Class schedule for March 9

- Short lecture on magnets for accelerators.
- Will take a break
- Then, you will be divided into two groups
- Group A will learn about magnets in the experiment (~50 min)
- Group B will continue last week's exercise
- You will switch
- Please email me your project paper (three preferences). My email: diktys@bnl.gov

Magnets for particle accelerators

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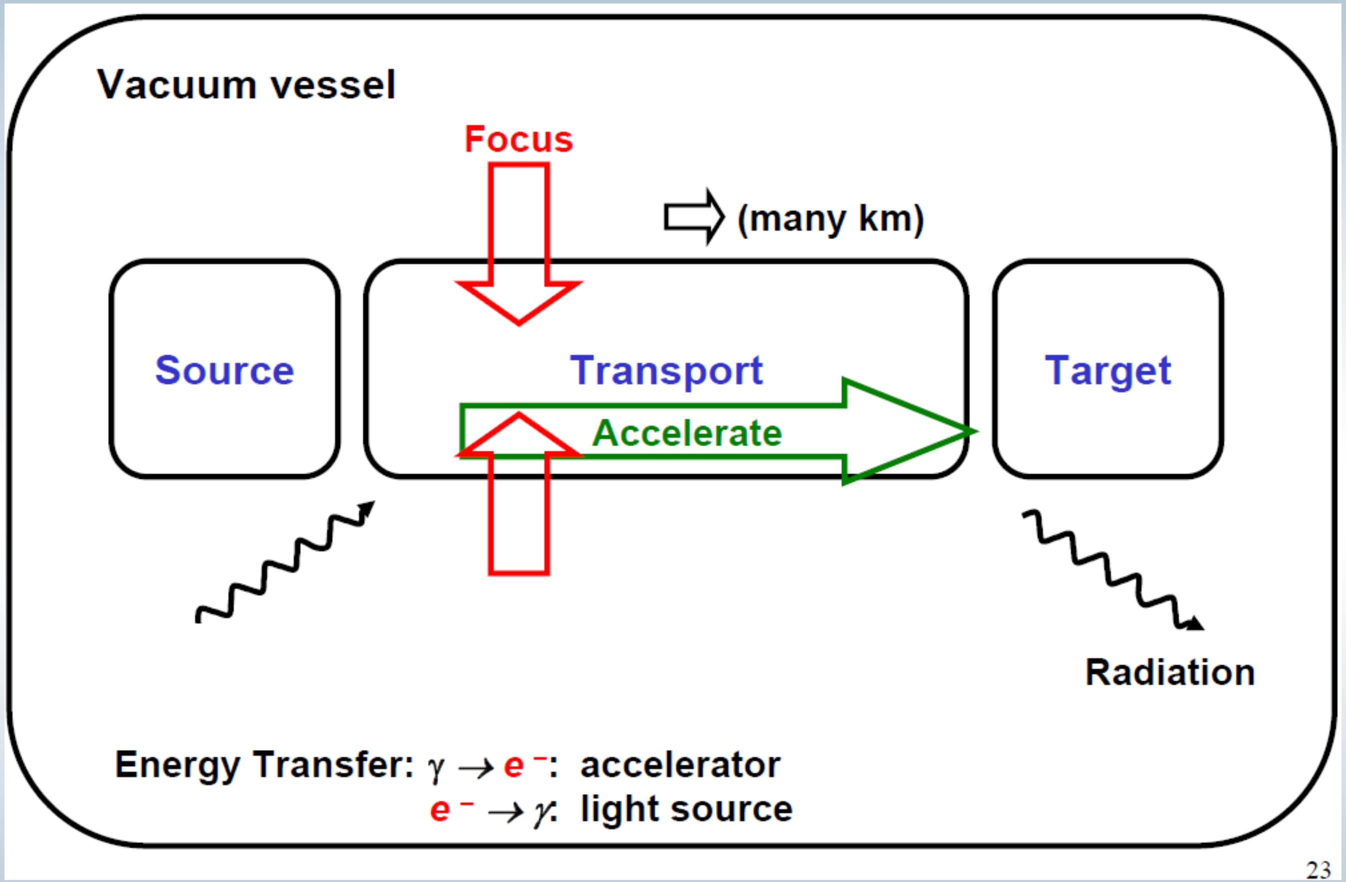
PHY 542
March 09, 2015

Accelerator simplified schematic



- Three main components: Source, transport, target
- Today we will focus on transport

Beam Transport

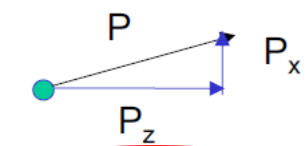
- Each particle is defined by position and momentum:

$$\vec{x} = (x, p_x, y, p_y, z, p_z)$$

- More convenient is to use position and divergence

$$\vec{x} = (x, p_x, y, p_y, z, p_z)$$

$$\vec{x} = (x, x' = \frac{p_x}{p_z}, y, y' = \frac{p_y}{p_z}, z, \frac{\Delta p}{p})$$

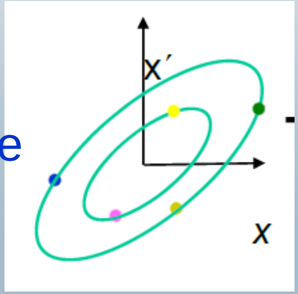


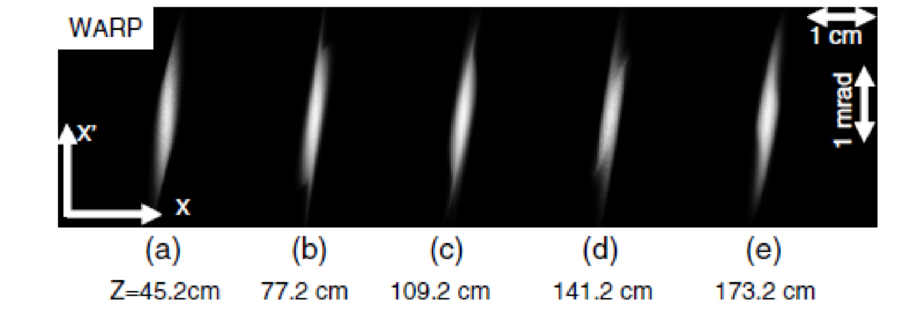
Paraxial approximation

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \approx v_z$$

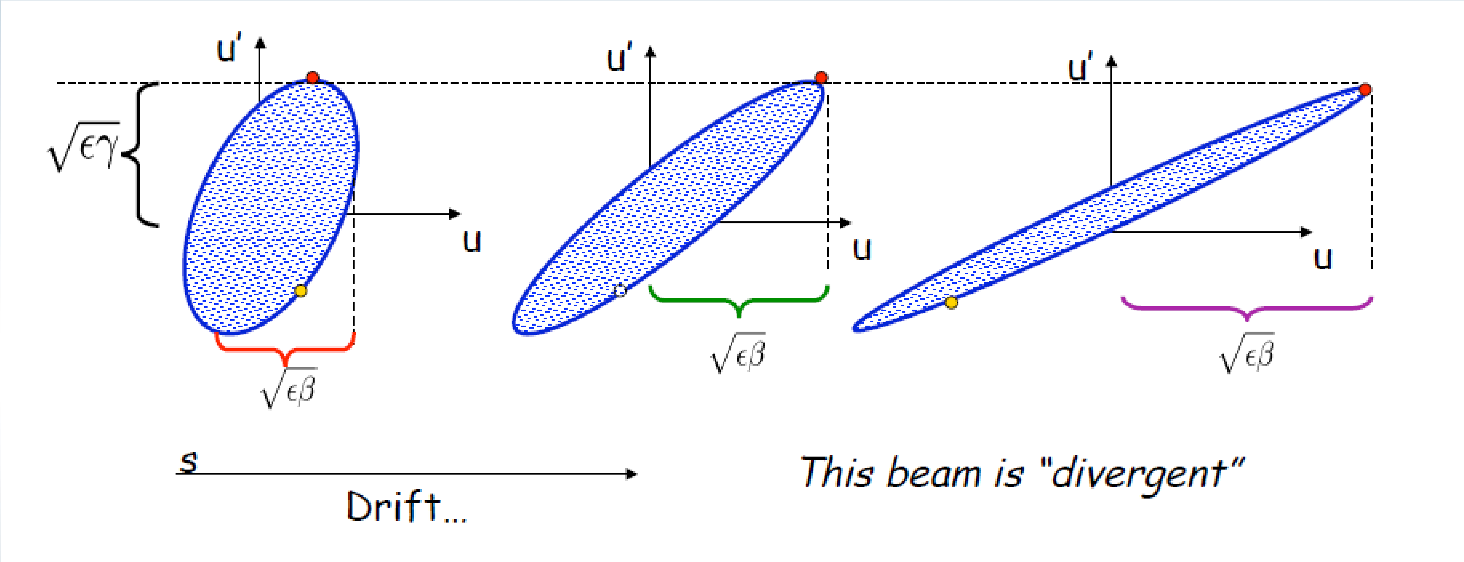
- Assuming no coupling the transverse motion can be represented by two dimensional vectors u=(x,x’) and v=(y,y’)

Phase-space





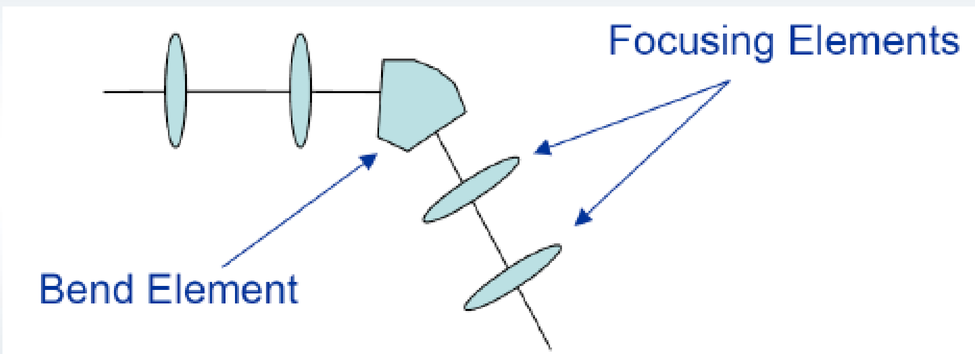
Beam phase-space in a drift



- Observation: Without focusing any beam would spread
- Magnets: Solenoids, quadrupoles, bends
- Why magnetic focusing? Why not electrostatic focusing?

Magnetic lattice

- An array of magnets that is guiding a charged particle beam from A to B using magnets is a transport system or a magnetic lattice

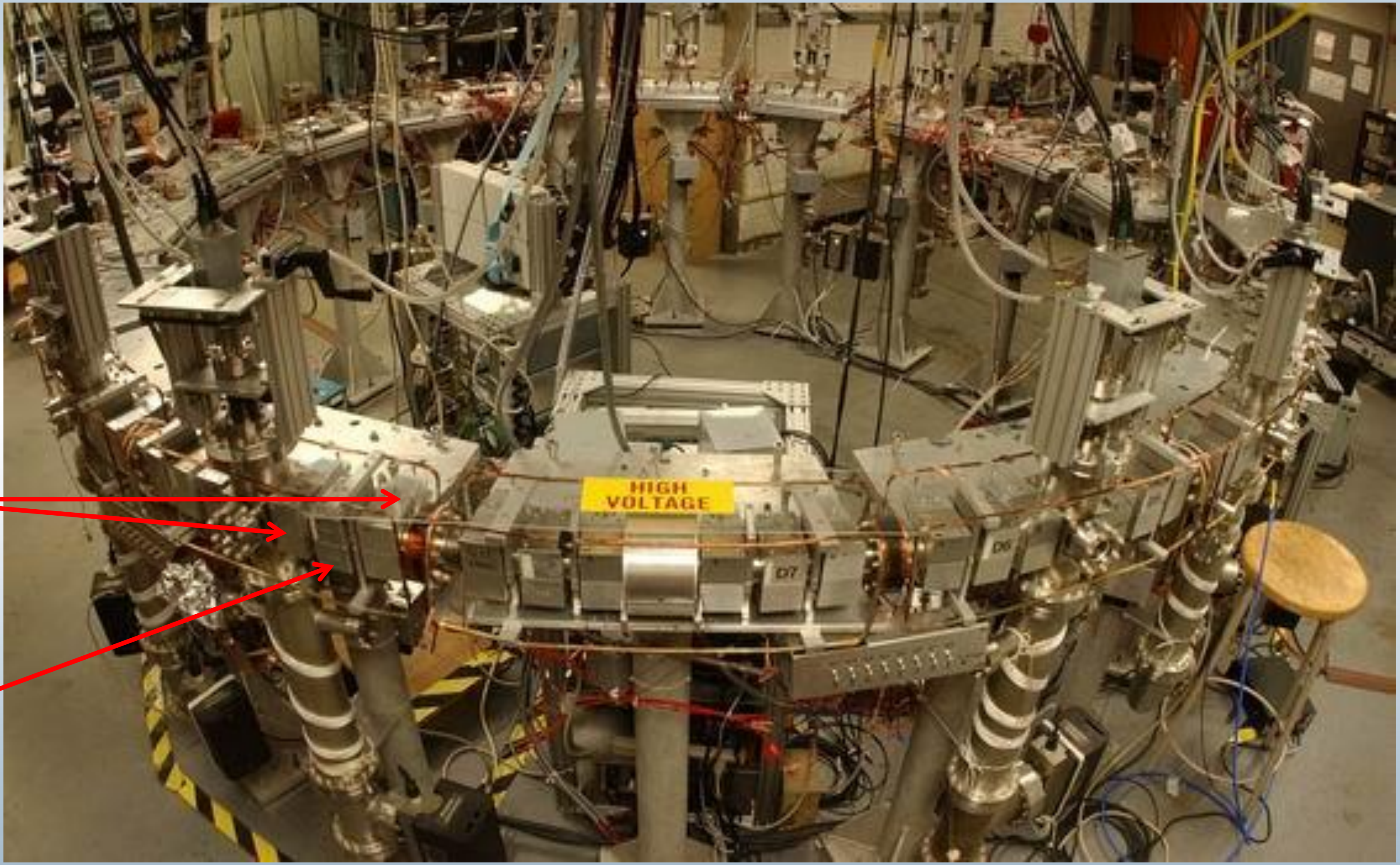


Bending Magnet

Quadrupole

Solenoid

Example: University Maryland Ring (UMER)

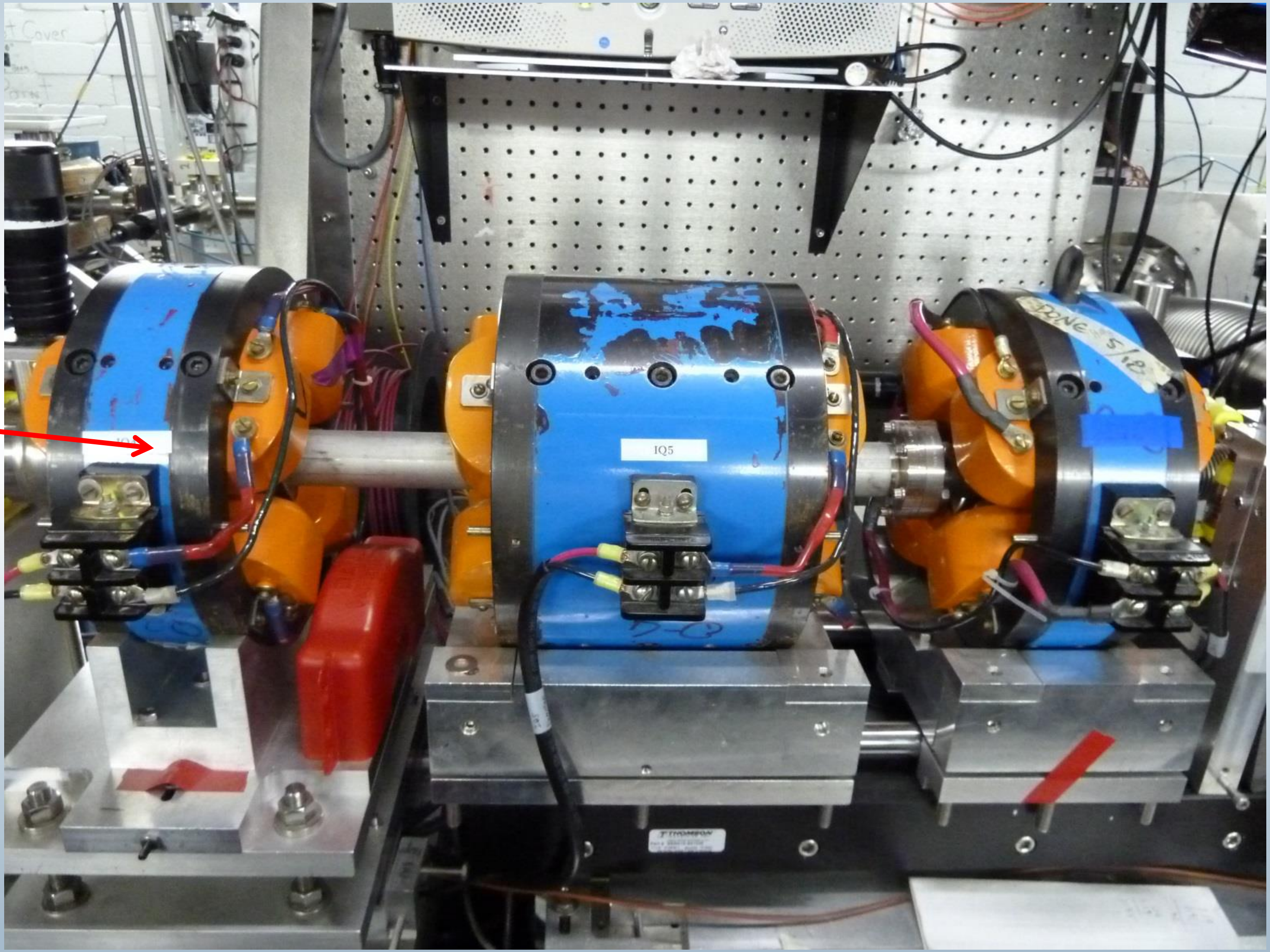


Quadrupole

Bend

Example: ATF

Quadrupole



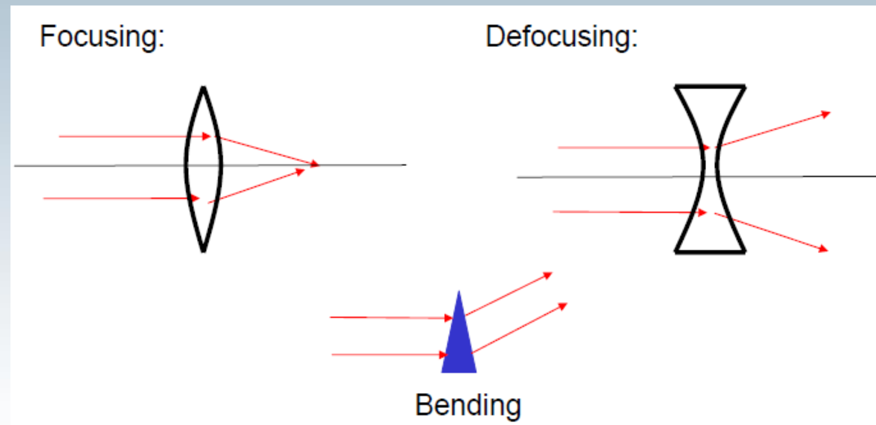
Magnetic focusing

- Accelerators rely on magnetic focusing. Can you guess why?
- Ratio of magnetic and electric forces:

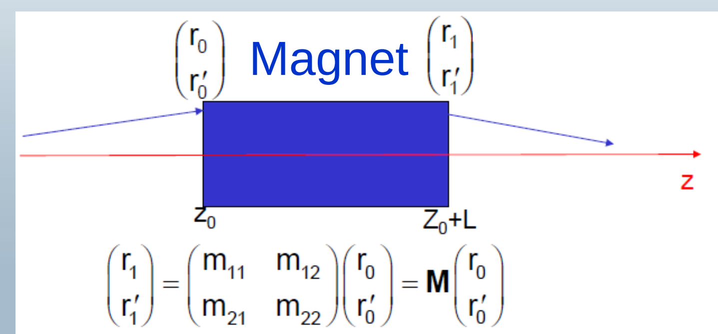
$$F = q(E + v \times B) \Rightarrow \frac{F_M}{F_E} = \frac{vB}{E} \Rightarrow \frac{F_M}{F_E} = 1 \Rightarrow \boxed{E = vB}$$

- Consider the case of high-energy particles ($v \sim c$)
  - B=1 T, the equivalent E-field is E=300 MV/m
- Consider the case for low energy particles ($v = 0.01c$)
  - For B=1 T, the equivalent E field is E=3 MV/m
- Summary: Magnets are better at high energy!

The good news…

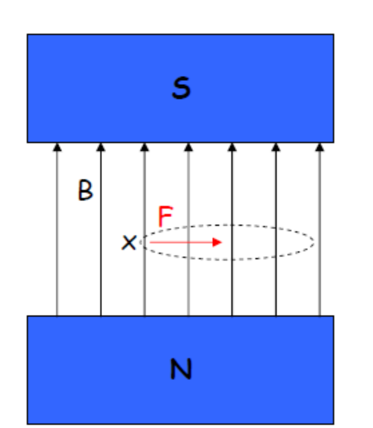


- Many of the concepts of conventional light optics can be carried over to describe accelerator optics
- We can use matrix concepts from light optics, i.e. a particle is a vector (x,x') and a lens is a matrix that multiplies (manipulates) vector
- Again no coupling assumed!



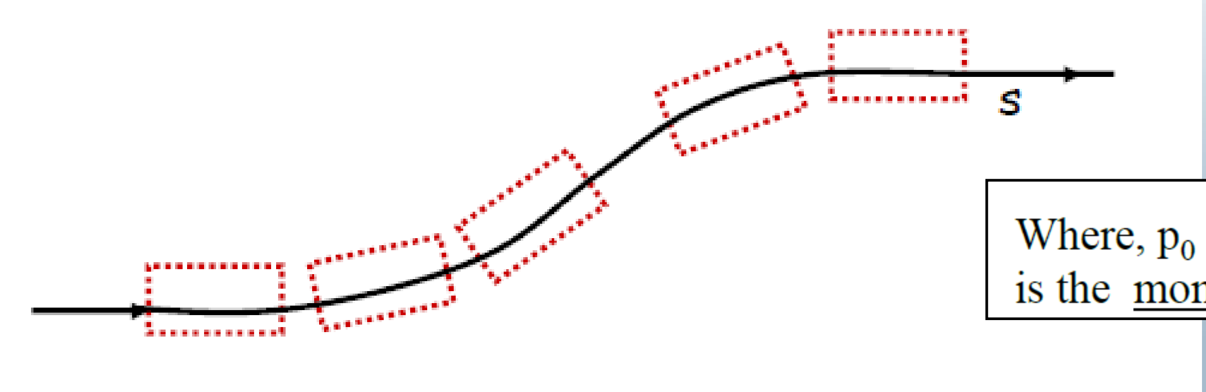
Bending magnets

- A dipole magnet gives us a constant field B
- For a positive particle traveling into the page, the force is to the right
- In an accelerator dipoles are used to bend the beam trajectory
- A useful quantity is the magnet rigidity



$$\frac{d\theta}{dt} = \omega_c = \frac{eB}{m} = \frac{eBv}{p_0}$$

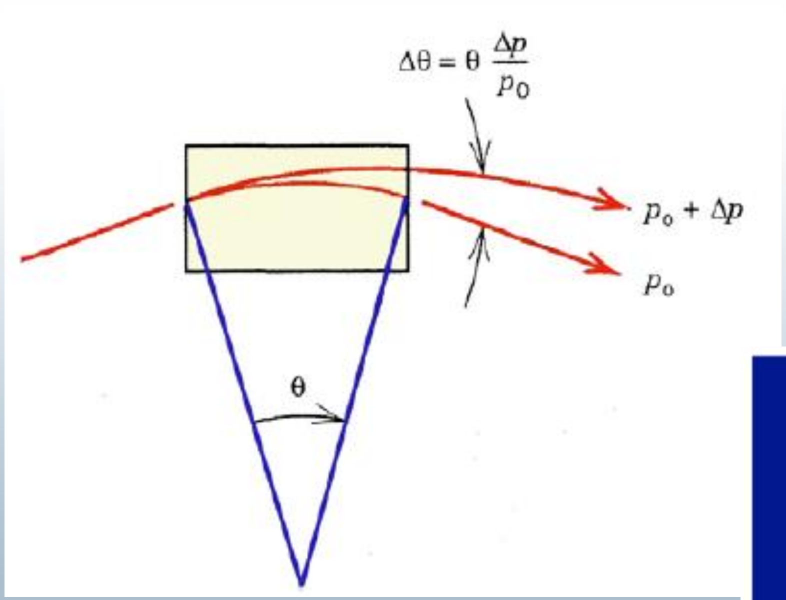
$$\Rightarrow \boxed{\theta = \frac{e}{p_0}\int_{s_1}^{s_2} Bdl = \frac{e}{B\rho}\int_{s_1}^{s_2} Bdl}$$

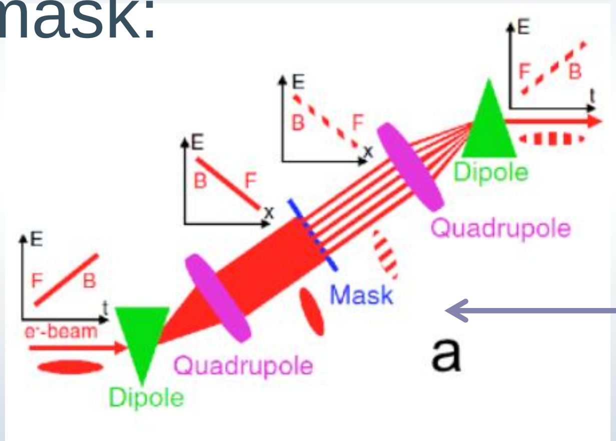


Where, p_0 is the momentum and $B\rho = p_0/e$ is the momentum 'rigidity' of the beam.

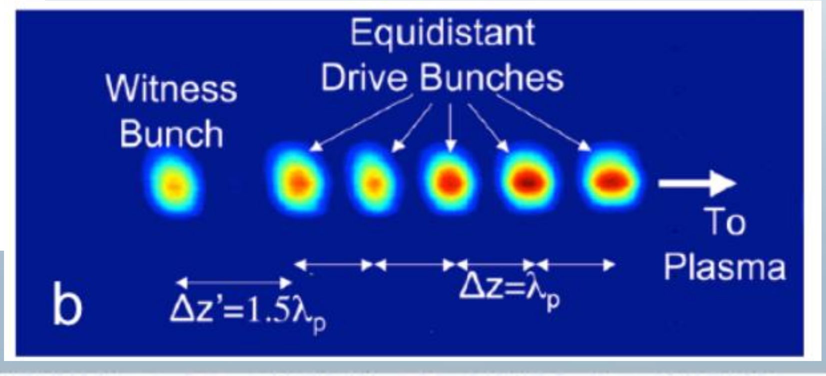
Dispersion

- Particles with different momentum follow different paths in the magnet
- Can cause beam degradation, BUT sometimes is useful. One example is the ATF mask:



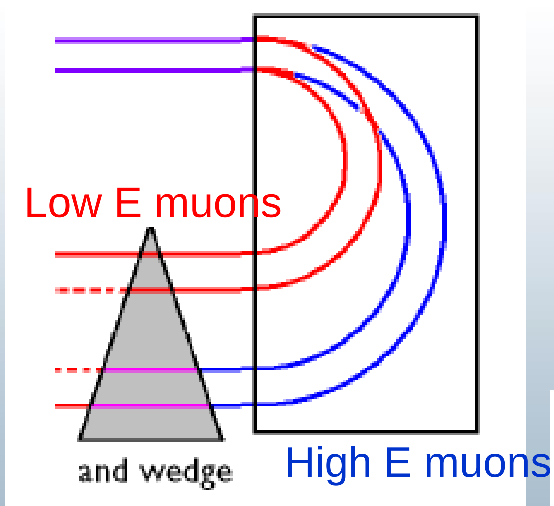


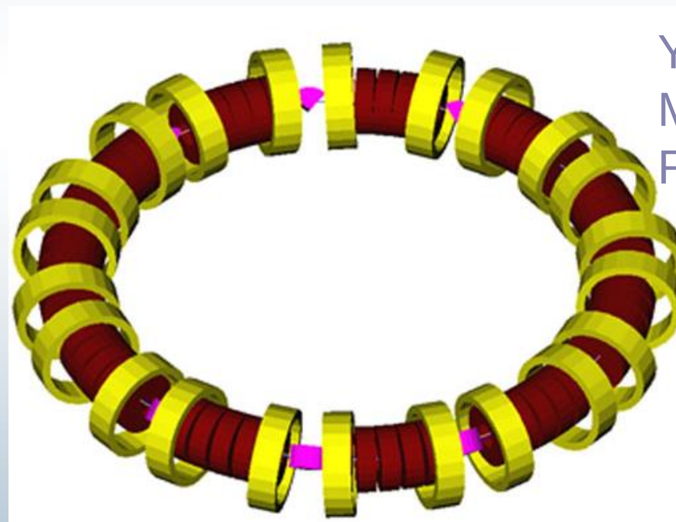
Dispersion Mask at the ATF



Dispersion for beam cooling

- Beam cooling through particle-matter interaction
- Introduce dispersion before entering absorber
- Pass through wedge material to cool the beam





Yellow: Bends
Magenta: Absorber
Red: Cavities

Emitance reduces by 5 orders of magnitude!

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Tapered channel for six-dimensional muon cooling towards micron-scale emittances

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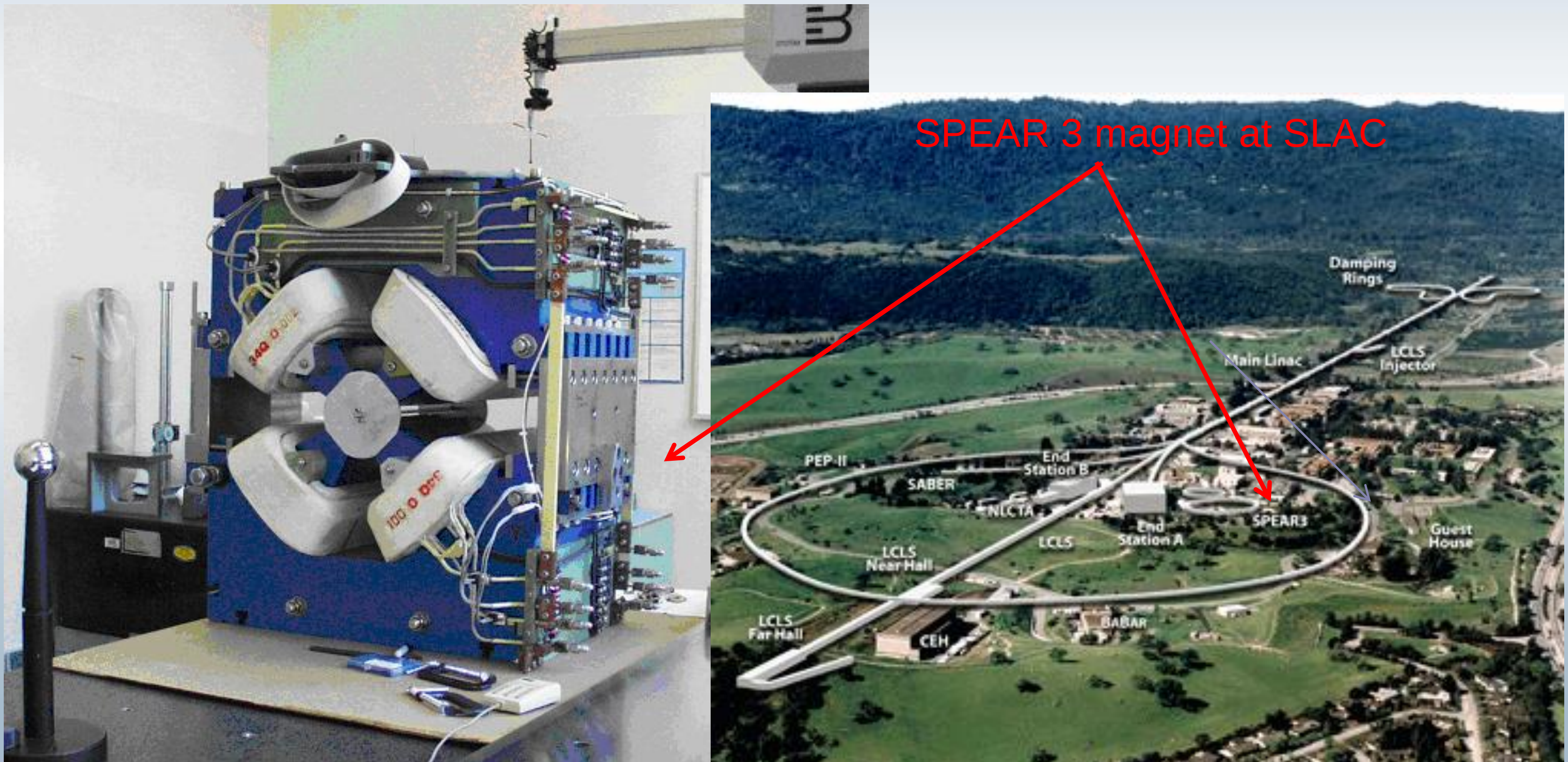
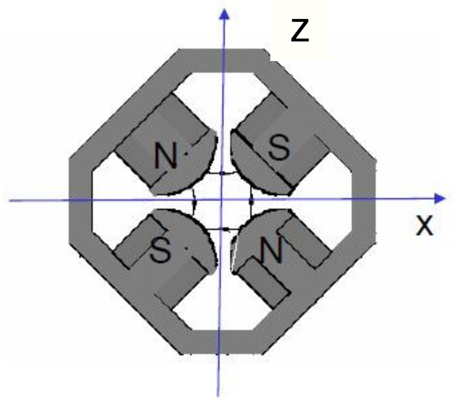
(Received 19 June 2013; published 23 September 2013)

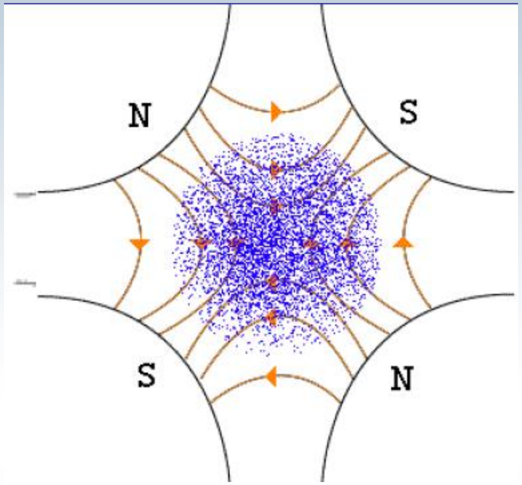
Quadrupoles

- Bends are only bending…
- Now we need something to focus the beam



Quadrupoles (cont.)





- Magnetic field on ideal quadrupole is: $\boxed{B = B_1 \left(z\hat{x} + x\hat{z} \right)}$
- Particles aligned on the center of the magnet receive no force from the quadrupole field
- Off-axis the force is pushing inward and is linear

Quadrupoles: Some simple math

- Particles displaced by (x,z) from the center receive a force:

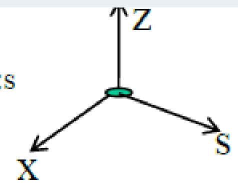
$$\boxed{B = B_1\left(z\hat{x} + x\hat{z}\right)}$$

$$\vec{F} = evB_1\hat{s}\times\left(z\hat{x} + x\hat{z}\right) = -evB_1 z\hat{z} + evB_1 x\hat{x}$$

the equations of motion become:

$$\frac{1}{v^2}\frac{d^2x}{dt^2} = \frac{eB_1}{\gamma m v}x, \quad \frac{1}{v^2}\frac{d^2z}{dt^2} = -\frac{eB_1}{\gamma m v}z$$

Particle Coordinates
- Moving along trajectory :s
- Vertical displacement: z
- Horizontal: x



- Or $\frac{d^2x}{ds^2} = x'' = \kappa x$ and $\frac{d^2z}{ds^2} = -\kappa z$ where $\kappa = \frac{eB_1}{\gamma m v}$
- Recall that orbits can be represented by two dimensional vectors u=(x,x') and v=(z,z')
- Thus:

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos\sqrt{\kappa}L & \frac{1}{\sqrt{\kappa}}\sin\sqrt{\kappa}L \\ -\sqrt{\kappa}\sin\sqrt{\kappa}L & \cos\sqrt{\kappa}L \end{pmatrix}\begin{pmatrix} x_0 \\ x0' \end{pmatrix}$$

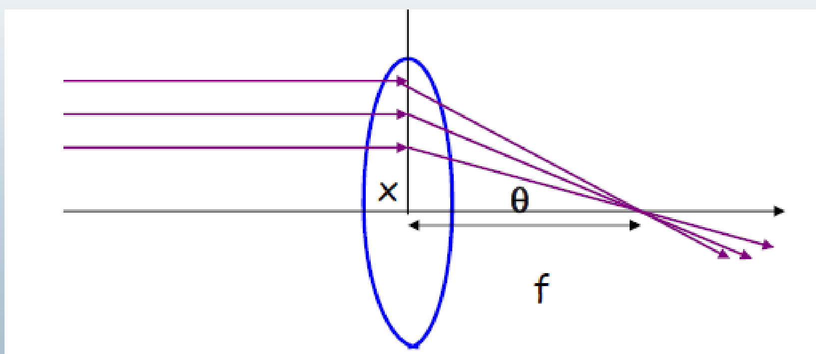
$$\begin{pmatrix} z \\ z' \end{pmatrix} = \begin{pmatrix} \cosh\sqrt{\kappa}L & \frac{1}{\sqrt{\kappa}}\sinh\sqrt{\kappa}L \\ \sqrt{\kappa}\sinh\sqrt{\kappa}L & \cosh\sqrt{\kappa}L \end{pmatrix}\begin{pmatrix} z_0 \\ z0' \end{pmatrix}$$

Thin lens approximation

- Take limit as L→0, while κL remains finite:

$$\begin{pmatrix} \cos(\sqrt{K}L) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}L) \\ -\sqrt{K}\sin(\sqrt{K}L) & \cos(\sqrt{K}L) \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ -KL & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{F} & 1 \end{pmatrix}$$

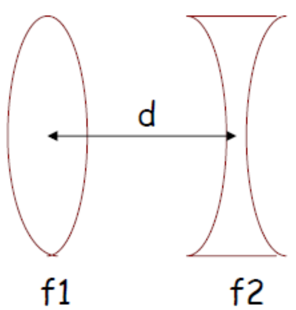
- Thus, if the quadrupole is thin enough, particles offset through the quad does not change much, but the slope of trajectory does – acts like a thin lens in geometrical optics!



Change in trajectory: $\Delta x' = \frac{x}{f}$

Quadrupole doublets

- Quadrupoles focus in one plane while defocusing in the other. So, how can this be used to provide net focusing in an accelerator?
- Consider again the optical analogy of two lenses, with focal lengths f1 and f2, separated by a distance d:



The combined f is:

$$\frac{1}{f_{combined}} = \frac{1}{f1} + \frac{1}{f2} - \frac{d}{f1 f2}$$

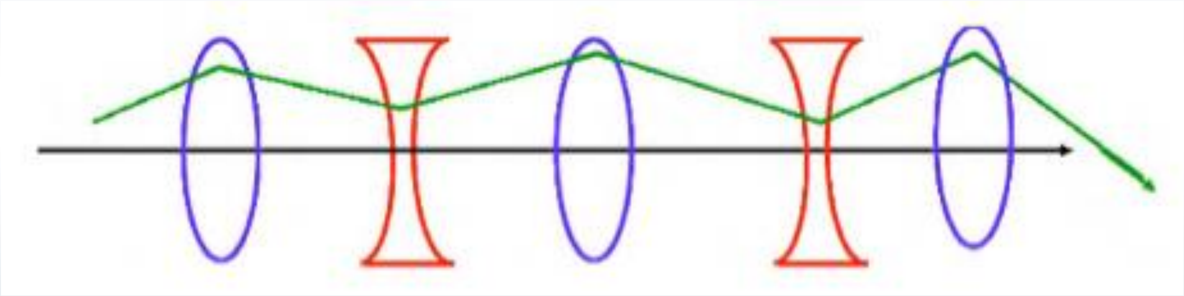
What if f1 = -f2?

$$f_{combined} = \frac{f1^2}{d}$$

- A quadrupole double is focusing on both planes!

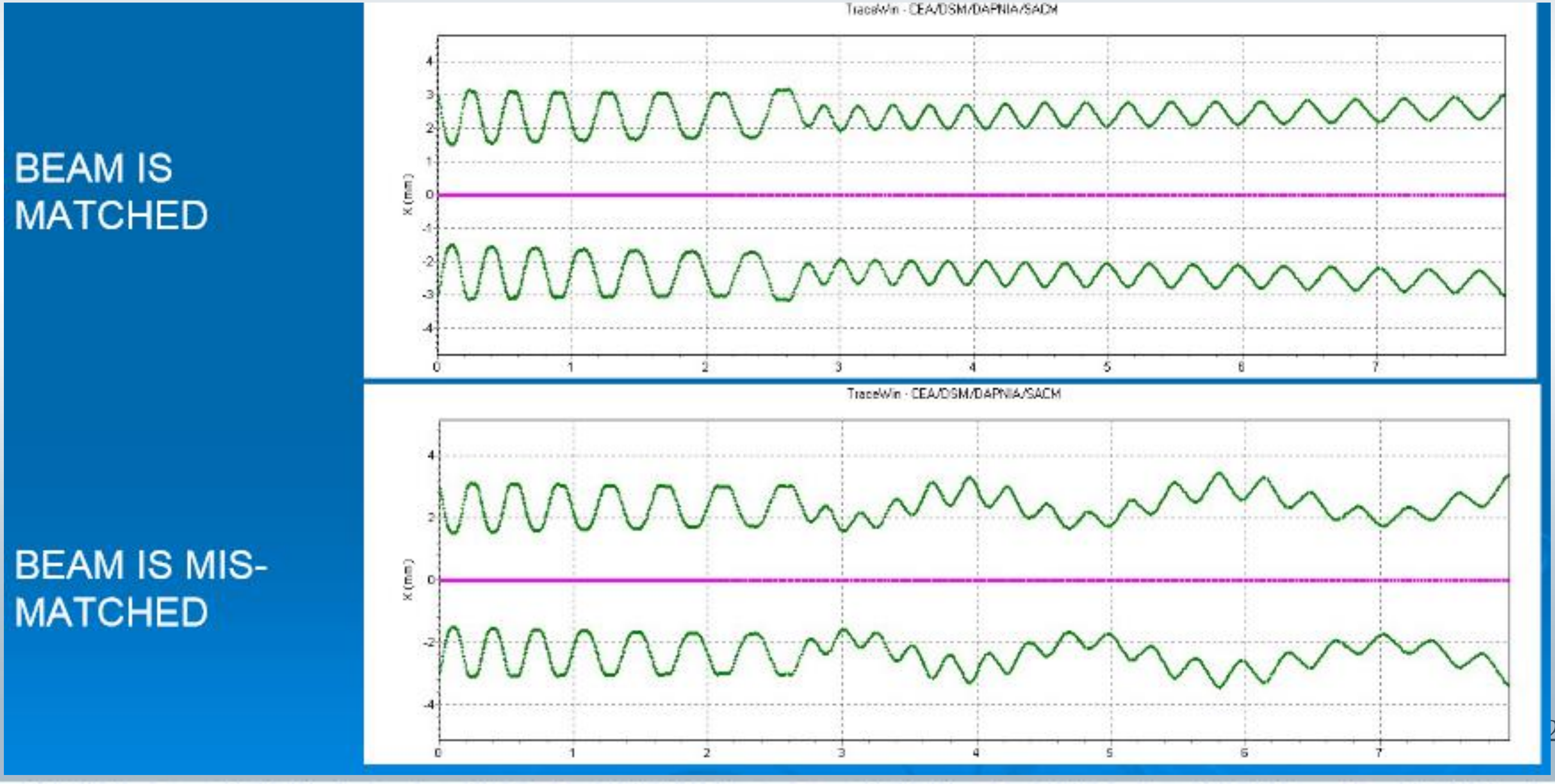
Quadrupole doublets (cont.)

- Strong focusing by sets of quadrupole doublets with alternating gradient

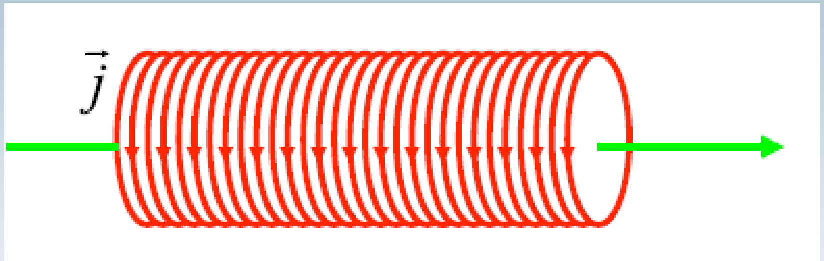


Beam matching

- Proper matching is a key for maintaining good quality beams!



Solenoids



- A *solenoid* is a tightly wound helical coil of wire whose diameter is small compared to its length.
- The magnetic field generated in the center, or *core*, of a current carrying solenoid is essentially *uniform*, and is directed along the axis of the solenoid.
- Solenoids are preferred at low energies