## Home Work PHY 554 #7. Due – March 4, 2020

HW 1 (5 points): RF cavity beam loading/unloading.

A short ultra-relativistic (1-v/c <<1) bunch with charge of 5 nC is passing through a 0.3 meter long 500 MHz pillbox accelerating cavity operating at the fundamental  $TM_{010}$  with peak accelerating field of 5 MV/m.

(1) Find the change of the cavity voltage  $\Delta V/V$  (accelerating field) after the beam passes through it as function of the phase of the beam passing the cavity. What are the maximum and minimum  $\Delta V/V$ ?

(2) How the beam loading  $\Delta V/V$  depends on the accelerating field? At what level of accelerating it reaches  $\Delta V/V$  1%?

- (a) Assume that beam does not change velocity in the cavity;
- (b) Hint use energy conservation law
- (c) Assume that relative change of the voltage  $\Delta V/V$  is small, e.g. the beam loading can be treated as a perturbation.

**HW 2 (3 points):** Cavities filled with ferrite material are used for RF system requiring large frequency tuning range. The frequency is controlled by applying external magnetic field,  $B_{ext}$ , to the ferrite material and by doing so to change it magenta permeability  $\mu(B_{ext})$ . A 300 m in circumference AGS synchrotron accelerates polarized protons from total energy of 2.5 GeV to 25 GeV.

- (a) Calculate the range of the beam revolution frequency in AGS;
- (b) Assuming 100% filling by ferrite, what should be ratio of  $\mu_{max}$  to  $\mu_{min}$ . Where  $\mu$  should have maximum value?

Note: RF systems operate on a fixed integer harmonic of the revolution frequency.

**HW 3 (2 points):** In RF cavity operating at 500 MHz, amplitude of the magnetic field at the part surface is 500 Gs or 500 Oe. Find power losses per square meter of the surface for:

- (a) Cu cavity\*
- (b) SRF cavity with surface resistance,  $R_s = 5 \ 10^{-9}$  Ohm.

How much water you can heat from 20 C° to 40 C° in one hour (3,600 second) by cooling such Cu cavity?

\*Hint: you may use the conductivity of Cu or scale  $R_s$  from results shown in lectures. Thermal capacitance of water is 4,179 J/kg/ C°.