

Homework 2. PHY 564

Problem 1. 10 points Motion of non-radiating charged particle in constant uniform magnetic field is a well known spiral:

$$\frac{d\vec{p}}{dt} = \frac{e}{c} [\vec{v} \times \vec{H}] = \frac{e}{c} H [\hat{e}_x v_y - \hat{e}_y v_x]; \vec{H} = \hat{e}_z H$$

$$E = c\sqrt{m^2 c^2 + \vec{p}^2} = \text{const}; \gamma = \text{const}; v = \text{const};$$

$$p_z = \text{const}; z = v_{oz} t + z_o;$$

$$p_x^2 + p_y^2 = \text{const}; p_x + ip_y = p_{\perp} e^{i\varphi(t)} = m\gamma v_{\perp} e^{i\varphi(t)}$$

simple substitution gives:

$$m\gamma v_{\perp} \frac{de^{i\varphi(t)}}{dt} = \frac{e}{c} [\vec{v} \times \vec{H}] = -i \frac{e}{c} H v_{\perp} e^{i\varphi(t)}$$

$$r_{\perp} = x + iy = i\omega m\gamma v_{\perp} \frac{de^{i\varphi(t)}}{dt}$$

$$\varphi(t) = \omega t + \varphi_o; \omega = -\frac{eH}{m\gamma c}$$

and trajectory: $z = v_{oz} t + z_o; x + iy = v_{\perp} / \omega \cdot e^{i\omega t}$. Do not forget to apply Re or Im to all necessary formulae. Use analytical extension of the Lorentz transformation to complex values by going into a reference frame with x-velocity going approaching infinity $\beta \Rightarrow \infty; \chi \rightarrow 0; \chi\beta \rightarrow 1$. Show that transverse electric field becomes a magnetic field (with an imaginary value) and visa versa. Follow this path and transfer 4-coordinates to that frame. Use analytical extension of *exp, sin, cos* to complex values and transform the solution above in that for motion in constant magnetic field. Compare it with known solution is your favorite EM book .

Problem 2. 4 points

Find maximum energy of a charged particle (with unit charge e !) which can be circulating in Earth's largest possible storage ring: the one going around Earth equator with radius of 6,384 km.

First, find it for storage ring using average bending magnetic field of a super-conducting magnet with strength of 10 T (100 kGs).

Second, find it for a very strong DC electric dipole fields of 10 MV/m.

Compare these energies with current largest (27 km in circumference) circular collider, LHC, circulating 6.5 TeV (1 TeV = 10^{12} eV).

Hint: assume that particles move with speed of the light. Check the final result for protons having rest mass of 938.27 MeV/c²