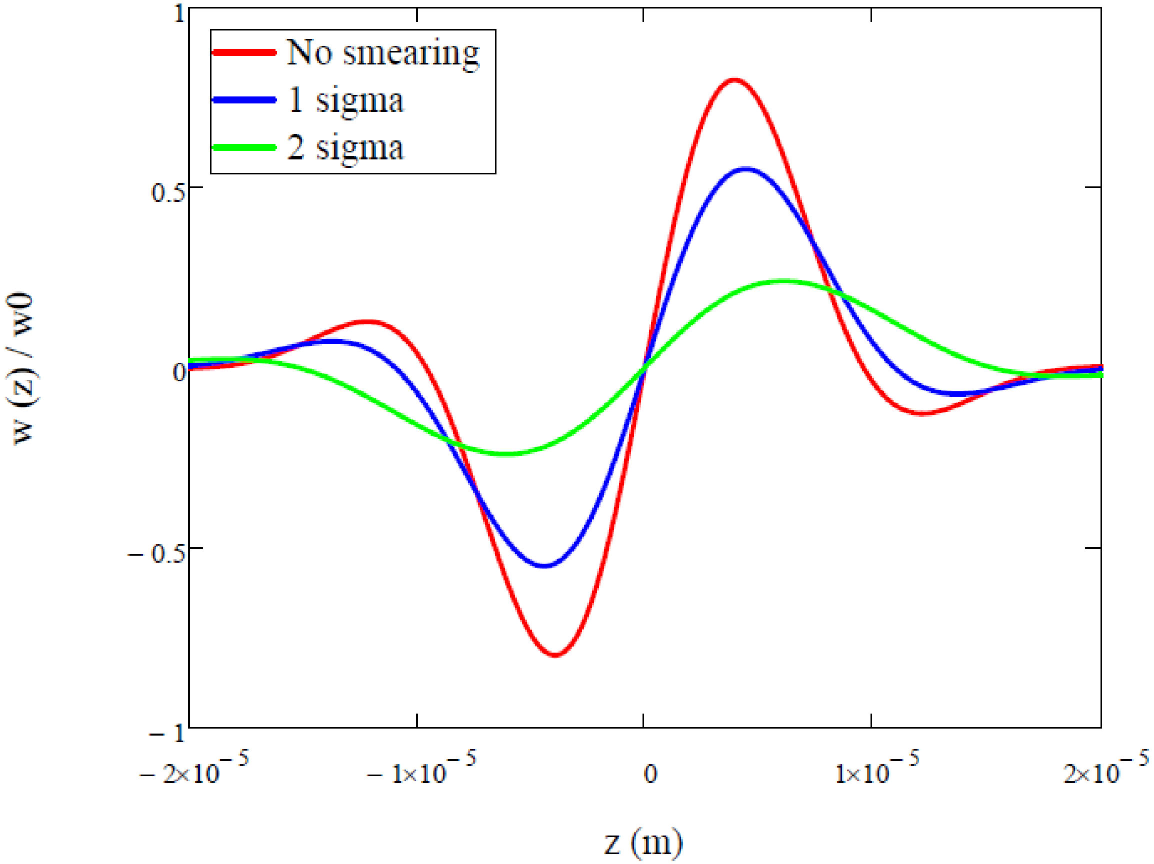
For an estimate, I used the same wavelength and coherent length since the shape of the wake are similar to that of the CeC experiment.

Noticing that there is no horizontal shift among curves, which is likely not included in the formula. If I remember correctly, Will used tracking code to get his curve, not the formula.

$$L_{k,m} = 60m$$

$$\sigma_p = 9.7 \times 10^{-4}$$





Backup slides

Will's equation

$$w(z_k - z_m, \delta) \approx \int_{-L_m/2}^{L_m/2} \frac{ds_m}{L_m} \int_{-L_k/2}^{L_k/2} \frac{ds_k}{L_k} w(z_k - z_m + (s_k - s_m)\delta/\gamma^2)$$

$$\begin{aligned}
w(\Delta z, \delta) &= \int_{-L_m/2}^{L_m/2} \frac{ds_m}{L_m} \int_{-L_k/2}^{L_k/2} \frac{ds_k}{L_k} w\left(z_k - z_m + \left(s_k - s_m\right)\delta / \gamma^2\right) \\
&= \int_{-1/2}^{1/2} d\overline{s}_m \int_{-1/2}^{1/2} d\overline{s}_k w\left(z_k - z_m + \left(\overline{s}_k L_k - \overline{s}_m L_m\right)\delta / \gamma^2\right) \\
&= \int_{-1/2}^{1/2} d\overline{s}_m \int_{-1/2}^{1/2} d\overline{s}_k w\left(z_k - z_m + \left(\overline{s}_k - \overline{s}_m\right) L_m \delta / \gamma^2\right) \\
&= \int_{-1/2}^{1/2} d\overline{s}_m \int_{-1/2-\overline{s}_m}^{1/2-\overline{s}_m} w\left(\Delta z + \alpha \cdot \Delta\overline{s}\right) d\Delta\overline{s} \\
&= \frac{1}{\alpha} \int_{-1/2}^{1/2} d\overline{s}_m \int_{-\alpha(1/2+\overline{s}_m)}^{\alpha(1/2-\overline{s}_m)} w\left(\Delta z + x\right) dx \\
&= \frac{1}{\alpha} \int_{-1/2}^{1/2} d\overline{s}_m \int_{\Delta z-\alpha(1/2+\overline{s}_m)}^{\Delta z+\alpha(1/2-\overline{s}_m)} w\left(y\right) dy \\
&= \frac{1}{k\alpha} \int_{-1/2}^{1/2} d\overline{s}_m \int_{k[\Delta z-\alpha(1/2+\overline{s}_m)]}^{k[\Delta z+\alpha(1/2-\overline{s}_m)]} \exp\left(-\frac{\eta^2}{2k^2\sigma_c^2}\right) \sin(\eta)\, d\eta
\end{aligned}$$

$$\begin{aligned}
w(\Delta z, \delta) &= \frac{1}{k\alpha} \int_{-1/2}^{1/2} d\overline{s}_m \int_{k[\Delta z-\alpha(1/2+\overline{s}_m)]}^{k[\Delta z+\alpha(1/2-\overline{s}_m)]} \exp\left(-\frac{\eta^2}{2k^2\sigma_c^2}\right) \sin(\eta)\, d\eta \\
&= -\frac{1}{k\alpha} \frac{\sqrt{\pi}}{4} a \exp\left(-\frac{a^2}{4}\right) \int_{-1/2}^{1/2} \left[erfi\left(\frac{a}{2} - \frac{ix_+}{a}\right) + erfi\left(\frac{a}{2} + \frac{ix_+}{a}\right) - erfi\left(\frac{a}{2} - \frac{ix_-}{a}\right) - erfi\left(\frac{a}{2} + \frac{ix_-}{a}\right) \right] d\overline{s}_m \\
&= \frac{i}{k\alpha} \frac{\sqrt{\pi}}{4} a \exp\left(-\frac{a^2}{4}\right) \int_{-1/2}^{1/2} \left[erf\left(\frac{ia}{2} + \frac{x_+}{a}\right) + erf\left(\frac{ia}{2} - \frac{x_+}{a}\right) - erf\left(\frac{ia}{2} + \frac{x_-}{a}\right) - erf\left(\frac{ia}{2} - \frac{x_-}{a}\right) \right] d\overline{s}_m
\end{aligned}$$