Homework 18 solution:

1. The longitudinal wakefield is given by the following equation

$$w_{//}(s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Z_{//}(\omega) e^{-i\omega s/c} d\omega. \tag{1}$$

Inserting the resistive wall impedance given in the problem into eq. (1) yields

$$
\begin{aligned}
w_{//}(s) &= \frac{c}{2\pi} \frac{(1-i)}{2\pi b} \sqrt{\frac{Z_0}{2\sigma}} \int_{-\infty}^{\infty} k^{1/2} e^{-iks} dk \\
&= \frac{Z_0 c}{2\pi b} \frac{1}{2\pi} \int_{-\infty}^{\infty} (1-i) \sqrt{\frac{k}{2\sigma Z_0}} e^{-iks} dk \\
&= \frac{Z_0 c}{2\pi b} \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{k}{\lambda} e^{-iks} dk \\
&= -\frac{c}{4\pi^{3/2} b |z|^{3/2}} \sqrt{\frac{Z_0}{\sigma}}
\end{aligned}
\tag{2}
$$

where the following relations

$$\lambda = i\left(1 - i\,\mathrm{sgn}(k)\right)\sqrt{\frac{\sigma|k|Z_0}{2}} = i(1-i)\sqrt{\frac{\sigma k Z_0}{2}}, \tag{3}$$

and

$$\frac{k}{\lambda} = \frac{\sqrt{2k}}{i(1-i)\sqrt{\sigma Z_0}} = (1-i)\sqrt{\frac{k}{2\sigma Z_0}} \tag{4}$$

are used.

2. Following the definition of the loss factor and that of the impedance, we obtain

$$k_{//} = \int_{-\infty}^{\infty} V_{//}(z)\lambda(z)dz$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \lambda(z_1)w_{//}(z_1-z)\lambda(z)dz_1dz$$

$$= \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Z_{//}(\omega_2)e^{i(\omega_2-\omega)z/c}\tilde{\lambda}(\omega)\tilde{\lambda}(\omega_1)e^{-i(\omega_1+\omega_2)z_1/c}d\omega_2 d\omega_1 d\omega dz_1 dz$$

$$= \frac{c^2}{(2\pi)}\int_{-\infty}^{\infty} Z_{//}(\omega)\tilde{\lambda}(\omega)\tilde{\lambda}(-\omega)d\omega$$

$$= \frac{c^2}{2\pi}\int_{-\infty}^{\infty} Z_{//}(\omega)\tilde{\lambda}(\omega)\tilde{\lambda}^*(\omega)d\omega$$

$$= \frac{c^2}{2\pi}\int_{-\infty}^{\infty} Z_{//}(\omega)\left|\tilde{\lambda}(\omega)\right|^2 d\omega$$

, (5)

where I used $\tilde{\lambda}^*(\omega) = \tilde{\lambda}(-\omega)$ since $\lambda(z)$ is real.