Coherent electron Cooling

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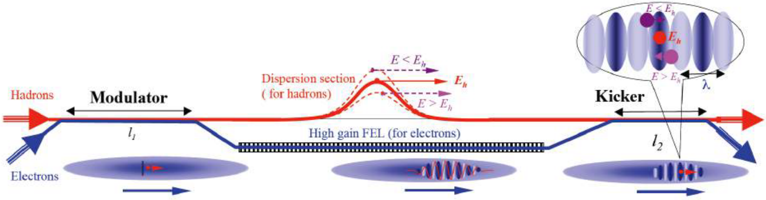
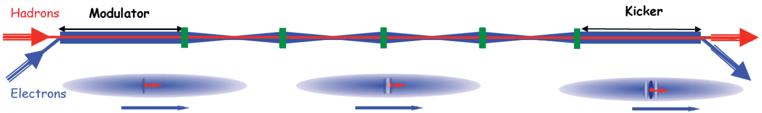
- In the Electron-Ion Collider (EIC), Strong Hadron Cooling (SHC) is needed to reach high luminosity. Present baseline approach for SHC is based on Coherent electron Cooling (CeC).
- A general CeC scheme consists of three main sections: Modulator, Amplifier, Kicker

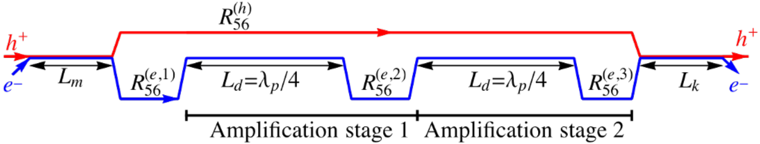


(a) CeC with free electron laser (FEL) amplifier

Other implementations of amplifier



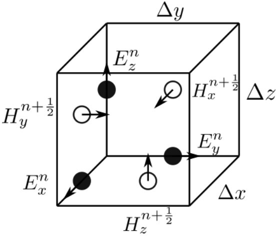
(a) Plasma cascade amplifier (PCA)



(b) Microbunched coherent electron cooling (MBEC)

Simulation tool

- The SPACE code is a parallel, relativistic, three-dimensional (3D) electromagnetic (EM) Particle-in-Cell (PIC) code.
  Finite-difference time-domain (FDTD) or Yee's method



  Uniform mesh, adaptive mesh, adaptive Particle-in-Cloud
- The GENESIS code is a three-dimensional, time-dependent code developed for high-gain FEL simulations.

Analytical tools for modulation process

Cold uniform electron beam ©V. N. Litvinenko

$$q = -Ze \cdot (1 - \cos\varphi_1) \qquad \varphi_1 = \omega_p l_1 / c\gamma_0$$

(a) Density modulation

$$\left\langle \frac{\delta E}{E} \right\rangle \cong -2Z \frac{r_e}{a^2} \cdot \frac{L_{pol}}{\gamma} \cdot \left(\frac{z}{|z|} - \frac{z}{\sqrt{a^2/\gamma^2 + z^2}} \right)$$

(b) Energy modulation

Analytical tools for modulation process

G. Wang, and M. Blaskiewicz. Physical Review E 78.2 (2008): 026413.
Linearized Vlasov Equation

$$\frac{\partial}{\partial t} f_1(\vec{x},\vec{v},t) + \vec{v} \cdot \frac{\partial}{\partial \vec{x}} f_1(\vec{x},\vec{v},t) - \frac{e\vec{E}}{m_e} \cdot \frac{\partial}{\partial \vec{v}} f_0(\vec{v}) = 0$$

$$\vec{\nabla} \cdot \vec{E}(\vec{x},t) = \frac{\rho(\vec{x},t)}{\epsilon_0}$$

$$\rho(\vec{x},t) = Z_i e\,\delta(\vec{x}) - e\tilde{n}_1(\vec{x},t)$$

$$\tilde{n}_1(\vec{x},t) = \int f_1(\vec{x},\vec{v},t) d^3 v$$

Analytical tools for modulation process

Fourier transform

$$\frac{\partial}{\partial t} f_1(\vec{k},\vec{v},t) + i\vec{k}\cdot\vec{v} f_1(\vec{k},\vec{v},t) + i\frac{e\Phi(\vec{k},t)}{m_e}\vec{k}\cdot\frac{\partial}{\partial\vec{v}} f_0(\vec{v}) = 0$$

$$\Phi(\vec{k},t) = \frac{e}{\epsilon_0 k^2}[Z_i - \tilde{n}_1(\vec{k},t)]$$

Multiply both sides by $e^{i\vec{k}\cdot\vec{v}t}$

$$\frac{\partial}{\partial t}[e^{i\vec{k}\cdot\vec{v}t} f_1(\vec{k},\vec{v},t)] = -i\frac{e}{m_e}\Phi(\vec{k},t)e^{i\vec{k}\cdot\vec{v}t}\left(\vec{k}\cdot\frac{\partial}{\partial\vec{v}} f_0(\vec{v})\right)$$

Analytical tools for modulation process

Initial condition $f_1(\vec{k}, 0) = 0$

$$f_1(\vec{k},\vec{v},t) = -i\frac{e}{m_e}\int_0^t \Phi(\vec{k},t_1)e^{i\vec{k}\cdot\vec{v}(t_1-t)}\vec{k}\cdot\frac{\partial}{\partial\vec{v}}f_0(\vec{v})dt_1$$

Note relation

$$i\int\frac{\vec{k}}{k^2}\cdot\frac{\partial}{\partial\vec{v}}f_0(\vec{v})e^{i\vec{k}\cdot\vec{v}\tau}d^3v = \int f_0(\vec{v})e^{i\vec{k}\cdot\vec{v}\tau}\tau\, d^3v$$

Analytical tools for modulation process

We have

$$\tilde{n}_1(\vec{k},t) = \omega_p^2 \int_0^t [\tilde{n}_1(\vec{k},t_1) - Z_i](t_1 - t) g(\vec{k}(t - t_1)) dt_1$$

$$g(\vec{u}) \equiv \frac{1}{n_0} \int f_0(\vec{v}) e^{-i\vec{u}\cdot\vec{v}} d^3v$$

$$\omega_p = \sqrt{n_0 e^2 / m_e \epsilon_0}$$

Analytical tools for modulation process

For cold electrons, the velocity distribution in the rest frame of the ion reads $f_0(\vec{v}) = n_0 \delta^3(\vec{v})$, which gives $g(\vec{u}) = 1$
The integral equation reduces to 2nd order ODE

$$\frac{d^2}{dt^2}\tilde{n}_1(\vec{k}, t) = -\omega_p^2 \tilde{n}_1(\vec{k}, t) + Z_i \omega_p^2$$

Analytical tools for modulation process

Without ion, with initial perturbation

$$\frac{d^2}{dt^2}\tilde{n}_1\left(\vec{k},t\right) = -\omega_p^2\tilde{n}_1\left(\vec{k},t\right)$$

$$\Rightarrow \tilde{n}_1\left(\vec{k},t\right) = \tilde{n}_1\left(\vec{k},0\right)\cos\left(\omega_p t\right) + \frac{\dot{\tilde{n}}_1\left(\vec{k},0\right)}{\omega_p}\sin\left(\omega_p t\right)$$

With ion, without initial perturbation

$$\tilde{n}_1\left(\vec{k},t\right) = Z_i\left[1-\cos\left(\omega_p t\right)\right]$$

Analytical tools for modulation process

Warm uniform electron beam with $\kappa - 2$ velocity distribution:

$$f_0(\vec{v}) = \frac{1}{\pi^2 \beta_x \beta_y \beta_z} \left(1 + \frac{v_x^2}{\beta_x^2} + \frac{v_y^2}{\beta_y^2} + \frac{v_z^2}{\beta_z^2} \right)^{-2}$$

(a) $\kappa - 2$

G. Wang, and M. Blaskiewicz. Physical Review E 78.2 (2008): 026413.

$$\tilde{n}_1(\vec{x},t) = \frac{Z_i}{\pi^2 a_x a_y a_z} \int_0^{\omega_p t} \frac{\tau \sin\tau \cdot d\tau}{\left[\tau^2 + \left(\frac{x}{a_x} + \frac{v_{0,x}}{\beta_x}\tau \right)^2 + \left(\frac{y}{a_y} + \frac{v_{0,y}}{\beta_y}\tau \right)^2 + \left(\frac{z}{a_z} + \frac{v_{0,z}}{\beta_z}\tau \right)^2 \right]^2}$$

(a) Density modulation

Analytical tools for modulation process

Warm uniform electron beam with $\kappa - 2$ velocity distribution. G. Wang, V. N. Litvinenko, and M. Blaskiewicz. "Energy Modulation in Coherent Electron Cooling." Proceedings of IPAC (2013).

$$\left\langle \frac{\delta E}{E_0} \right\rangle = \frac{\langle v_z \rangle}{c} = -\frac{1}{e n_0 \pi a^2 c} I_d\left(\gamma_0 z_l, \frac{L_{\mathrm{mod}}}{\beta_0 \gamma_0 c} \right)$$

(a) Energy modulation

$$I_d(z,t) = -\frac{Z_i e \omega_p^2}{\pi} \int_0^t d\tau \left(z + v_{0,z}\tau\right) \left\{ \frac{a_z \sin(\omega_p \tau)}{\left[\bar{\beta}^2 \tau^2 + \left(z + v_{0,z}\tau\right)^2\right]\left[1 + \bar{\beta}^2 \tau^2 + \left(z + v_{0,z}\tau\right)^2 / a^2\right]} \right.$$
$$\left. - \cos(\omega_p \tau) \left[\frac{\arctan\left(\left|z + v_{0,z}\tau\right| / (\bar{\beta}\tau) \right)}{\left|z + v_{0,z}\tau\right|} - \frac{\arctan\left(\sqrt{\left(z + v_{0,z}\tau\right)^2 + a^2} / (\bar{\beta}\tau) \right)}{\sqrt{\left(z + v_{0,z}\tau\right)^2 + a^2}} \right] \right\}$$

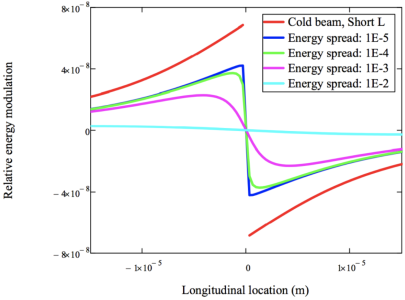
(b) Energy modulation

Analytical tools for modulation process

The warm beam result reduces to the previously derived cold beam result at the corresponding limits

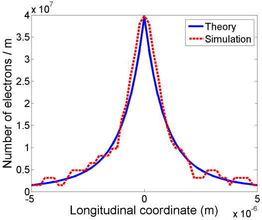
$$\bar{\beta} = 0 \qquad v_{0,z} = 0 \qquad L_{\text{mod}} << \beta_0 \gamma_0 c / \omega_p$$

(a)

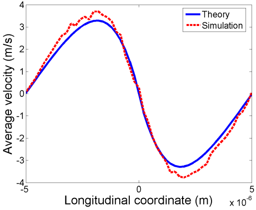


(b) Energy modulation

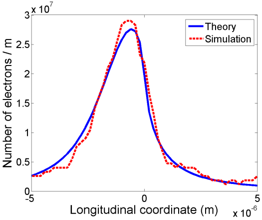
Simulation using uniform beam



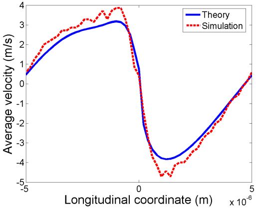
(a) Density, stationary ion



(b) Velocity, stationary ion



(c) Density, moving ion



(d) Velocity, moving ion

Simulation using Gaussian beam

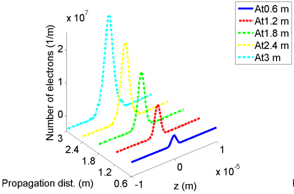
Continuous focusing field

$$\vec{E}_1(\vec{r}) = \frac{m_e}{e}\frac{\sigma_v^2}{\sigma_r^2}(\vec{r} - \vec{r}_0)$$

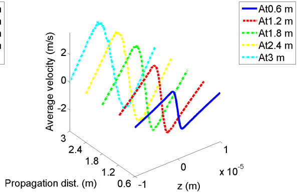
$$\vec{E}_2(\vec{r}) = \frac{q}{2\pi\varepsilon_0|\vec{r} - \vec{r}_0|}\left(1 - e^{-|\vec{r}-\vec{r}_0|^2/2\sigma_r^2}\right)$$

where $\vec{r} = (x, y)$ is the radial coordinate in transverse plane, $\vec{r}_0 = (x_0, y_0)$ is the center of the Gaussian distribution, σ_r is the RMS of the Gaussian distribution in both horizontal and vertical directions and σ_v is the RMS velocity of the electron distribution.
Transverse beam size is constant in the modulator.

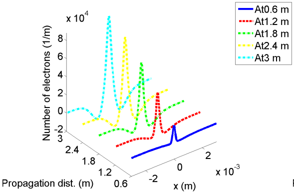
Simulation using Gaussian beam, continuous focusing



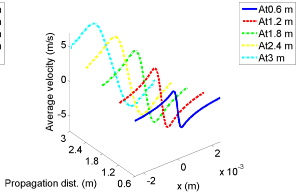
(a) Longitudinal density



(b) Longitudinal velocity

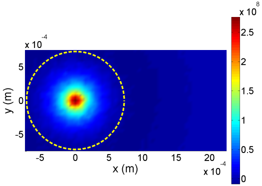


(c) Transverse density

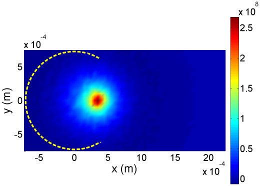


(d) Transverse velocity

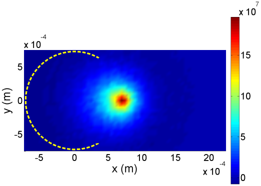
Simulation using Gaussian beam, continuous focusing



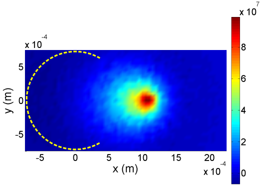
(a) Ion at center



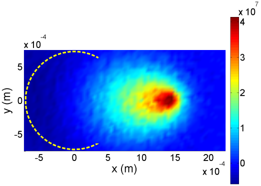
(b) Ion 0.5σ off center



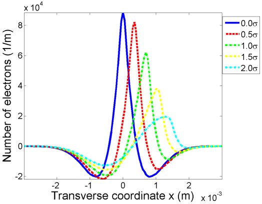
(c) Ion 1.0σ off center



(d) Ion 1.5σ off center

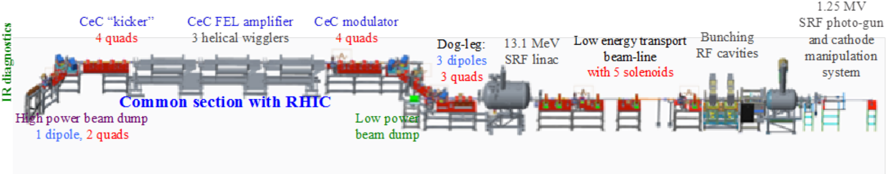


(e) Ion 2.0σ off center

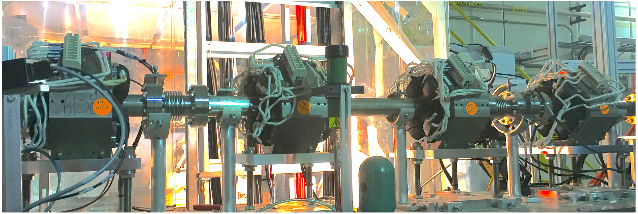


(f) Transverse density

FEL-based CeC experiment



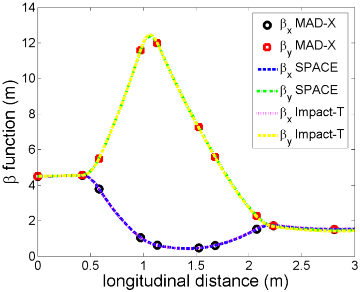
Modulator of FEL-based CeC experiment



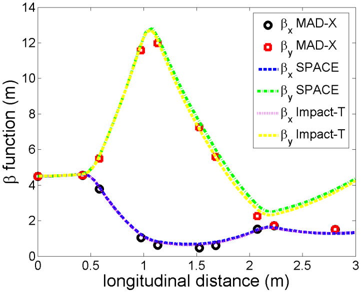
Q4 Q3 Q2 Q1

$$
\begin{aligned}
B_x &= G \cdot y \\
B_y &= G \cdot x \\
\kappa &= \frac{G}{B\rho} \\
B\rho(T \cdot m) &= 3.3356pc(GeV)
\end{aligned}
$$

Modulator, quadrupole beam line

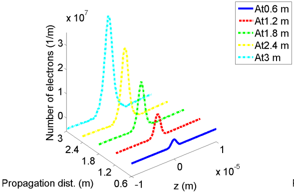


(a) No space charge

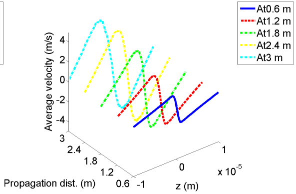


(b) With space charge

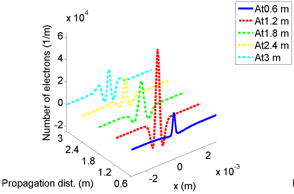
Modulation, quadrupole beam line



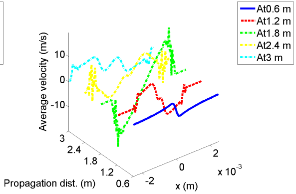
(a) Longitudinal density



(b) Longitudinal velocity



(c) Transverse density



(d) Transverse velocity

Transport in quadrupole channel

$$\langle x_o \delta x'_o \rangle = -\varepsilon, \varepsilon > 0.$$

(a) Initial correlation

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \begin{pmatrix} a(s) & b(s) \\ c(s) & d(s) \end{pmatrix} \begin{pmatrix} x_o \\ x'_o \end{pmatrix}, \quad ad - bc = 1$$

(b) Transport

$$\begin{pmatrix} \delta x(s) \\ \delta x'(s) \end{pmatrix} = \begin{pmatrix} a(s) & b(s) \\ c(s) & d(s) \end{pmatrix} \begin{pmatrix} 0 \\ \delta x'_o \end{pmatrix}$$

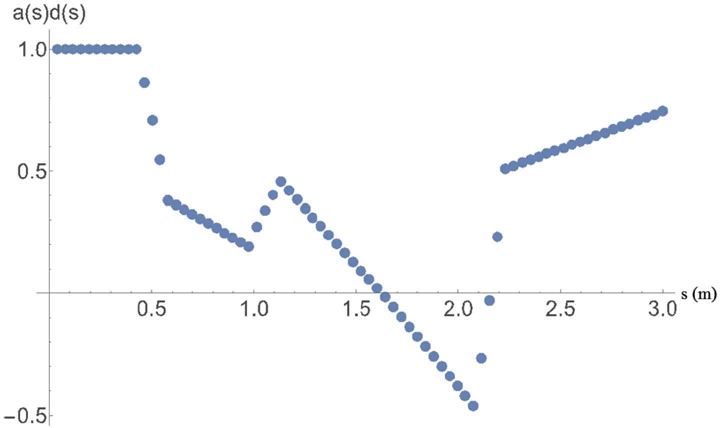
(c) Transport

$$\begin{aligned} x &= a x_o + b x'_o \\ \delta x' &= d \delta x'_o \\ \langle x \delta x' \rangle &= ad \cdot \langle x_o \delta x'_o \rangle \\ &= -ad \cdot \varepsilon \end{aligned}$$

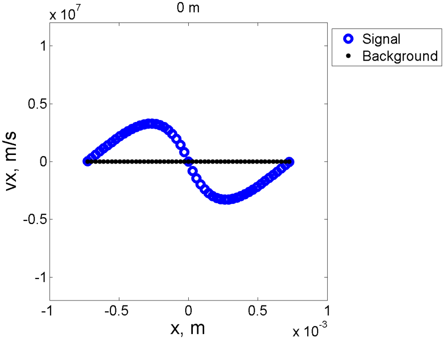
(d) Final correlation

Transport in quadrupole channel

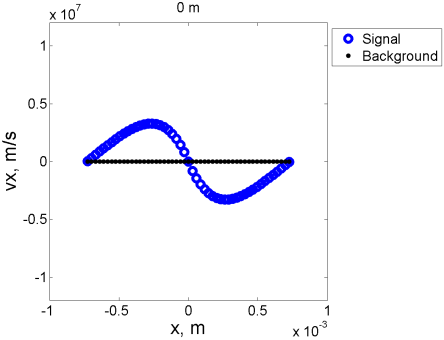
J. Ma, et al. Physical Review Accelerators and Beams 21.11 (2018): 111001.



Transverse phase advance in quadrupole beam line



(a) No space charge



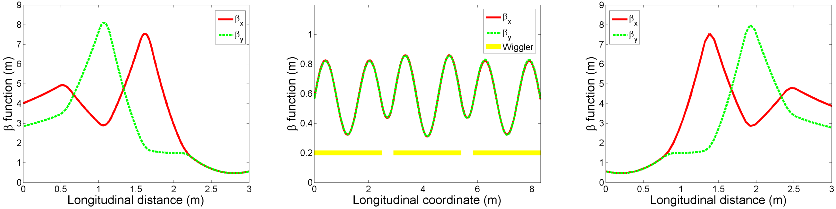
(b) With space charge

Bunching factor

$$b \equiv \frac{1}{N_\lambda} \sum_{k=1}^{N_\lambda} e^{i \frac{2\pi}{\lambda_{opt}} z_k}, -\frac{\lambda_{opt}}{2} \leq z_k \leq \frac{\lambda_{opt}}{2},$$

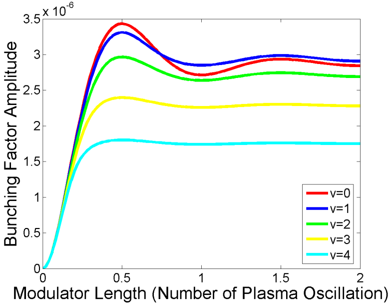
where λ_{opt} is the optical wavelength, the sum is taken over a slice of λ_{opt} width, centered at the location of the ion, and N_λ is the total number of electrons within that slice.

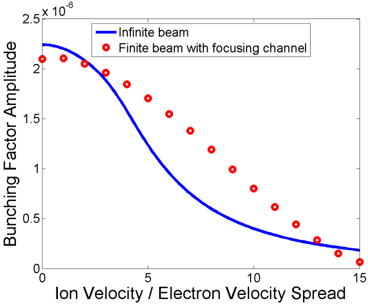
Beam envelope in FEL-based CeC



(a) Modulator (b) FEL amplifier (c) Kicker

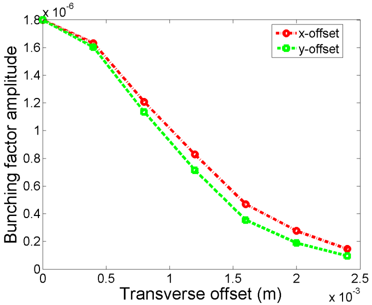
Dependence on ion velocity and modulator length

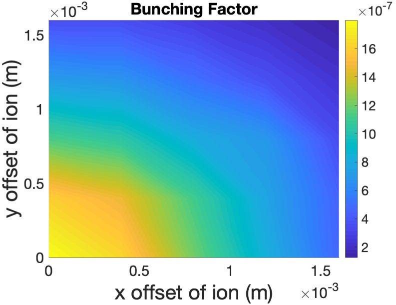




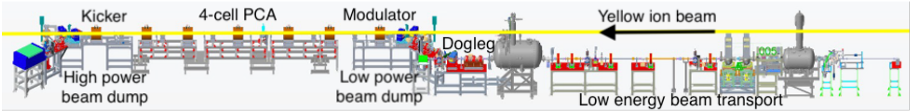
The ion velocity is in unit of electron longitudinal velocity spread.

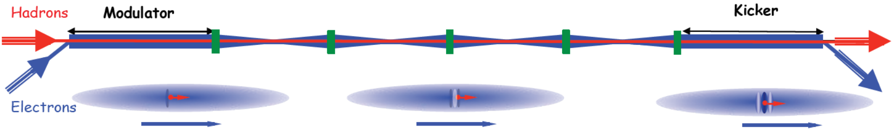
Dependence on ion transverse offset





PCA-based CeC





Solenoid field

An example of on-axis magnetic field:

$$B_{z,0} = \frac{B_0}{2}\left(\frac{L/2 - z}{\sqrt{(z - L/2)^2 + R^2}} + \frac{L/2 + z}{\sqrt{(z + L/2)^2 + R^2}}\right)$$

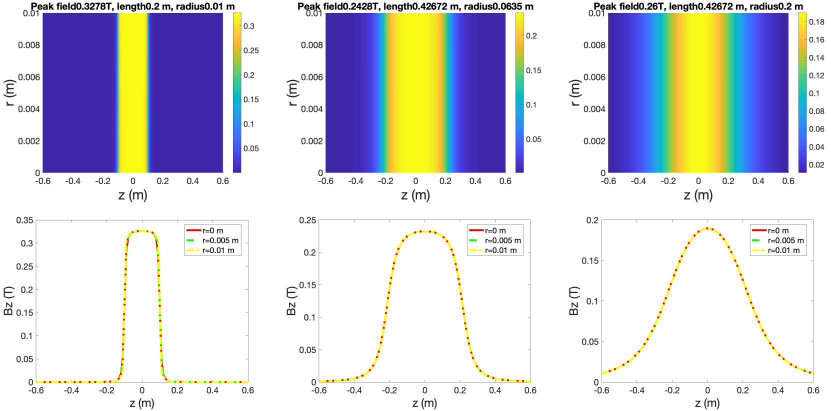
Off-axis magnetic field:

$$\begin{aligned} B_z(r) &= B_{z,0} - \frac{r^2}{4}B''_{z,0} + \frac{r^4}{64}B''''_{z,0} - \frac{r^6}{2304}B''''''_{z,0} \cdots \\ B_r(r) &= -\frac{r}{2}B'_{z,0} + \frac{r3}{16}B'''_{z,0} - \frac{r^5}{384}B'''''_{z,0} \cdots \end{aligned}$$

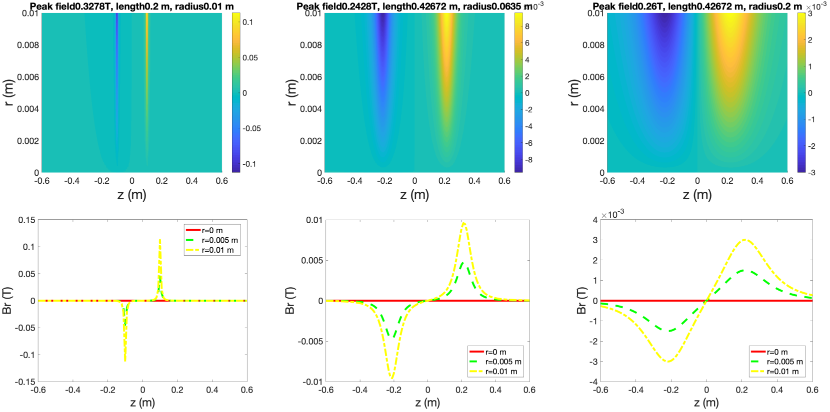
Lorentz transformation of the fields

$$
\begin{aligned}
E_x^* &= \gamma E_x - \gamma\beta c B_y \\
E_y^* &= \gamma E_y + \gamma\beta c B_x \\
E_z^* &= E_z \\
B_x^* &= \gamma B_x + \frac{\gamma\beta}{c} E_y \\
B_y^* &= \gamma B_y - \frac{\gamma\beta}{c} E_x \\
B_z^* &= B_z
\end{aligned}
$$

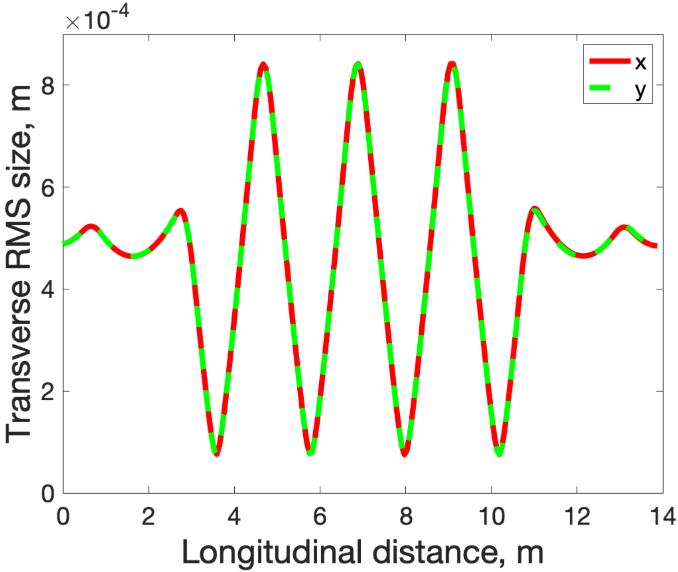
Solenoid field B_z



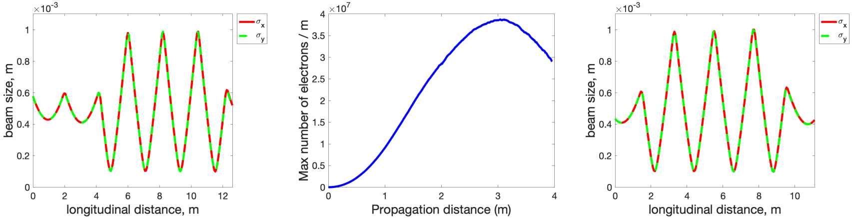
Solenoid field B_r



Beam envelope in PCA-based CeC

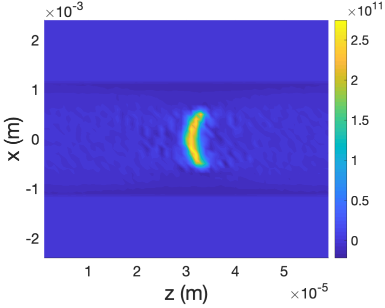


Modulator in PCA-based CeC

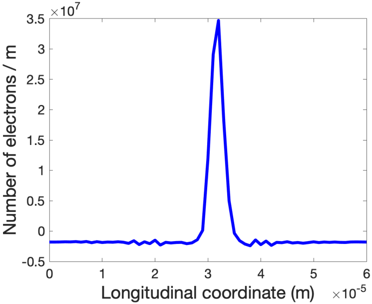


(a) Modulator length 4 m (b) Density modulation (c) Modulator length 1.5 m

Density modulation in PCA-based CeC

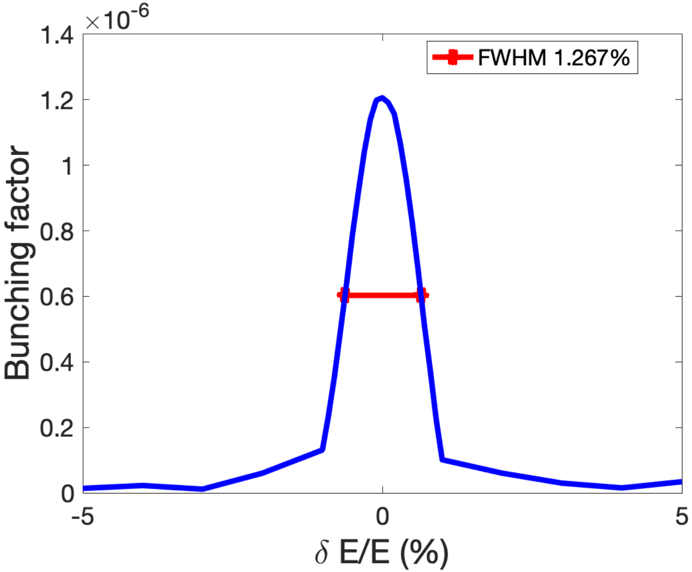


(a) 2D plot

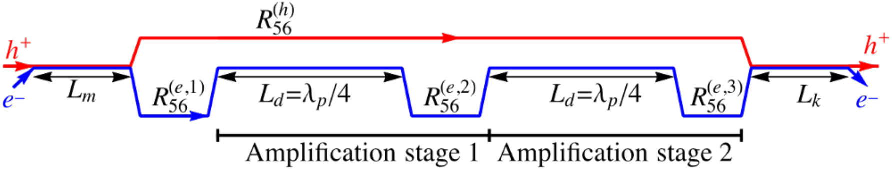


(b) 1D plot

Dependence on energy difference

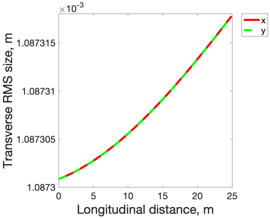


MBEC

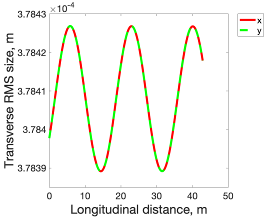


$R_{56}^{(h)}$
h^+
e^-
L_m
$R_{56}^{(e,1)}$
$L_d = \lambda_p/4$
$R_{56}^{(e,2)}$
$L_d = \lambda_p/4$
$R_{56}^{(e,3)}$
L_k
h^+
e^-
Amplification stage 1
Amplification stage 2

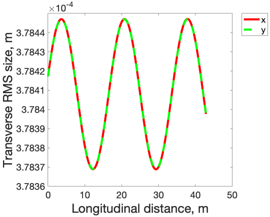
Beam envelope in MBEC



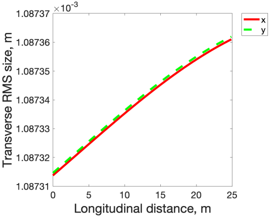
(a) Modulator



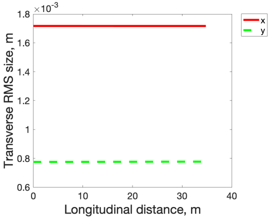
(b) First stage



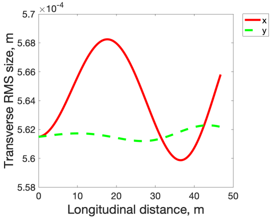
(c) Second stage



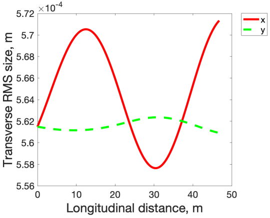
(d) Kicker



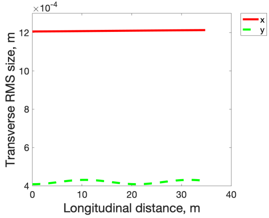
(a) Modulator



(b) First stage

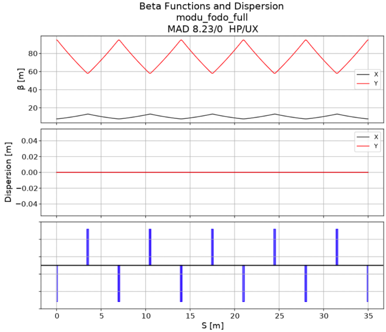


(c) Second stage

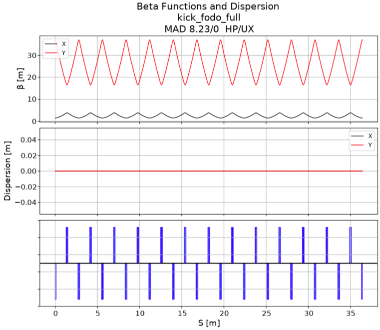


(d) Kicker

Beam envelope in MBEC

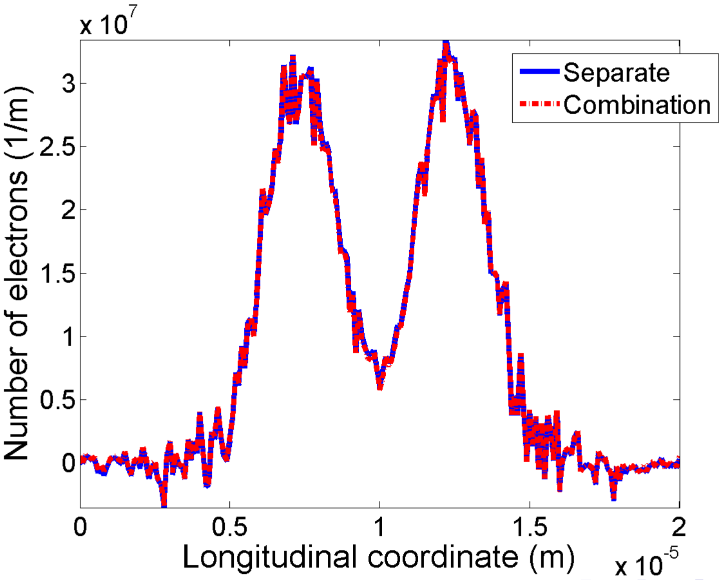


(a) Modulator

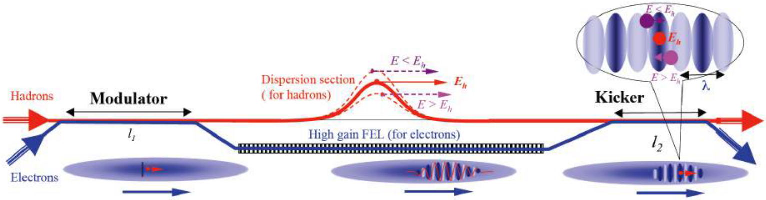


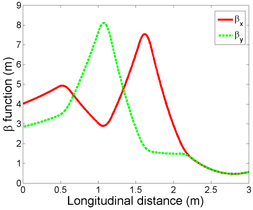
(b) Kicker

Superposition principle in density modulation

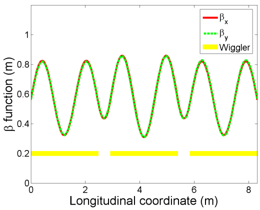


FEL-based CeC

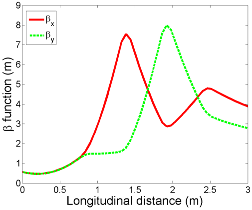




(a) Modulator

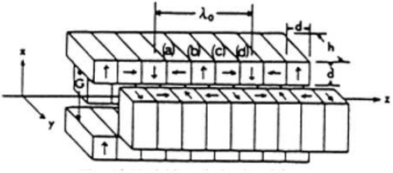


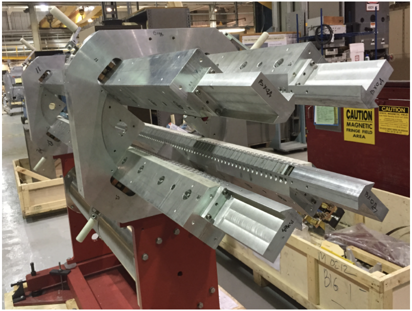
(b) FEL amplifier



(c) Kicker

Helical undulator





$$
\begin{aligned}
B_x(x,y,z) &= B_0\cos(k_u z)\\
B_y(x,y,z) &= B_0\sin(k_u z)
\end{aligned}
$$

Electron motion in helical wiggler without radiation field

$$\vec{B}_w(x,y,z) = B_w\left[\cos(k_u z)\hat{x} - \sin(k_u z)\hat{y}\right]$$

$$\vec{F}(x,y,z) = -e\vec{v}\times\vec{B} = -ev_z\hat{z}\times\vec{B} = -ev_z B_w\left[\cos(k_u z)\hat{y} + \sin(k_u z)\hat{x}\right]$$

$$\frac{d(m\gamma v_x)}{dt} = m\gamma\frac{dv_x}{dt} = -ev_z B_w\sin(k_u z) \qquad \frac{d(m\gamma v_y)}{dt} = m\gamma\frac{dv_y}{dt} = -ev_z B_w\cos(k_u z)$$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} \qquad v = \sqrt{v_x^2+v_y^2+v_z^2} \qquad \tilde{v} \equiv v_x + iv_y$$

$$m\gamma\frac{d\tilde{v}}{dt} = -iev_z B_w\left(\cos(k_u z) - i\sin(k_u z)\right) = -iev_z B_w e^{-ik_u z}$$

$$m\gamma\frac{d\tilde{v}}{dt} = m\gamma\frac{dz}{dt}\frac{d\tilde{v}}{dz} = -iev_z B_w e^{-ik_u z} \Rightarrow m\gamma\frac{d\tilde{v}}{dz} = -ieB_w e^{-ik_u z}$$

Electron motion in helical wiggler without radiation field

$$\frac{\tilde{v}(z)}{c} = \frac{-ieB_w}{mc\gamma}\int e^{-ik_u z_1} dz_1 = \frac{eB_w}{mc\gamma k_u} e^{-ik_u z} = \frac{K}{\gamma} e^{-ik_u z}$$

$$\vec{v}_\perp(z) = \frac{cK}{\gamma}\left[\cos(k_u z)\hat{x} - \sin(k_u z)\hat{y}\right] \qquad v_z = const.$$

$$K \equiv \frac{eB_w \lambda_w}{2\pi mc} \qquad \theta_s = K/\gamma$$

Energy change of electrons due to radiation field

$$\vec{v}_\perp(z) = \frac{cK}{\gamma}\left[\cos(k_u z)\hat{x} - \sin(k_u z)\hat{y}\right]$$

$$\vec{E}_\perp(z,t) = E\left[\cos(kz-\omega t)\hat{x} + \sin(kz-\omega t)\hat{y}\right] \qquad E_z = 0$$

$$= E\left[\cos(k(z-ct))\hat{x} + \sin(k(z-ct))\hat{y}\right] \qquad \omega = kc$$

$$\frac{d\mathcal{E}}{dt} = \vec{F}\cdot\vec{v} = -e\vec{v}_\perp \cdot \vec{E}_\perp$$

$$\frac{d\mathcal{E}}{dz} = -eE\theta_s \cos\left[\left(k_w + k - k\frac{c}{v_z}\right)z + \psi_0\right]$$

Resonant radiation wavelength

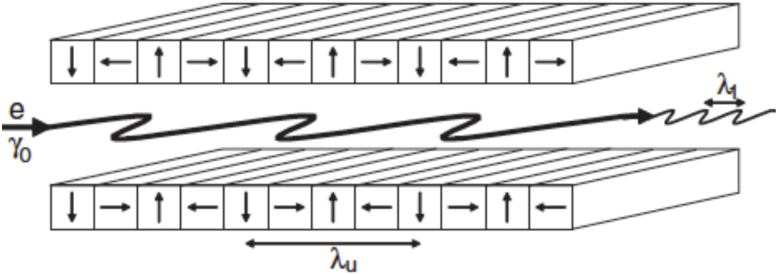
$$k_w + k_0 - k_0 \frac{c}{v_z} = 0 \Rightarrow \lambda_0 = \lambda_w \left(\frac{c}{v_z} - 1 \right) \approx \frac{\lambda_w}{2\gamma_z^2}$$

$$\gamma_z^{-2} \equiv 1 - v_z^2 / c^2 = 1 - \left(v_z^2 + v_\perp^2 \right) / c^2 + v_\perp^2 / c^2 = \gamma^{-2} + \theta_s^2 = \gamma^{-2} \left(1 + K^2 \right)$$

$$\lambda_0 \approx \frac{\lambda_w \left(1 + K^2 \right)}{2\gamma^2}$$

$$K \equiv \frac{eB_w \lambda_w}{2\pi mc}$$

Planar undulator



$$
\begin{aligned}
B_y(x, y, z) &= B_0 \sin(k_u z) \\
\lambda_0 &= \frac{\lambda_w}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)
\end{aligned}
$$

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