USPAS'23: Hadron Beam Cooling in Particle Accelerators
Homework#3: Stochastic Cooling

Problem #1: Numerical model of SC (Total: 8 points)

In this problem we will use a simple model to simulate the effects of Stochastic Cooling. You can use your favourite tool (Python, Matlab, Mathematica, MathCad, etc.) to follow along this exercise.

1. Develop a simple SC model (2 points):

   - Generate an array of N random numbers $[x_1, x_2, .., x_N]$ - these are the initial positions of your particles. Generate at least several thousands of particles.
   - Calculate and record the variance of the array.
   - Slice your bunch into $N_s = 20$ equal samples and calculate the average of each sample (i.e. errors to be corrected).
   - Subtract sample average from the position of its respective sample (i.e. apply correction).
   - Randomize the order of the elements in the corrected array to get a new series.
   - Repeat at least 2000 times.

2. Characterize your system and discuss:

   - (2 points) Plot the particle distribution before and after cooling. Check how the beam variance changes with time.
   - (2 points) Repeat the procedure for $N_s = 10, 300, 600$. How does it affect the cooling process and why?
   - (2 points) How can you change this model to take the gain into account? Implement that and study how gain affects the cooling time.

Problem #2: Mixing, cooling rate, and bandwidth (Total: 8 points)

In the lectures, we have introduced an expression for the r.m.s. cooling rate that includes Kicker-to-Pick-Up mixing, M, and Pick-Up-to-Kicker mixing, $\tilde{M}$:

$$\frac{1}{\tau} = \frac{W}{N} \left[2g \left(1 - \tilde{M}^{-2}\right) - g^2 \left(M + U\right) \right], \tag{1}$$

1. (3 points) Using Eq. 1, find the expressions for the maximum achievable cooling rate and the respective optimum gain. If you have an ideal system, how long would it take to cool $N = 10^9$ particles if the system bandwidth is $W = 1$ GHz? How about $N = 4 \times 10^{13}$?

2. (2 points) Assuming that the time-of-flight dispersion between Pick-Up and Kicker and between Kicker and Pick-Up are such that the unwanted mixing is 1/2 of the wanted mixing, plot cooling time as a function of number of particles ($N \in [10^5, 10^{13}]$) for a system with $W = 1$ GHz. Explore all combinations of the following parameters: $M = 1$, 10, 50; $U = 0$, 10. Discuss your observations.

3. (3 points) It appears that the unwanted mixing imposes a limit on the upper frequency of the cooling band. Using the parameters of the "first generation cooling experiment" at the Antiproton Accumulator Ring at CERN (1984), obtain the upper frequency of the cooling band for that machine by following the steps below:

   - Assume a band-pass with flat response from $f_{\min}$ to $f_{\max}$. Find an expression for the useful width of the correction pulse T_c.
   - Express the time of flight error δt_{PK} in terms of the momentum spread $\Delta p/p$ and local Pick-Up-to-Kicker slip factor η_{PK}.
   - What is the upper frequency of the cooling band for given momentum spread $\Delta p/p = 2 \times 10^{-2}$, slip factor $\eta_{\mathrm{PK}} \approx \eta = 0.1$, flight time (Pick-Up-to-Kicker/circumference) $\alpha_{\mathrm{T}} = 0.5$, and revolution frequency $f_{\mathrm{rev}} = 1.5$ MHz?

Problem #3: Optical Stochastic Cooling (Total: 9 points)

Let us consider the application of optical stochastic cooling to three types of particles: electrons/positrons, protons/antiprotons, and heavy ions.

1. **Electrons:** (3 points) Since electrons already have a good damping mechanism due to synchrotron radiation, examine what OSC can do in low energy regime.
   Consider a 150 MeV ring of 60 m circumference with 2 cooling insertions: one for longitudinal-horizontal cooling and one for longitudinal-vertical. Assume the following beam parameters: $N = 5 \times 10^9$, normalized transverse emittances $\varepsilon_{x,n} = \varepsilon_{y,n} = 5 \times 10^{-4}$ m, bunch length $l_b = 2.5$ cm, and relative energy spread of 10^{-3}. For the amplifier with central wavelength of 0.8 μm and a bandwidth of 10%, calculate the optimal amplification factor g, the damping time for betatron oscillations $\tau_{x,y}$, and the damping time for energy oscillations τ_δ. Verify that with one bunch in the ring and with the obtained amplification factor, the average output power of the amplifier is about 5 W in each cooling insertion.

2. **Protons:** (3 points) Assume once again a machine with two cooling insertions, but this time we'll be cooling six bunches with 1×10^{11} protons/bunch with relative momentum spread of 3×10^{-4}, and a revolution frequency of 47.7 kHz. Consider an amplifier with an average output power of 100 W and a central wavelength $\lambda = 0.8 \mu$m. The undulator radiation with this wavelength could be obtained in an undulator with a peak magnetic field of 8 T and $\lambda_u = 1.5$m. Estimate the damping times for betatron and synchrotron oscillations.

3. **Heavy Ions:** (3 points) Consider damping of lead ions at an energy of 32.8 TeV. Assume 124 bunches of 1×10^8 ions/bunch, a relative momentum spread of 3×10^{-4}, and revolution frequency of 43 kHz. With two cooling insertions, and an undulator with a peak magnetic field of 8 T and $\lambda_u = 0.3$m, and the same optical amplifier as above, calculate the damping times for betatron and synchrotron oscillations.