Homework 18

1. (10 points) The gain of a FEL oscillator (i.e. FEL working in low gain regime) depends on the energy of the electron beam. Using the results derived in the lecture, find the electron beam energy for maximal gain in a FEL oscillator. Assume that the undulator length is l_w, the undulator period is λ_w, the undulator parameter is K and the radiation wavelength is λ_0.

2. (10 points) Show that for $\hat{C} << 1$, the eigenvalue of the growing mode for the 1-D FEL (cold beam) can be approximated as

$$\lambda = a_0 + a_1\hat{C} + a_2\hat{C}^2$$

with

$$a_0 = \frac{\sqrt{3}}{2} + i\frac{1}{2},$$

$$a_1 = -i\frac{2}{3},$$

and

$$a_2 = -\frac{1}{9}\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right).$$

Hint: Insert the expansion, $\lambda = a_0 + a_1\hat{C} + a_2\hat{C}^2$, into the polynomial equation for the eigenvalues in a cold beam (lecture 23, slide # 10) and request coefficients of $\hat{C}^0$, $\hat{C}^1$ and $\hat{C}^2$ to vanish.