

Solutions for HW # 7

1. The longitudinal impedance is defined as

$$Z_{//}(\omega) = \frac{1}{c} \int_0^{\infty} w_{//}(s) e^{i\omega s/c} ds . \quad (1)$$

Taking the complex conjugate of eq. (1) and noticing that

$$w_{//}^*(s) = w_{//}(s) , \quad (2)$$

it follows that

$$Z_{//}^*(\omega) = \frac{1}{c} \int_0^{\infty} w_{//}(s) e^{-i\omega s/c} ds = \frac{1}{c} \int_0^{\infty} w_{//}(s) e^{i(-\omega)s/c} ds = Z_{//}(-\omega) . \quad (3)$$

The transverse impedance is defined as

$$Z_{\perp}(\omega) = -\frac{i}{c} \int_0^{\infty} w_{\perp}(s) e^{i\omega s/c} ds . \quad (4)$$

Taking the complex conjugate of eq. (4) and noticing that

$$w_{\perp}^*(s) = w_{\perp}(s) \quad (5)$$

yield

$$Z_{\perp}^*(\omega) = \frac{i}{c} \int_0^{\infty} w_{\perp}(s) e^{-i\omega s/c} ds = -\left[-i \frac{1}{c} \int_0^{\infty} w_{\perp}(s) e^{i(-\omega)s/c} ds \right] = -Z_{\perp}(-\omega) . \quad (6)$$

2. Using the relation

$$F(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikz} \tilde{F}(k) dk ,$$

we obtain

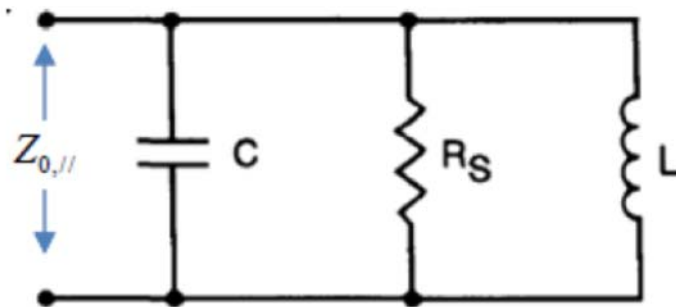
$$\begin{aligned}
\sum_{l=-\infty}^{\infty} F(lC) &= \frac{1}{2\pi} \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{iklC} \tilde{F}(k) dk \\
&= \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \tilde{F}(k) \sum_{l=-\infty}^{\infty} e^{i2\pi l \frac{kC}{2\pi}} \\
&= \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \tilde{F}(k) \sum_{p=-\infty}^{\infty} \delta\left(\frac{kC}{2\pi} - p\right) \quad , \\
&= \frac{1}{2\pi} \sum_{p=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{2\pi \delta\left(k - \frac{2\pi p}{C}\right)}{C} \tilde{F}(k) dk \\
&= \frac{1}{C} \sum_{p=-\infty}^{\infty} \tilde{F}\left(\frac{2\pi p}{C}\right)
\end{aligned}$$

where we also used

$$\delta(g(x)) = \sum_i \frac{\delta(x - x_i)}{|g'(x_i)|}$$

with x_i being the roots of $g(x)$.

3.



The impedance is determined by

$$\begin{aligned}
\frac{1}{Z_{0, //}} &= \frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C} \\
&= \frac{1}{R_s} + \frac{1}{j\omega L} + j\omega C \\
&= \frac{1 + jR_s\sqrt{\frac{C}{L}}\left(\omega\sqrt{LC} - \frac{1}{\omega\sqrt{LC}}\right)}{R_s}, \quad (7) \\
&= \frac{1 + jQ\left(\frac{\omega}{\omega_R} - \frac{\omega_R}{\omega}\right)}{R_s} \\
&= \frac{1 + iQ\left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)}{R_s}
\end{aligned}$$

i.e.

$$Z_{0, //} = \frac{R_s}{1 + iQ\left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)}$$

where $j = -i$, $Q \equiv R_s\sqrt{\frac{C}{L}}$ and $\omega_R \equiv \frac{1}{\sqrt{LC}}$.