Homework 2. PHY 564 September 2, 2015

Due September 9, 2015

Problem 1. 10 points Motion of non-radiating charged particle in constant uniform magnetic field is a well known spiral:

$$\frac{d\vec{p}}{dt} = \frac{e}{c}\left[\vec{v}\times\vec{H}\right] = \frac{e}{c}H\left[\hat{e}_x v_y - \hat{e}_y v_x\right]; \vec{H} = \hat{e}_z H$$

$$\mathrm{E} = c\sqrt{m^2c^2 + \vec{p}^2} = conts; \gamma = const; v = const;$$

$$p_z = const;\ z = v_{oz}t + z_o;$$

$$p_x^2 + p_y^2 = const;\ p_x + ip_y = p_\perp e^{i\varphi(t)} = m\gamma v_\perp e^{i\varphi(t)}$$

simple substitution gives:

$$m\gamma v_\perp \frac{de^{i\varphi(t)}}{dt} = \frac{e}{c}\left[\vec{v}\times\vec{H}\right] = -i\frac{e}{c}Hv_\perp e^{i\varphi(t)}$$

$$r_\perp = x + iy = i\omega m\gamma v_\perp \frac{de^{i\varphi(t)}}{dt}$$

$$\varphi(t) = \omega t + \varphi_o;\ \omega = -\frac{eH}{m\gamma c}$$

and trajectory: $z = v_{oz}t + z_o; x + iy = v_\perp/\omega \cdot e^{i\omega t}$. Do not forget to apply Re or Im to all necessary formulae. Use analytical extension of the Lorentz transformation to complex values by going into a reference frame with x-velocity going approaching infinity $\beta \Rightarrow \infty; \chi \to 0; \chi\beta \to 1$. Show that transverse electric field becomes a magnetic field (with an imaginary value) and visa versa. Follow this path and transfer 4-coordinates to that frame. Use analytical extension of *exp, sin, cos* to complex values and transform the solution above in that for motion in constant magnetic field. Compare it with known solution is your favorite EM book .

Problem 2. 4 points

Find maximum energy of the a charged particle (with unit charge *e*!) which can be circulating in Earth's larges possible storage ring: the one going around Earth equator with radius of 6,384 km.

First, find it for storage ring using average bending magnetic field of a super-conducting magnet with strength of 10 T (100 kGs).

Second, find it for a very strong DC electric dipole fields of 10 MV/m.

Compare these energies with current largest (27 km in circumference) circular collider, LHC, circulating 6.5 TeV (1 TeV = 10^{12} eV).

Hint: assume that particles move with speed of the light. Check the final result for protons having rest mass of 938.27 MeV/c^2