Homework 5. Due September 23

Problem 1. 5 points. Following up HW4: your proved that simple combination of field multipoles can describe the edge of a magnet. You also learned that we can used Laplacian equation on effective field potential:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\varphi=0$$

Let expand the potential in transverse direction while keeping arbitrary dependence along the beam propagating axis (s=z)

$$\varphi=\sum_{n+m=k}^{\infty} a_{nm}(z)x^n y^m$$

Derive the condition (connections) between functions $a_{nm}(z)$.

Problem 2. 8 points. Prove that

$$\det\left[I+\varepsilon A\right]=1+\varepsilon\cdot Trace[A]+O\left(\varepsilon^2\right)$$

where I is unit *nxn* matrix, A is an arbitrary *nxn* matrix and ε is infinitesimally small real number. Term $O\left(\varepsilon^2\right)$ means that it contains second and higher orders of ε.

Hint: first, look on the diagonal elements $\prod_{m=1}^{n}\left(1+\varepsilon a_{mm}\right)$ first, then see what contribution to determinant comes from non-diagonal terms $a_{km}; k\neq m$.