PHY542. Emittance Measurements using solenoid scan

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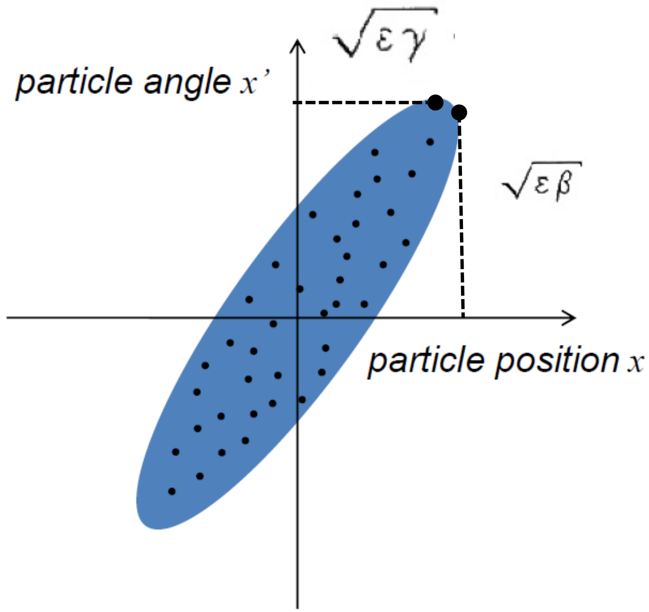
Emittance, what is it ?



ε = Area in *x, x'* plane occupied by beam particles divided by π

Beam ellipse and its orientation is described by 4 parameters

$$\varepsilon = \gamma x^2 + 2\,\alpha\, x\, x' + \beta\, x'^2$$

$\sqrt{\beta\varepsilon}$ Is the beam half width

$\sqrt{\gamma\varepsilon}$ Is the beam half divergence

α Describes how strong x and x' are correlated

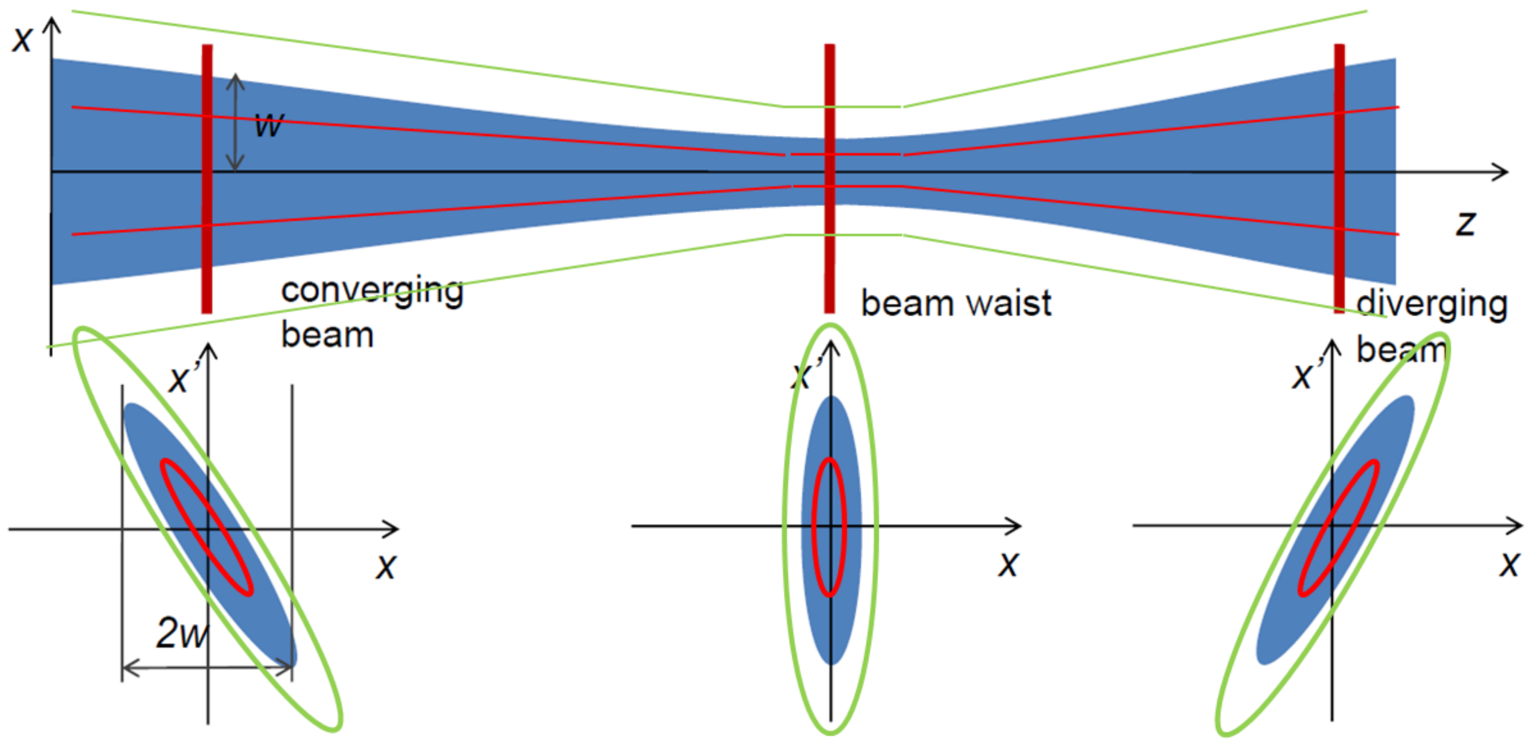
a<0 beam diverging

a>0 beam converging

a=0 beam size is maximum or minimum(waist)

The three orientation parameters are connected by the relation $\gamma = \dfrac{1+\alpha^2}{\beta}$

Beam envelope along a beamline.



Along a beamline the orientation and aspect ratio of the beam ellipse in x, x' changes, but area (emittance) remains constant.

Alike initial beam distributions have similar phase space dynamics

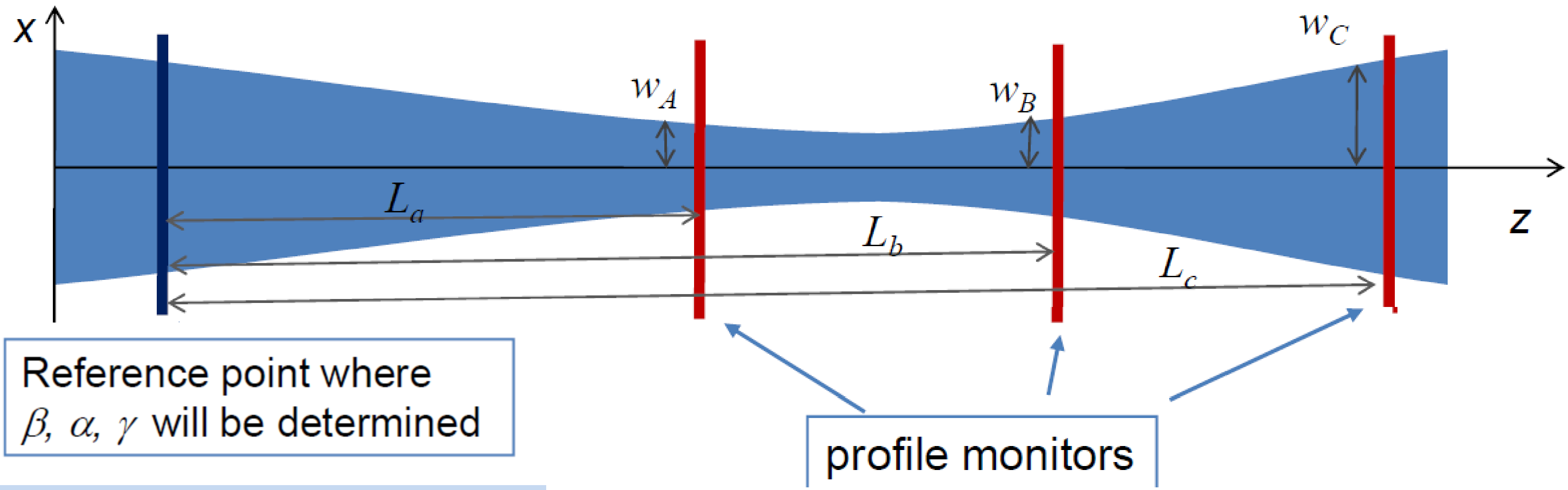
Beam width along Z is described $$w(z) = \sqrt{\beta(z)\,\varepsilon}$$

$\beta(z)$ describes the beam line, ε – describes beam quality

Emittance measurement in transfer line or linac

Twiss parameters α,β,γ are a priori not known, they have to be determined together with emittance ε

Method A



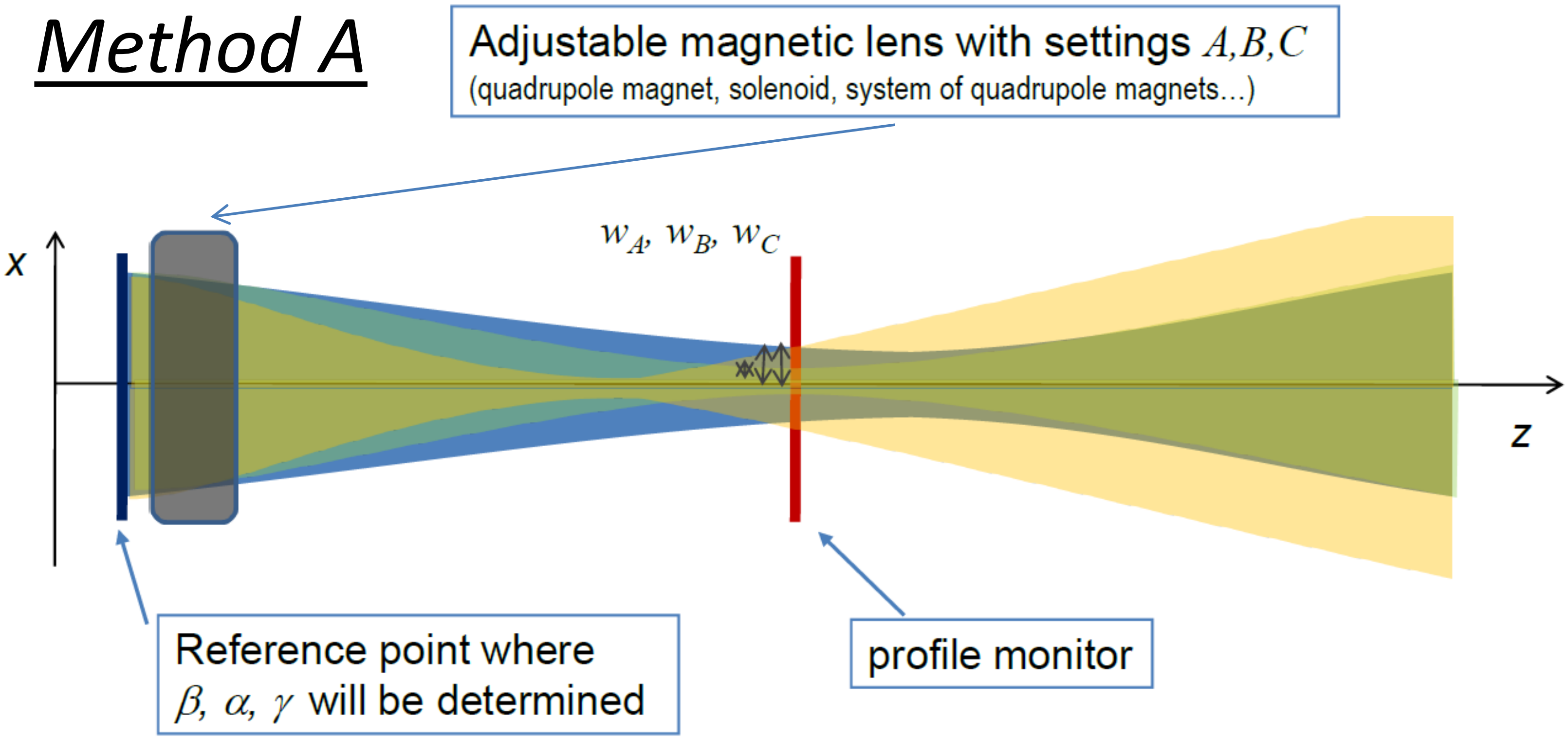
$$w_A^2 = \beta\,\varepsilon - 2\,L_A\,\alpha\,\varepsilon + L_A^2\,\gamma\,\varepsilon$$
$$w_B^2 = \beta\,\varepsilon - 2\,L_B\,\alpha\,\varepsilon + L_B^2\,\gamma\,\varepsilon$$
$$w_C^2 = \beta\,\varepsilon - 2\,L_C\,\alpha\,\varepsilon + L_C^2\,\gamma\,\varepsilon$$

3 linear equations, 3 independent variable
Solved by inverting matrix.

$$\beta\varepsilon\cdot\gamma\varepsilon-(\alpha\varepsilon)^2=\varepsilon^2\left(\beta\cdot\gamma-\alpha^2\right)=\varepsilon^2 \quad\Rightarrow\quad \sqrt{\beta\varepsilon\cdot\gamma\varepsilon-(\alpha\varepsilon)^2}=\varepsilon,\quad \beta=\frac{\beta\varepsilon}{\varepsilon},\quad \alpha=\frac{\alpha\varepsilon}{\varepsilon}$$

Emittancemeasurement in transfer line or linac, (count.)

- ***Method A***



$$w^2 = c^2\ \beta\varepsilon - 2cs\ \alpha\ \varepsilon + s^2\ \gamma\varepsilon, \qquad \begin{pmatrix} c & s \\ c' & s' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} m_{11}(I_{mag}) & m_{12}(I_{mag}) \\ m_{21}(I_{mag}) & m_{22}(I_{mag}) \end{pmatrix}$$

$$w_A^2 = c_A^2 \beta\varepsilon - 2c_A s_A\ \alpha\ \varepsilon + s_A^2\ \gamma\ \varepsilon$$
$$w_B^2 = c_B^2 \beta\varepsilon - 2c_B s_B\ \alpha\ \varepsilon + s_B^2\ \gamma\ \varepsilon$$
$$w_C^2 = c_C^2 \beta\varepsilon - 2c_C s_C\ \alpha\ \varepsilon + s_C^2\ \gamma\ \varepsilon$$

3 linear equations, 3 independent variable
Solved by inverting matrix.

$$\beta\varepsilon\cdot\gamma\varepsilon - (\alpha\varepsilon)^2 = \varepsilon^2\left(\beta\cdot\gamma - \alpha^2\right) = \varepsilon^2 \quad \Rightarrow \quad \sqrt{\beta\varepsilon\cdot\gamma\varepsilon - (\alpha\varepsilon)^2} = \varepsilon, \quad \beta = \frac{\beta\varepsilon}{\varepsilon}, \quad \alpha = \frac{\alpha\varepsilon}{\varepsilon}$$

Summary beam profile technics

- To determine ε, β, α at a reference point in a beamline one needs at least 3 *w* measurements with different transfer matrices between the reference point and the *w* measurements location.
- Different transfer matrices can be achieved with different profile monitor locations, different focusing magnet settings or combinations of both.
- Once β, α at one reference point is determined the values of β, α at every point in the beamline can be calculated.

Measurements for more different matrix helps to reconstruct emittance better

Emittance reconstruction from beam profiles measurements in the drift space

If space charge effects are neglectable then rms beam radius $x_{rms}^2(s)$ in drift space is parabola

1. Collect data from several BPMs at different location s
2. Fit parabola

$$x_{rms}^2(s) = as^2 + bs + c$$

3. If A,B,C defines in the follow form:

$$x_{rms}^2(s) = \varepsilon\beta(s) = As^2 - 2ABs + (C + AB^2)$$

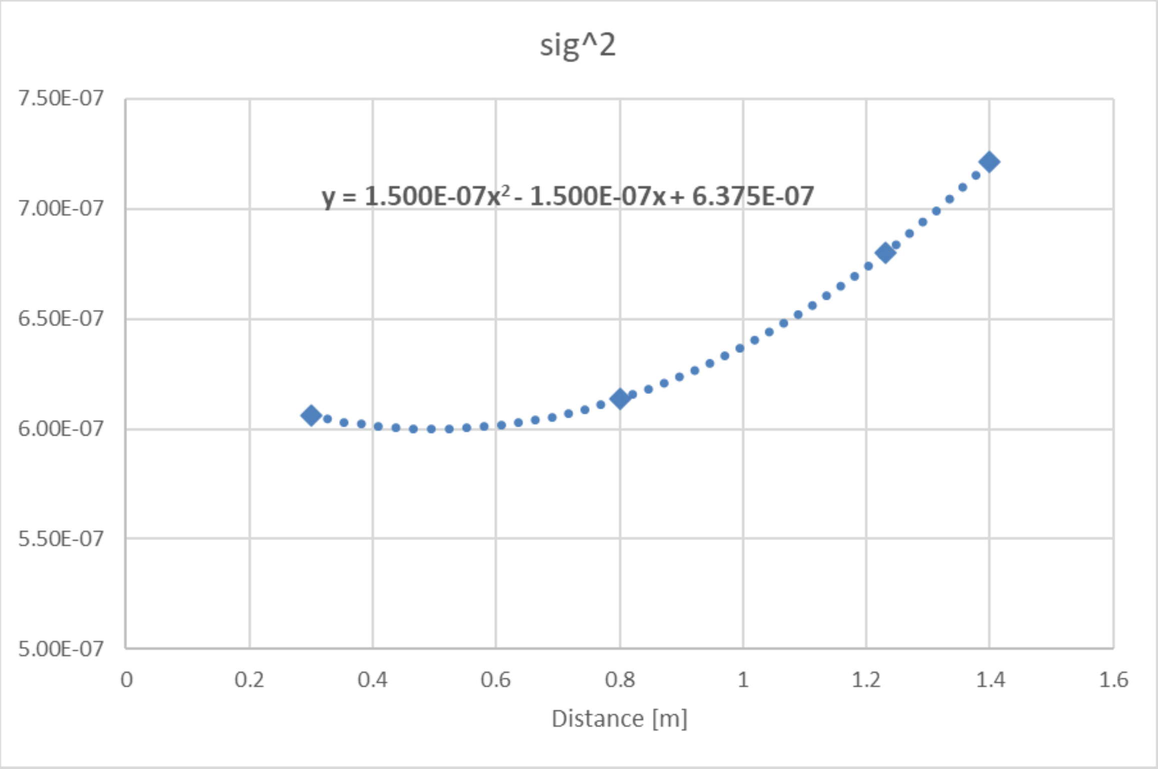
4. then emittance can be calculated :

$$\varepsilon = \sqrt{AC}$$

5. Please note that value of B gives the location of the beam envelope waist

Example of 4BPMS data analysis

s	sig	sig^2
0.3	0.0007785	0.000000606
0.8	0.0007833	6.135E-07
1.23	0.0008246	6.79935E-07
1.4	0.0008494	7.215E-07



	a	b	c	A	B	C	emit
x	1.50E-07	-1.50E-07	6.38E-07	1.500E-07	5.000E-01	6.000E-07	3E-07

Quadrupole or solenoid scan

The quadrupole scan technique is a standard technique used in accelerator facilities to measure the transverse emittance. It is based on the fact that the squared rms beam radius (x_{rms}^2) is proportional to the quadrupole "strength" or inverse focal-length f squared, so

$$x_{rms}^2 =< x^2 >= A\left(\frac{1}{f^2}\right) - 2AB\left(\frac{1}{f}\right) + (C + AB^2) \qquad (1)$$

where A, B, C are constants and f is the focal length defined as

$$\frac{1}{f} = \kappa l, \qquad (2)$$

<u>For quadrupole:</u>

$$k\left[\frac{1}{cm^2}\right] = \frac{G\left[\frac{kGauss}{cm}\right]}{Brho[kGauss*cm]}$$

<u>For solenoid:</u>

$$k\left[\frac{1}{cm^2}\right] = \left[\frac{B_z[kGauss]}{Brho[kGauss*cm]}\right]^2$$

here κ is the magnet focusing strength in units of 1 over length squared and l is the effective length of the magnet.

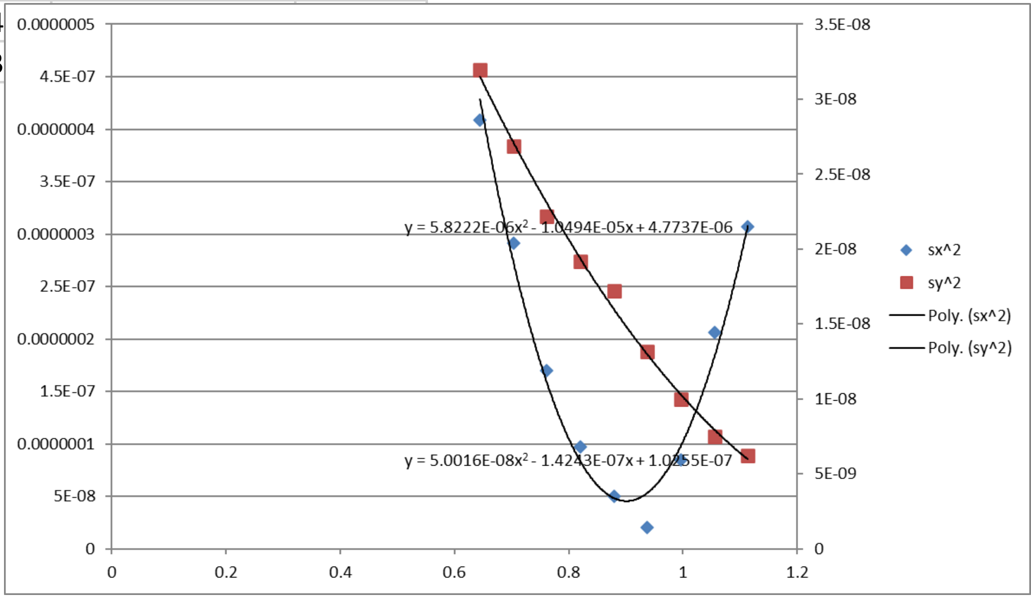
The emittance can be estimated according to

$$\varepsilon = \frac{\sqrt{AC}}{d^2} \qquad (3)$$

where d is the distance from the magnet you scan to the point you calculate the beam rms radius.

Example of Quadrupole scan data analysis from previous year

I, A	sx, pix	sy, pix	sx, m	sy, m	p, 1/m	sx^2	sy^2
5.5	45	12	0.000639	0.0001788	0.644596	4.08321E-07	3.2E-08
6	38	11	0.0005396	0.0001639	0.703195	2.91168E-07	2.69E-08
6.5	29	10	0.0004118	0.000149	0.761795	1.69579E-07	2.22E-08
7	22	9.3	0.0003124	0.0001386	0.820395	9.75938E-08	1.92E-08
7.5	15.7	8.8	0.0002229	0.0001311	0.878994	4.97022E-08	1.72E-08
8	10	7.7	0.000142	0.0001147	0.937594	2.0164E-08	1.32E-08
8.5	20.5	6.7	0.0002911	9.983E-05	0.996193	8.47392E-08	9.97E-09
9	32	5.8	0.0004544	8.642E-05	1.054		
9.5	39	5.3	0.0005538	7.897E-05	1.113		



	a	b	c	A	B	C	e	en
y	**5.002E-08**	**-1.424E-07**	**1.026E-06**	5.002E-08	1.424E+00	9.241E-07	9.216E-09	1.039E-06
x	**5.822E-06**	**-1.049E-05**	**4.774E-06**	5.822E-06	9.012E-01	4.507E-08	2.196E-08	2.475E-06