Homework 9. Due October 10

Problem 1. 10 points. Two frequency RF system. Consider a storage ring negative η_τ and the RF system operating at two frequencies:

$$\frac{d\mathrm{E}}{ds} = \frac{eV_o}{C}\left(\sin\left(h_{rf}k_o\tau\right) - \sin\left(2h_{rf}k_o\tau\right)\right)$$

Find stationary point on the phase diagram, draw characteristic phase-space trajectories (approximately is fine) and show the direction of the motion by arrows.

Problem 2. 4x5 points.

For a single frequency RF system with Hamiltonian with α indicating an energy loss/gain,

$$\langle \mathcal{H}_s \rangle = \eta_\tau \frac{\pi_\tau^{\ 2}}{2} + \frac{1}{C}\frac{eV_{RF}}{p_o c}\frac{\cos\left(k_o h_{rf}\tau\right)}{k_o h_{rf}} + \alpha \cdot \tau; \quad \eta_\tau < 0.$$

1. Define the stationary points (RF phases) in the phase space and indicate level of α when stationary points are no longer exists.
2. Draw phase space trajectories for $\alpha = \frac{1}{2} \cdot \frac{1}{C}\frac{eV_{RF}}{p_o c}$. Show the direction of the motion by arrows.
3. Define the depth of the “RF bucket”, e.g. the difference between the maximum and minimum π_τ staying within a single RF separatrix (e.g. being localized). Express it through the RF voltage, the slip factor and the value of stationary phase.
   Note – consider the central separatrix around $\tau = 0$.
4. Find period of the oscillation as function of $\langle \mathcal{H}_s \rangle$ inside the central separatrix (around $\tau = 0$).