Homework PHY 554 #15 **Due April 29, 2020**

HW 1 (5 points)

Show that for $\hat{C} << 1$, the eigenvalue of the growing mode for the 1-D FEL (cold beam) can be approximated as

$$\lambda = a_0 + a_1\hat{C} + a_2\hat{C}^2$$

with

$$a_0 = \frac{\sqrt{3}}{2} + i\frac{1}{2} \ ,$$

$$a_1 = -i\frac{2}{3} \ ,$$

and

$$a_2 = -\frac{1}{9}\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) \ .$$

HW 2 (5 points)

Assuming the saturation of a FEL takes place at the condition

$$\Omega_{p,sat} L_G \approx 1 \ ,$$

where $\Omega_{p,sat} = \sqrt{\frac{eE_{sat}\theta_s\omega}{\gamma_z^2 cE_0}}$ is the small-amplitude angular frequency of an electron oscillating in the radiation fields, E_{sat} is the amplitude of the radiation field at saturation and $L_G = \frac{1}{\sqrt{3}\Gamma}$ is the 1-D gain length of the radiation power, show that the radiation power at saturation is given by

$$P_{sat} = \varepsilon_0 cE_{sat}^2 A = \chi\rho\frac{E_0}{e}I_e \ ,$$

where A is the cross-section of the radiation fields (which is equal to the cross-section of the electron beam for 1-D model) and I_e is the peak current of the electron beam, find the numerical coefficient χ .