Homework 14

Problem 1. 10 points, 2D distribution function and RMS beam sizes

For the case of fully coupled transverse oscillations with eigen vectors

$$Y_1 = \begin{bmatrix} w_{1x} e^{i\varphi_{1x}} \\ \left(u_{1x} + i\dfrac{q}{w_{1x}} \right) e^{i\varphi_{1x}} \\ w_{1y} e^{i\varphi_{1y}} \\ \left(u_{1y} + i\dfrac{1-q}{w_{1y}} \right) e^{i\varphi_{1y}} \end{bmatrix}; \quad Y_2 = \begin{bmatrix} w_{2x} e^{i\varphi_{2x}} \\ \left(u_{2x} + i\dfrac{1-q}{w_{2x}} \right) e^{i\varphi_{2x}} \\ w_{2y} e^{i\varphi_{2y}} \\ \left(u_{2y} + i\dfrac{q}{w_{2y}} \right) e^{i\varphi_{2y}} \end{bmatrix}$$

and known values of eigen emittances $\varepsilon_{1,2} \equiv I_{1,2} = \dfrac{\left\langle a_{1,2}^2 \right\rangle}{2}$ of stationary Gaussian distribution (solution of Fokker-Plank equation)

(a) **6 points**; Write explicit expression for the distribution function in terms of x, Px, y and Py.

(b) **4 points**; Write expression of the RMS beam sizes

$$\sigma_x = \sqrt{< x^2 >}; \sigma_y = \sqrt{< y^2 >}$$

using beam emittances and necessary components of eigen vectors.

Problem 2. 10 points, 3D distribution function and RMS beam sizes

(a) 5 points: For the case of fully coupled transverse oscillations with eigen vectors

$$Y_k(s) = \begin{bmatrix} \mathrm{w}_{kx} e^{i\chi_{kx}} \\ \left(\mathrm{v}_{kx} + i\dfrac{q_{kx}}{\mathrm{w}_{kx}} \right) e^{i\chi_{kx}} \\ \mathrm{w}_{ky} e^{i\chi_{ky}} \\ \left(\mathrm{v}_{ky} + i\dfrac{q_{ky}}{\mathrm{w}_{ky}} \right) e^{i\chi_{ky}} \\ \mathrm{w}_{k\tau} e^{i\chi_{k\tau}} \\ \left(\mathrm{v}_{k\tau} + i\dfrac{q_{k\tau}}{\mathrm{w}_{k\tau}} \right) e^{i\chi_{k\tau}} \end{bmatrix}; k = 1,2,3$$

and known values of eigen emittances $\varepsilon_k \equiv I_k = \dfrac{\left\langle a_k^2 \right\rangle}{2}; k = 1,2,3$ of stationary Gaussian distribution, write expression of the RMS beam sizes

$$\sigma_x = \sqrt{<x^2>}; \sigma_y = \sqrt{<y^2>}; \sigma_\tau = \sqrt{<\tau^2>}$$

using the beam emittances and necessary components of eigen vectors.

(b) 5 points: For the case of slow synchrotron oscillations and approximate expressions for the eigen vectors:

$$Y_k = \begin{bmatrix} Y_{k\beta} \\ \mathrm{y}_{k\tau} \\ 0 \end{bmatrix} = \begin{bmatrix} \mathrm{w}_{kx} e^{i\chi_{kx}} \\ \left(\mathrm{v}_{kx} + \dfrac{iq_k}{\mathrm{w}_{kx}} \right) e^{i\chi_{kx}} \\ \mathrm{w}_{ky} e^{i\chi_{ky}} \\ \left(\mathrm{v}_{ky} + \dfrac{i(1-q_k)}{\mathrm{w}_{ky}} \right) e^{i\chi_{ky}} \\ \mathrm{y}_{k\tau} = \eta^T S Y_{k\beta} \\ 0 \end{bmatrix}; k = 1,2; \; Y_\delta = \begin{bmatrix} \eta \\ \chi_\tau \\ 1 \end{bmatrix} = \begin{bmatrix} \eta_x \\ \eta_{px} \\ \eta_y \\ \eta_{py} \\ \chi_\tau \\ 1 \end{bmatrix};$$

and known values of eigen emittances $\varepsilon_k \equiv I_k = \dfrac{\langle a_k^2 \rangle}{2}; k = 1,2$ and RMS values of the relative energy spread $\sigma_\delta = \sqrt{\langle \delta^2 \rangle}$ write expressions for transverse beam sizes:

$$\sigma_x = \sqrt{<x^2>}; \sigma_y = \sqrt{<y^2>}$$