Chapter 10
Strong Focusing Synchrotron

Abstract This Chapter introduces the strong focusing synchrotron, alternating gradient (AG) and separated focusing, and the theoretical material needed for the simulation exercises. It begins with a brief reminder of the historical context, and continues with beam optics, chromaticity, and acceleration. It relies on basic charged particle optics and acceleration concepts introduced in the previous Chapters, and further addresses the following aspects:
- resonances and resonant extraction,
- stochastic energy loss by synchrotron radiation.

The simulation of a strong focusing synchrotron requires just two, possibly three, optical elements from `zgoubi` library: DIPOLE, BEND, or MULTIPOL to simulate (possibly combined function) dipoles, DRIFT to simulate straight sections, and MULTIPOL to simulate lenses (which can be otherwise simulated using QUADRUPO, SEXTUPOL, OCTUPOLE, etc.). A fourth element, CAVITE, is required for acceleration. Particle monitoring requires keywords introduced in the previous Chapters, including FAISCEAU, FAISTORE, possibly PICKUPS, and some others. Spin motion computation and monitoring resort to SPNTRK, SPNPRT, FAISTORE. Optics matching and optimization use FIT[2]. INCLUDE is used, mostly here in order to shorten the input data files. SYSTEM is used to, mostly, resort to gnuplot so as to end simulaitons with some specific graphs obtained by reading data from output files such as zgoubi.fai (resulting from the use of FAISTORE), zgoubi.plt (resulting from IL=2), or other zgoubi.*.out files resulting from a PRINT command.

Notations used in the Text

B; $\mathbf{B}$, $B_{\mathrm{x,y,s}}$	field value; field vector, its components in the moving frame
$B\rho = p/q$; $B\rho_0$	particle rigidity; reference rigidity
C; C_0	orbit length, $C = 2\pi R + \left[\begin{smallmatrix}\text{straight}\\ \text{sections}\end{smallmatrix}\right.$; reference, $C_0 = C(p = p_0)$
E	particle energy
EFB	Effective Field Boundary
f_{rev}, $f_{\mathrm{rf}} = f_{\mathrm{rev}}$	revolution and accelerating voltage frequencies
G	gyromagnetic anomaly, $G = 1.792847$ for proton
G; $K = G/B\rho$	quadrupole gradient; focusing strength
m; m_0; M	mass, $m = \gamma m_0$; rest mass; in units of MeV/c^2
$\mathbf{p}$; p; p_0	momentum vector; its modulus; reference
P_i, P_f	beam polarization, initial, final
q	particle charge
r, R	orbital radius ; average radius, $R = C/2\pi$
s	path variable
v	particle velocity
$V(t)$; $\hat{V}$	oscillating voltage; its peak value
x, x', y, y', l, $\frac{dp}{p}$	horizontal, vertical, longitudinal coordinates in moving frame

α	momentum compaction
α	trajectory angle
$\beta = v/c$; β_0; β_{s}	normalized particle velocity; reference; synchronous
β_{u}	betatron functions ($u:\ x, y, Y, Z$)
$\gamma = E/m_0$	Lorentz relativistic factor
δp	momentum offset or Dirac distribution
Δp	momentum offset
ε	wedge angle
ε_{u}	Courant-Snyder invariant ($u:\ x, r, y, l, Y, Z, s$, etc.)
ϵ_R	strength of a depolarizing resonance
μ_{u}	betatron phase advance, $\mu_{\mathrm{u}} = \int_{\mathrm{period}} ds/\beta_{\mathrm{u}}(s)$ ($u:\ x, y, Y, Z$)
ν_{u}	wave numbers, horizontal, vertical, synchrotron ($u:\ x, y, Y, Z, l$)
ρ, ρ_0	curvature radius; reference
σ	beam matrix
ϕ; ϕ_{s}	particle phase at voltage gap; synchronous phase
ϕ_{u}	betatron phase advance, $\phi_{\mathrm{u}} = \int ds/\beta_{\mathrm{u}}$ ($u:\ x, y, Y, or Z$)
φ	spin angle to the vertical axis

10.1 Introduction

In the very manner that the 1930s-1940s cyclotron, betatron, microtron, weak focusing synchrotron, still in use today, have since essentially not changed in their concepts, design principles, magnet gap profile, today's gap profile, yoke and current coil geometry of combined function alternating-gradient (AG) dipoles remain essentially as patented in 1950 (Fig. 10.1) [1].

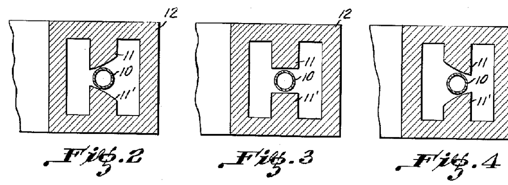


Fig. 10.1 Bending magnet pole profiles for a *focussing system for ions and electrons* [1]. Assuming curvature center to the left, the right (respectively left) profile is defocusing (resp. focusing), the middle profile has zero index

In 1952, in the context of studies relative to the Cosmotron, strong focusing was devised at the Brookhaven National Laboratory (BNL): *"Strong focusing forces result from the alternation of large positive and negative n-values in successive sectors of the magnetic guide field in a synchrotron. This sequence of alternately converging and diverging magnetic lenses* [...] *leads to significant reductions in oscillation amplitude"* [2]. It led to the construction of the first two high-energy proton AG synchrotrons, in the 30 GeV range, in the late 1950s: the proton-synchrotron (PS) at CERN, and the AGS at BNL, major pieces 60 years later still, of the respective injection chains of the two largest colliders in operation, the LHC and RHIC. Early works at BNL provided theoretical formalism, still at work today, for the analyzis of beam dynamics in synchrotrons [3].

The optical principle behind the AG concept is that a doublet of focusing and defocusing lenses with proper stengths results in a, possibly quite strong, very short focal distance, converging system. The dramatic effect of strong-index AG on transverse beam size allows small dipole gaps, thus small magnets: from lowest energies (medical synchrotrons in the 100 MeV range for instance) to the highest ones (particle physics and nuclear physics colliders, hundreds of GeV to multi-TeV range), beams are essentially confined in a centimeter scale transverse space, making a synchrotron a string of dipole magnets containing beam in a ring vacuum pipe of cm to 10 cm diameter; the size of the ring is essentially determined by its circumference, proportional to the magneitc rigidity. This revolutionized the race to high energies, from an upper 10 GeV about of the prior weak focusing synchrotrons and their huge magnets, to todays 7 TeV at the LHC with magnets transverse size in the meter range, and with further plans for 100 TeV rings [5]. It fostered as well the development of high energy synchrotron light sources around the world, with electron beam energies up to 8 GeV.

The original AG dipole design (that of the PS and AGS rings), whereby gradient dipoles combine beam guiding and beam focusing, has the benefit of compactness. It is still prised today and resorted to, for instance in hadrontherapy applications (Fig. 10.3); light source lattice vertical focusing [7], etc. Seperated function AG focusing, whereby beam guiding is ensured by uniform field dipoles while focusing is ensured separately by quadrupoles, followed from the development of the latter

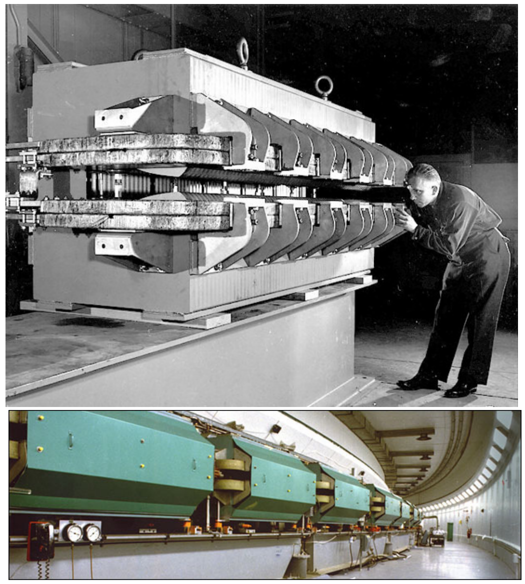


Fig. 10.2 Top: the AGS combined function main magnet - one of 240 steering the beam around the ring, bottom: the 809 m circumference AGS synchrotron [4]. The hyperbolic profile poles are visible on the top photo, partly hidden by the field coils

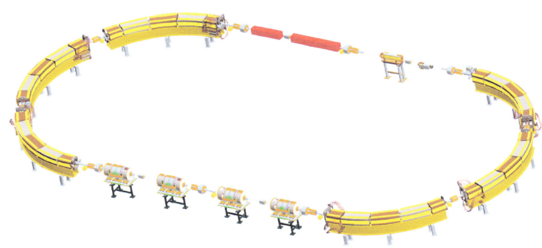


Fig. 10.3 The ion rapid cycling medical synchrotron (iRCMS) [6], an RCS aimed at providing ion beams for the treatment of cancer tumours

(Fig.10.4), a spin-off of the strong index technology [8]. Separated function optics has the merit of flexibility, allowing modular functions in complex rings such as bending-free dispersion suppression sections, low-beta collision or insertion device sections, long straights, etc. Low-emittance, high-brightness light source lattices have complicated focusing further, by introducing longitudinal field gradient bending systems, aimed at minimizing the chromatic invariant [9].

Due to the necessary ramping of the field in order to maintain a constant orbit, synchrotrons accelerators are pulsed, some storage rings species are pulsed as well, high energy colliders in particular to bring beams to highest store energy. The acceleration is cycled and the accelerating voltage fequency as well in ion accelerators, from injection to top energy. If the ramping uses a constant electromotive force, then (Eq. 9.3)

$$B(t) \approx \frac{t}{\tau} \tag{10.1}$$

Fig. 10.4 A quadrupole magnet at LBL in 1957, used for beam lines at the 184-inch cyclotron. An early specimen here, obviously, being a spin-off of the early 1950s concept of strong focusing [10]

$\dot{B} = dB/dt$ does not exceed a few Tesla/second, thus the repetition rate of the acceleration cycle if of the order of a Hertz. If instead the magnet winding is part of a resonant circuit then the field oscillate,

$$B(t) = B_0 + \frac{\hat{B}}{2}(1 - \cos \omega t) \tag{10.2}$$

so that, in the interval of half a voltage repetition period (*i.e.*, $t : 0 \rightarrow \pi/\omega$) the field increases from an injection threshold value to a maximum value at highest rigidity, $B(t) : B_0 \rightarrow B_0 + \hat{B}$. The latter determines the highest achievable energy: $\hat{E} = pc/\beta = q\hat{B}\rho c/\beta$. The repetition rate with resonant magnet cycling can reach a few tens of Hertz, a species known as a rapid-cycling synchrotron (RCS). In both cases anyway B imposes its law and the other quantities comprising the acceleration cycle (RF frequency in particular) will follow B(t).

Rapid cycling allows high intensity beams. Instances are the Cornell 12 GeV, 60 Hz, electron synchrotron, commissioned in 1967, today the injector of Cornell 5 GeV synchrotron light source (CHESS); Fermilab 8 GeV, 60 Hz, booster which provides protons for the production of neutrino beams; the 30 GeV 500 kW beam J-PARC facility in Japan. Rapid cycling is also considered in ion-therapy applications, Fig. 10.3.

10.2 Basic Concepts and Formulæ

Alternating gradient focusing is sketched in Fig. 10.5.

The focusing index value can be estimated from the fields met in these structures: a maximum $B \sim 1$ Tesla in the dipole gap, and as well at pole tip in quadrupoles ~ 10 cm off axis. The latter results in $\frac{\Delta B}{\Delta x} \sim 10$ T/m, the former in ~meters to tens of meters dipole curvature radius. All in all,

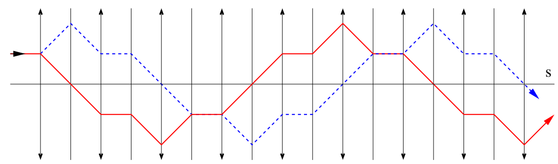


Fig. 10.5 Horizontally focusing lenses (field index $n \gg 0$, the solid red trajectory) are vertically defocusing ($n \ll 0$, the dashed blue trajectory), and vice versa. This imposes alternating gradients in order for a sequence to be globally focusing.

$$n = \frac{\rho}{B}\frac{\partial B}{\partial x} \sim \frac{10^{0\sim 2}_{[\mathrm{m}]}}{1_{[\mathrm{T}]}} \times 10_{[\mathrm{T/m}]} \sim 10^{1\sim 3} \quad \gg 1 \tag{10.3}$$

much greater than in a weak focusing structure, characterized by $0 < n < 1$.

10.2.1 Components of the Strong Focusing Optics

Combined function (AG) optics

This is, typically, the BNL AGS and CERN PS optics, using dipoles that ensure both beam guiding and focusing (Fig. 10.2). Separate quadrupole and multipole lenses have later been introduced in these lattices as they provide knobs for the adjustment of optical functions and parameters.

AG optics is still at work in modern designs, as in th iRCMS whose six 60 deg arcs are comprised of a sequence of five focusing and defocusing combined function dipoles [6], Fig. 10.3.

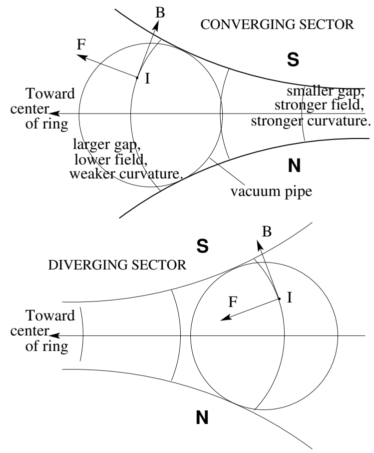
Field

Referring to the normal conducting magnet technology, an hyperbolic pole profile (Fig. 10.1): equipotential $V(x, y) = A\,xy$ (A a constant, typically $\sim 10\,$T/m, *cf.* Eq. 10.3), results in $B_y = \frac{\partial V}{\partial y} = A\,x$, *i.e.* a radial field index $n = \frac{\rho}{B_y}\frac{\partial B}{\partial x}|_{\mathrm{y=0}}$, responsible for the focusing; the pole profile opens up either inward (toward the center of curvature, a horizontally focusing dipole, vertically defocusing) or outward (a vertically focusing dipole, horizontally defocusing), Fig. 10.6.

In a straight AG dipole a line of constant field is a straight line; an instance is the AGS main magnet (Fig. 10.2). Another instance is the Fermilab recycler arcs permanent magnet dipole, which includes quadrupole and sextupole components [11, 12]. The modeling of the field can be derived from the Laplace potential $V(s, x, y)$, see below; the AGS on-line model uses that technique [13].

In a bent AG dipole a line of constant field is an arc of a circle; the field guides the reference particle along the arc in the median plane. The mid-plane field can be expressed as

Fig. 10.6 Beam focusing in combined function dipoles. The center of curvature is to the left. The pole profile follows an equipotential $V = axy$. Top: the pole profile opens up towards the center of curvature → the dipole is horizontally converging (vertically diverging: current I comes out of the page, force **F** results from field **B**). Bottom: pole profile closing toward the center of curvature → the dipole is horizontally diverging, vertically converging



$$B_y(r,\theta) = \mathcal{G}(r,\theta)\, B_0 \left(1 + n\,\frac{r-r_0}{r_0} + n_2 \left(\frac{r-r_0}{r_0}\right)^2 + n_3 \left(\frac{r-r_0}{r_0}\right)^3 + \,...\right) \quad (10.4)$$

with r_0 the eference radius. Higher order indices, sextupole n_2, octupole n_3, ..., may be residual effects: fabrication tolerance, saturation, magnetic permeability, deformation of yoke with years, ..., as in the AGS dipoles, or included by design.

In a straight AG dipole a line of constant field is a straight line; an instance is the AGS main magnet (Fig. 10.2). Another instance is the Fermilab recycler arcs permanent magnet dipole, which includes quadrupole and sextupole components [11, 12]. The modeling of the field in a straight combined function dipole can be derived from the scalar potential of Eq. 10.5.

Separated function optics

Main bends have zero index and ensure beam guiding. In smaller rings though, bending may contribute horizontal focusing; wedge angles in addition may be introduced and contribute some horizontal and vertical focusing/defocusing. Quadrupole lenses, alternately focusing and defocusing, ensure the essential of the focusing.

Higher order multipole lenses are used for the compensation of adverse effects: coupling, aberrations, space charge, impedance, etc., and for beam manipulations: coupling, resonant extraction, etc.

The field in a multipole of order n (n = 1, 2, 3, ...: dipole, quadrupole, sextupole, ...) derives, via $\mathbf{B} = \mathbf{grad}V$, from the Laplace potential [14]

$$V_n = (n!)^2 \left\{ \sum_{\mathrm{q}=0}^{\infty} (-)^q \alpha_{\mathrm{n},0}^{(2q)}(s) \frac{(x^2+y^2)^q}{4^q q!(n+q)!} \right\} \left\{ \frac{x^{n-m} y^m}{m!(n-m)!} \sin m\frac{\pi}{2} \right\} \qquad (10.5)$$

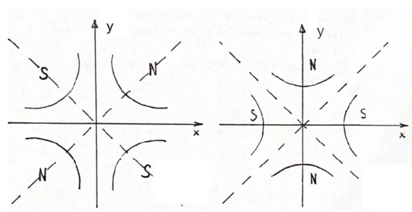
wherein $\alpha^{(2q)}_{\mathrm{n},0} = d^{2q}\alpha_{\mathrm{n},0}/ds^{2q}$ accounts for the s-dependence of the field form factor. Technologies for multipoles and combined multipoles include warm magnets and pole profiling (Figs. 10.2, 10.4), permanent magnets [11, 15], superconducting magnets and $\cos\theta$ windings as in RHIC and LHC colliders, and variants.

In a hard-edge model the left sum in Eq. 10.5 is reduce to the $q = 0$ term, with the following outcomes.

Quadrupole

The equipotential (the pole profile) is an hyperbola: Gxy =constant, in an upright quadrupole (left), $G(x^2 - y^2)$ =constant in a $\pi/4$ skewed quadrupole (right); the resulting field writes

$$B_x = \frac{\partial V}{\partial x} = Gy \qquad B_y = \frac{\partial V}{\partial y} = Gx$$



$$B_x = Gx \qquad B_y = -Gy$$

Upright quadrupoles are used for focusing, skew quadrupoles are used to compensate, or introduce, transverse coupling. Their focusing strength

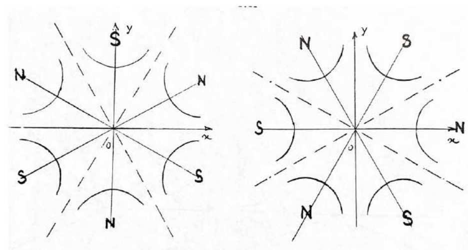
$$K = \frac{1}{L}\frac{\int G(s)\,ds}{p/q}$$

is momentum-dependent.

Sextupole

The equipotential satisfies $H(3x^2y - y^3)$ =constant in an upright sextupole (left), $H(x^3 - 3xy^2)$ =constant in a $\pi/6$ skewed sextupole (right), with resulting field

$B_x = 2Hxy$
$B_y = H(x^2 - y^2)$



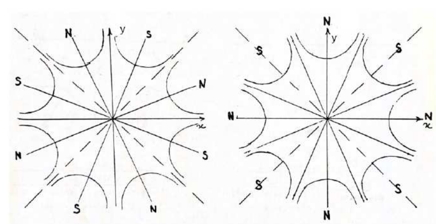
$B_x = H(x^2 - y^2)$
$B_y = -2Hxy$

Upright sextupoles introduce a vertical field component $B_y \propto x^2$, they are used to correct optical aberrations, to modify the momentum dependence of the wave numbers ν_x, ν_y, and in beam manipulations such as resonante extraction. Skew sextupoles introduce a radial field component $B_x \propto y^2$, they are used to correct optical aberrations.

Octupole

The equipotential pole profile satisfies $O(x^3y - xy^3)$ =constant in an upright octupole (left), $O(x^4 - 6x^2y^2 + y^4)$ =constant in a $\pi/8$ skewed octupole (right), yielding the field

$B_x = O(3x^2y - y^3)$
$B_y = O(x^3 - 3xy^2)$



$B_x = O(x^3 - 3xy^2)$
$B_y = -O(3x^2y - y^3)$

Upright octupoles are used to introduce a vertical field componnet $B_y \propto x^3$; skew octupoles introduce a vertical field component $B_y \propto y^3$ Octupoles are used to correct aberrations, or to modify the amplitude dependence of wave numbers.

10.2.2 Transverse motion

The transverse motion of a particle in the periodic lattice of a ring acceleratior satisfies Hill's equations

$$\frac{d^2x}{ds^2} + K_x(s)x = \frac{1}{\rho_0}\frac{\Delta p}{p_0}, \qquad \frac{d^2y}{ds^2} + K_y(s)y = 0 \tag{10.6}$$

wherein $K_x(s)$, $K_y(s)$ have the periodicity of the lattice, and depend locally on the nature of the optical elements:

$$\begin{array}{lll}
- \text{dipole}: & \begin{cases} K_x = \dfrac{1-n}{\rho_0^2} \\ K_y = \dfrac{n}{\rho_0^2} \end{cases} & \left(n = -\frac{\rho_0}{B_0}\frac{\partial B_y}{\partial x}\right) \\
- \text{a wedge at } \ s = s_0: & \left\{ K_{\substack{x\\y}} = \pm\dfrac{\tan\varepsilon}{\rho_0}\delta(s-s_0) \ \left(\varepsilon \lesseqgtr 0: \begin{array}{l}\text{focusing}\\ \text{defocusing}\end{array}\right);\ \frac{1}{\rho_0} = 0 \right. & \\
- \text{quadrupole} & \left(\text{gradient } G = \frac{\text{field at pole tip}}{\text{radius at pole tip}}\right): K_{\substack{x\\y}} = \dfrac{\pm G}{B\rho};\ \dfrac{1}{\rho_0} = 0 & \\
- \text{drift space}: & K_x = K_y = 0;\ \dfrac{1}{\rho_0} = 0 &
\end{array} \tag{10.7}$$

By contrast with the single index ($0 < n < 1$) betatron and weak focusing technologies, strong focusing with its independent focusing ($G > 0$) and defocusing ($G < 0$) families allows separate adjustment of the horizontal and vertical focusing strengths, and wave numbers as a consequence.

The on-momentum ($p = p_0$) closed orbit coincides with the reference axis of the optical structure. The betatron motion for an on-momentum particle, *i.e.* the excursion x, y around the closed orbit, satisfies Eq. 10.6 with $\Delta p = 0$. Solving the latter (see Sect. 9.2) requires introducing two independent solutions $u_{\substack{1\\2}}(s)$ (Eq. 9.12), the linear combination of which yields the pseudo harmonic motion (Eq. 9.15)

$$\left| \begin{array}{l} u(s) = \sqrt{\beta(s)\varepsilon/\pi}\, \cos\left(\int \dfrac{ds}{\beta(s)} + \varphi\right) \\ u'(s) = -\sqrt{\dfrac{\varepsilon/\pi}{\beta(s)}}\, \sin\left(\int \dfrac{ds}{\beta(s)} + \varphi\right) + \alpha(s)\, \cos\left(\int \dfrac{ds}{\beta(s)} + \varphi\right) \end{array} \right. \tag{10.8}$$

The motion satisfies the Courant-Snyder invariant, namely (Fig. 9.10)

$$\gamma_u(s)u^2 + 2\alpha_u(s)uu' + \beta_u(s)u'^2 = \frac{\varepsilon_u}{\pi} \tag{10.9}$$

The form and the orientation of this phase space ellipse change along the period, its surface is constant.

Beam envelopes are given by the extrema,

$$\hat{x}_{\text{env}}(s) = \pm\sqrt{\beta_x(s)\frac{\varepsilon_x}{\pi}}, \qquad \hat{y}_{\text{env}}(s) = \pm\sqrt{\beta_y(s)\frac{\varepsilon_y}{\pi}} \tag{10.10}$$

Phase space motion

Write the two independent solutions $u_{\frac{1}{2}}(s)$ (Eq. 9.12) under the form

$$u_1(s) = \underbrace{F(s)}_{\text{S-periodic}} \times \underbrace{e^{i\mu\frac{s}{S}}}_{\frac{2\pi S}{\mu}\text{-periodic}} \quad \text{and} \quad u_2(s) = u_1^*(s) = F^*(s)\,e^{-i\mu\frac{s}{S}} \tag{10.11}$$

wherein $F(s) = \sqrt{\beta(s)}\,e^{i\left(\int_0^s \frac{ds}{\beta(s)} - \mu\frac{s}{S}\right)}$. Introduce $\psi(s) = \int_0^s \frac{ds}{\beta(s)} - \mu\frac{s}{S}$ so that $F(s) = \sqrt{\beta(s)}\,e^{i\psi(s)}$, Eq. 10.8 thus takes the form

$$\left|\begin{array}{l} u(s) = \overbrace{\sqrt{\beta(s)\varepsilon/\pi}}^{S\text{-periodic}}\ \overbrace{\cos\left[\nu\frac{s}{R} + \underbrace{\psi(s)}_{\text{S-per.}} + \varphi\right]}^{\frac{2\pi S}{\mu}\text{-periodic}} \\ u'(s) = -\sqrt{\frac{\varepsilon/\pi}{\beta(s)}}\ \sin\left[\nu\frac{s}{R} + \psi(s) + \varphi\right] + \alpha(s)\ \cos\left[\nu\frac{s}{R} + \psi(s) + \varphi\right] \end{array}\right. \tag{10.12}$$

wherein $\nu = \frac{N\mu}{2\pi}$. Thus, as $\beta(s)$ and $\psi(s)$ are S-periodic functions, the turn-by-turn motion observed at a given azimuth s (*i.e.*, $u(s)$, $u(s+\mathcal{S})$, $u(s+2\mathcal{S})$, ...) is sinusoidal with frequency $\nu = N\mu/2\pi$. Successive particle positions $(u(s), u'(s))$ in phase space lie on the Courant-Snyder invariant (Eq. 10.9).

The wave numbers ν_x and ν_y can be adjusted independently in a separated function lattice, by means of two independent quadrupole families. The working point (ν_x, ν_y) fully characterizes the first order optical setting of the ring.

Off-momentum motion

The motion of an off-momentum particle satisfies the inhomogeneous Hill's horizontal differential Eq. 10.6. The chromatic closed orbit

$$x_{\text{ch}}(s) = D_x(s)\frac{\delta p}{p} \tag{10.13}$$

is a particular solution of the equation, its periodicity is that of the cell.

By contrast with the weak focusing configuration, where the on-momentum closed orbit and chromatic closed orbits are parallel (Eq. 9.26: D_x=constant, independent of s), chromatic closed orbits in a strong focusing optical structure are distorted, their excursion depends on the distribution along the cell of (i) the dispersive elements which are the dipoles, and (ii) the focusing.

The horizontal motion of an off-momentum particle is a superposition of the particular solution (Eq. 10.13) and of the betatron motion, solution of the homogeneous Hill's equation (Eq. 9.22 with $\delta p/p = 0$), namely

$$x(s) = x_\beta(s) + x_{\mathrm{ch}}(s) = \sqrt{\beta_x(s)\frac{\varepsilon_x}{\pi}}\ \cos\left(\int \frac{ds}{\beta_x} + \varphi\right) + D_x(s)\frac{\Delta p}{p_0} \qquad (10.14)$$

whereas the vertical motion is unchanged (Eq. 10.12 taken for $u(s) \equiv y(s)$).

10.2.3 Resonances. Resonant Extraction

Consider the excitation of transverse beam motion by a generator of frequency Ω located at some azimuth along the ring [16]. The action of the excitation $S \times \sin\Omega t$ on the oscillating motion $u(t)$ can be written under the form

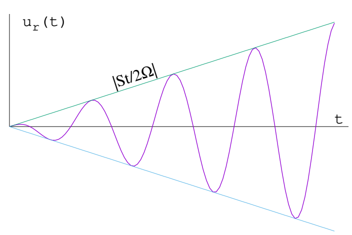
$$\frac{d^2u}{dt^2} + \omega^2 u = S\sin\Omega t \qquad (10.15)$$

The betatron motion is assumed harmonic for simplicity, case for instance of weak focusing. Take S constant, the solution (superposition of the solution of the homogeneous differential equation and of a particular solution of the inhomogeneous differential equation) writes

$$u(t) = U\cos(\omega t + \varphi) + \frac{S}{\omega^2 - \Omega^2}\sin\Omega t \qquad (10.16)$$

If betatron motion and excitation are in synchronism, *i.e.* on the resonance, $\omega = \Omega$, a particular solution of Eq. 10.15 is

$$u_r(t) = -\frac{S\,t}{2\Omega}\cos\Omega t$$



the amplitude of the oscillatory motion grows rapidely with time, at a rate $|St/2\Omega|$.

Assume S periodic instead, take its Fourier expansion $S(t) = \sum_{p=0}^{\infty} a_p \cos(p\omega' t + \varphi_p)$, the equation of motion thus writes

$$\frac{d^2u}{dt^2} + \omega^2 u = \sum_{p=0}^{\infty} a_p \cos(p\omega' t + \varphi_p)\ \sin\Omega t =$$

$$\sum_{p=0}^{\infty} \frac{a_p}{2} \Big[\sin[(\Omega - p\omega')t + \varphi_p] + \sin[(\Omega + p\omega')t + \varphi_p] \Big]$$

Resonance may occur at oscillator frequencies $\omega = \Omega \pm p\omega'$, their strength depends on the amplitude a_p of the excitation harmonics. If the generator is located at one point in the ring, it excites all harmonics.

Sextupole and octupole resonances

The horizontal motion in the presence of a sextupole component ($B_y(\theta)|_{y=0} = S(\theta)x^2$, see *Sextupole*, above) as part of the ring optical lattice satisfies

$$\frac{d^2x}{d\theta^2} + \nu_x^2 x = S(\theta)x^2 \tag{10.17}$$

Assume weak perturbation of the motion, so that $x(\theta) \approx \hat{x}\cos(\nu_x\theta + \varphi)$; the perturbation $S(\theta)$ is 2π-periodic thus substitute its Fourier series expansion $S(\theta) = \sum_{p=0}^{\infty} a_p \cos(p\omega'\theta + \varphi_p)$ in the differential equation; develop to get

$$\frac{d^2x}{d\theta^2} + \nu_x^2 x = \frac{\hat{x}^2}{2} \sum_{\mathrm{p}=0}^{\infty} a_p \Big[\cos(p\theta + \varphi_p) +$$

$$\cos[(p - 2\nu_x)\theta + \varphi_p - 2\varphi] \ + \ \cos[(p + 2\nu_x)\theta + \varphi_p + 2\varphi] \Big]$$

Thus resonance may occur at betatron frequency families $\nu_x = \pm p$, $\nu_x = \pm(p - 2\nu_x)$, and $\nu_x = \pm(p + 2\nu_x)$, *i.e.*,

$$\left[\begin{array}{l} \nu_x = \text{integer} \\ 3\nu_x = \text{integer} \end{array} \right.$$

In the case of a single sextupole in the ring, all the harmonics p are excited with the same amplitude a_p.

An octupole perturbation introduces a field component $B_y(\theta)|_{y=0} = O(\theta)x^3$ (see *Octupole*, above) in the optical lattice. In a similar way, assume weak perturbation so that $x(\theta) \approx \hat{x}\cos(\nu_x\theta + \varphi)$; to $O(\theta)$ substitute its Fourier expansion; this yields

$$\left[\begin{array}{l} \nu_x = \text{integer} \\ 2\nu_x = \text{integer} \\ 4\nu_x = \text{integer} \end{array} \right.$$

Resonances in a general manner occur at betatron frequencies satisfying

$$m\nu_x + n\nu_y = \text{integer}$$

with the property that

$$\frac{\varepsilon_x}{m} - \frac{\varepsilon_y}{n} = \text{constant}, \qquad \text{an invariant of the motion}$$

with the following consequences:

- if m and n have opposite signs the resonance causes energy exchange betwen the horizontal and vertical motions: $\frac{\varepsilon_x}{|m|} + \frac{\varepsilon_y}{|n|}$ = constant, an increase of ε_x correlates with a decrease of ε_y and vice-versa; in the presence of linear coupling for instance, $\nu_x - \nu_y$ = integer, $\varepsilon_x + \varepsilon_y$ = constant; an increase in motion amplitude anyway may cause particle loss, an issue in cyclotrons with the Walkinshaw resonance $\nu_x = 2\nu_y$ which causes vertical beam loss upon increase of ε_y;

- if m and n have the same sign the resonance induces motion instabilty: $\frac{\varepsilon_x}{m} - \frac{\varepsilon_y}{n}$ = constant, ε_x and ε_y may both increase with no limit.

Resonant Extraction

10.2.4 Synchrotron Motion

Paticle motion in the longitudinal phase space (phase, momentum) is determined by the lattice and by the acceleration parameters. The synchrotron acceleration technique has been discussed in Sect. 9.2.2, outcomes are leaned on, here.

Acceleration parameters include RF voltage $\hat{V}$, frequency $f_{\mathrm{rf}} = \omega_{\mathrm{rf}}/2\pi$, ******* Transition γ_{tr} is a property of the lattice and determines the synchronous phase region, $[0, \pi/2]$ or $[\pi/2, \pi]$.

Synchrotron angular frequency

$$\Omega_s = (\omega_{\mathrm{rev}}^2 \, |\eta| h_{\mathrm{RF}} e\hat{V} \cos\phi_s \; / \; 2\pi E_s)^{1/2}$$

with $\eta = 1/\gamma^2 - \alpha$ the phase slip factor (Eq. 9.33), h_{RF} the RF harmonic, $\omega_{\mathrm{rev}} = 2\pi/T_{\mathrm{rev}}$ the revolution angular frequency, $\hat{V}$ the RF peak voltage, ϕ_s the synchronous phase.

The bucket height, "momentum acceptance", satisfies

$$\pm\frac{\Delta p}{p} = \pm\frac{1}{\beta}\sqrt{\frac{q\hat{V}}{\pi h \eta E_s}\left[-(\pi - 2\varphi_s)\sin\varphi_s + 2\cos\varphi_s\right]} \tag{10.18}$$

$$\alpha = \frac{\Delta C}{C}\Big/\frac{\Delta p}{p_0} \equiv \frac{\Delta R}{R}\Big/\frac{\Delta p}{p_0} \tag{10.19}$$

The maximum extent in phase for small amplitude oscillations satisfies

$$\pm\Delta\varphi_{\mathrm{max}} = \frac{h\eta E_s}{p_s R_s \Omega_s} \times max\left(\frac{\Delta E}{E_s}\right) \tag{10.20}$$

****** separatrix **********

The motion of a particle with enegy offset $\delta E = E - E_s$ satisfies the longitudinal invariants

$$\epsilon_l = \frac{\alpha E_s}{2\Omega_s}\left[\left(\frac{\delta E}{E_s}\right)^2 + \frac{1}{\Omega_s^2}\left(\frac{d}{dt}\frac{\delta E}{E_s}\right)^2\right] \tag{10.21}$$

$$(\widehat{\delta E})^2 = (\delta E)^2 + \frac{1}{\Omega_s^2}\left(\frac{d\delta E}{dt}\right)^2 \tag{10.22}$$

Introducing the squared *rms* relative synchrotron amplitude $\sigma^2_{\widehat{\delta E}/E} \equiv (\widehat{\delta E}/E_s)^2$ this yields in addition

$$\epsilon_l = \frac{\alpha E_s}{2\,\Omega_s}\,\sigma^2_{\widehat{\delta E}/E} \tag{10.23}$$

10.2.5 Radiative Energy Loss

check what was said in betatron chapter ...

A particle of rest mass m_0 and charge e travelling in a magnetic field is subject to stochastic photon emission, which causes energy loss [19]. The phenomenon involves two random processes:

- the probability of photon emission over a trajectory arc δs, a Poisson law,

$$p(k) = \frac{\Lambda^k}{k!}e^{-\Lambda} \qquad \text{with} \quad \Lambda =< k >=< k^2 > \tag{10.24}$$

wherein k is the number of photons emitted over δs, $\Lambda = \frac{5er_0}{2\hbar\sqrt{3}}B\rho\frac{\delta s}{\rho}$ is its average value, $r_0 = e^2/\,4\pi\epsilon_0 m_0 c^2$ is the classical radius of the particle, $\epsilon_0 = 1\,/\,36\pi 10^9$, $\hbar$ is the Plank constant,

- the energy ϵ of the photon(s), following the probability law

$$\mathcal{P}\left(\frac{\epsilon}{\epsilon_c}\right) = \frac{3}{5\pi}\int_0^{\epsilon/\epsilon_c}\frac{d\epsilon}{\epsilon_c}\int_{\epsilon/\epsilon_c}^{\infty}K_{5/3}(x)dx \tag{10.25}$$

with $K_{5/3}$ the modified Bessel function, $\gamma = E/E_0$ with $E_0 = m_0c^2$ the rest energy, and ϵ_c the critical energy of the radiation,

$$\epsilon_c = \frac{3\hbar\gamma^3 c}{2\rho} \tag{10.26}$$

The average energy loss over δs is, assuming ultra-relativistic particles: $\beta = v/c \approx 1$,

$$\delta E = \frac{2}{3}r_0E_0\gamma^4\,\frac{\delta s}{\rho^2} = \frac{2}{3}r_0ec\gamma^3B\frac{\delta s}{\rho} \quad \underbrace{\approx 1.88\;10^{-15}\;\gamma^3\frac{\delta s}{\rho^2}}_{\text{for electrons}} \tag{10.27}$$

The energy spread resulting from the stochastic emission is

$$\sigma_{\delta E/E} = \frac{\sqrt{110\sqrt{3}\hbar c\ /\ \pi\epsilon_0}}{24E_0/e}\,\gamma^{5/2}\,\frac{\sqrt{\delta s}}{\rho^{3/2}} \quad \underbrace{\approx 3.80\ 10^{-14}\ \gamma^{5/2}\frac{\sqrt{\delta s}}{\rho}}_{\text{for electrons}} \tag{10.28}$$

In a storage ring the RF system restores on average the energy lost by SR. Usefull formulas are given in Tab. 10.1, in particular, assuming a flat ring the partition of energy between radial and longitudinal motions is determined by the partition numbers

$$\mathrm{J_x} = 1 - \mathcal{D},\ \ \mathrm{J_y} = 1,\ \ \mathrm{J_l} = 2 + \mathcal{D},\ \ \text{with } \mathcal{D} = \frac{\overline{\mathrm{D_x}(1-2\mathrm{n})/\rho^3}}{\overline{\rho^2}} \tag{10.29}$$

where $\overline{(*)}$ denotes an average over the ring circumference.

Table 10.1 Radiation parameters$^{(a)}$, energy loss and equilibrium quantities at the synchronous energy, E_s, in an isomagnetic ring

Critical photon energy, ϵ_c	keV	$\frac{3\hbar\gamma^3 c}{2\rho}$
Average photon energy, $\overline{\epsilon}$	keV	$\frac{8}{15\sqrt{3}}\,\epsilon_c$
rms energy spread, $\sqrt{\overline{(\epsilon-\overline{\epsilon})^2}}$	keV	$\frac{\sqrt{211}}{15\sqrt{3}}\,\epsilon_c$
Energy loss, U_s	MeV / turn	$C_\gamma\,\frac{E_s^4}{\rho}$
Nb. of average photons	/turn/particle	$U_s/\overline{\epsilon}$
Longitudinal:		
equil. emittance, $\varepsilon_{l,eq}$	μeV.s	$\frac{\alpha E_s}{\Omega_s}\,\frac{C_q\gamma^2}{J_l\,\rho}$
rms energy spread, $\sigma_{\delta E/E}$		$\frac{1}{\sqrt{2}}\sigma_{\widehat{\delta E/E}} = \sqrt{\frac{C_q}{J_l\rho}}\gamma$
rms bunch length, σ_l	mm	$\frac{\alpha c}{\Omega_s}\sigma_{\frac{\delta E}{E}}$
Radial:		
equil. emittance, $\varepsilon_{x,eq}$	nm	$= \frac{C_q\gamma^2}{J_x\rho}\bar{\mathcal{H}}$
rms width, $\sigma_x(s)^{(b)}$	m	$\left(\beta_x(s)\varepsilon_{x,eq} + D_x^2(s)\sigma^2_{\frac{\delta E}{E}}\right)^{1/2}$
Damping times, $\tau_{x,y,l}$	ms	$\frac{T_{rev}E_s}{U_s J_{x,y,l}}$

(a) Units are, c: m/s; ρ: m; E_s: GeV
$C_\gamma = \frac{4\pi}{3}\,\frac{r_0}{(m_0c^2)^3}$ (= 8.846276 10^{-5} m/GeV^3 for electrons).
$C_q = \frac{55}{32\sqrt{3}}\,\frac{\hbar}{m_0 c}$ (= 3.8319386×10^{-13} m for electrons).
(b) With $\varepsilon_{x,eq}, \beta(s)$ and dispersion $D_x(s)$ in meter.

Damping of accelerated motion

In an accelerator (a light source injector for instance), the RF voltage increases during acceleration in order to compensate the increasing energy loss. To first order in the invariant ε_u (with u standing for x or y) transverse damping in the presence of acceleration satisfies [**?**]

$$\frac{d\varepsilon_u}{dt} = -\frac{2}{\tau_u(t)}\varepsilon_u + C_u(t) - \frac{1}{p}\frac{dp}{dt}\varepsilon_u\ , \ \text{where } \tau_u^{-1} = J_u\frac{\overline{P}}{2E},\ \begin{cases} C_x = \overline{\mathcal{H}\frac{\dot{N}<\epsilon^2>}{E^2}} \\ C_y = \frac{\beta_y}{2\gamma^2}\frac{\dot{N}<\epsilon^2>}{E^2} \end{cases} \tag{10.30}$$

Longitudinal damping satisfies

$$\frac{d(\widehat{\delta E})^2}{dt} = -\frac{2(\widehat{\delta E})^2}{\tau_l(t)} + (\dot{N}\left\langle\epsilon^2\right\rangle)(t) + \frac{(\widehat{\delta E})^2}{2E}\frac{dE_s}{dt} \quad \text{with } \tau_l^{-1} = J_l\frac{\overline{U_s}}{2E_s} \tag{10.31}$$

******** Figures **??**, **??** display the evolution of horizontal and vertical emittance with time, respectively

$$\overline{\epsilon}_{\mathrm{x}}(\mathrm{t}) = \epsilon_{\mathrm{x},0}\left(\mathrm{e}^{\mathrm{t}/|\tau_{\mathrm{x}}|} - 1\right), \qquad \overline{\epsilon}_{\mathrm{y}}(\mathrm{t}) = \epsilon_{\mathrm{y,i}}\,\mathrm{e}^{-\mathrm{t}/\tau_{\mathrm{y}}} \tag{10.32}$$

with $\epsilon_{\mathrm{x},0}$ a constant and $\epsilon_{\mathrm{y,i}}$ an initial value.

10.2.6 Depolarizing esonances

By contrast with weak focusing optics where depolarizing resonances are weak because horizontal field components are weak (Sect. 9.2.3), the use of stong focusing field gradients in the combined function magnets and/or focusing lenses of strong focusing optics results in strong radial field components and therefore strong depolarizing resonances.

Spin precession and resonant spin motion in the magnetic components of a cyclic accelerator have been introduced in Sects. 4.2.5, 5.2.5. The general conditions for depolarizing resonance to occur have been introduced in Sect. 9.2.3. In a strong focusing synchrotron they assentially result from the radial field components in the focusing magnets and their strength is determined by the lattice optics, as follows.

Strength of imperfection resonances

Imperfection, or integer, depolarizing resonances are driven by a non-vanishing vertical closed orbit $y_{\rm co}(\theta)$ which causes spins to experience periodic radial fields in focusing magnets, dipoles in combined function lattices and quadrupoles in separated function lattices, namely,

$$B_x(\theta) = G\, y(\theta) = K(\theta) \times B_0\rho_0 \times y_{\rm co}(\theta)$$

with θ the orbital angle, $B_0\rho_0$ the lattice rigidity and $y_{\rm co}(\theta)$ the closed orbit excursion. Resonance occurs if the spin undergoes an integer number of precessions over a turn (it then undergoes 1-turn-periodic torques), so that spin tilts at field perturbations along the closed orbit add up coherently. Thus resonances occur at integer values

$$G\gamma_n = n$$

A Fourier development of these perturbative fields yields the strength of the $G\gamma_n$ harmonic [21, Sect. 2.3.5.1]

$$\epsilon_n^{\rm imp} = (1 + G\gamma)\frac{R}{2\pi} \oint K(\theta)\; y_{\rm co}(\theta)\, e^{-jG\gamma(\theta-\alpha)}\, e^{jn\theta}\, d\theta$$

In the thin-lens approximation this simplifies into a series over the quadrupole fields,

$$\epsilon_n^{\rm imp} = \frac{1 + G\gamma_n}{2\pi} \sum_{\rm Qpoles} \left[\cos G\gamma_n\, \alpha_i + \sin G\gamma_n\, \alpha_i\right] (KL)_i\;\, y_{\rm co}(\theta_i) \qquad (10.33)$$

with θ_i the quadrupole location, $(KL)_i$ the integrated strength (slice the dipoles as necessary in an AG lattice for this series to converge) and α_i the cumulated orbit deviation.

Orbit harmonics near the betatron tune ($n = G\gamma_n \approx \nu_y$) excite strong resonances. Imperfection resonance strength is further amplified in P-superperiodic rings, with m-cell superperiods, if the betatron tune $\nu_y \approx$ integer $\times\, m \times P$ [22, Chap.3-I].

Strength of imperfection resonances

Intrinsic depolarizing resonances are driven by betatron motion, which causes spins to experience strong radial field components in quadrupoles, namely

$$B_x(\theta) = G\, y(\theta) = K(\theta) \times B_0\rho_0 \times y_\beta(\theta) \qquad (10.34)$$

The effect of resonances on spin depends upon betatron amplitude and phase, their effect on beam polarization depends on beam emittance. Longitudinal fields from dipole ends are usually weak by comparison and ignored. The location of intrinsic resonances depends on betatron tune, it is given in an M-periodic structure by

$$G\gamma_n = nM \pm \nu_y$$

10.3 Exercises

In complement to the present exercises, an extensive tutorial on depolarizing resonances in a strong focusing synchrotron, considering poton, helion, or electron beams, using the lattice of the AGS Booster at BNL, can be found in Ref. [21, Chap. 14,"Spin Dynamics Tutorial: Numerical Simulations"]. The simulaitons include tune-jump quadrupoles, solenoid, snakes, electron beam polaization life time and spin rotators.

10.1 Construct SATURNE II synchrotron. Spin Dynamics With Snakes
Solution: page 361

Over the years 1978-1997 the 3 GeV synchrotron SATURNE II at Saclay (Fig. 10.7) delivered ion beams up to 1.1 GeV/nucleon, including polarized proton, deuteron and ^{6}Li beams, for intermediate energy nuclear physics research, including meson production [17, 18]. The separated function synchrotron was designed *ab initio* for the acceleration of polarized beams [20], and the first strong focusing synchrotron to do so - ZGS, first to accelerate polarized beams, protons and deuterons, was a weak focusing synchrotron (see Chap. 9).

SATUNE II is a FODO lattice with missing dipole. Its parameters are given in Tab. 10.2.

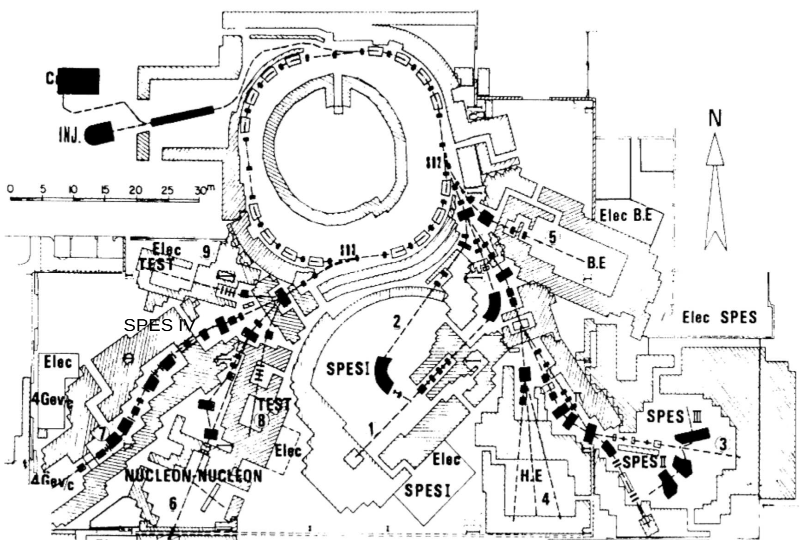


Fig. 10.7 SATURNE II synchrotron and its experimental areas [23], including mass spectrometers SPES I to SPES IV, a typical 1960-80s nuclear physics accelerator facility. Polarized ion sources are on the top left, followed by a 20 MeV linac

(a) Simulate the main dipole using BEND, include fringe fields assuming $\lambda = 8$ cm extent and the following Enge coefficient values (Eq. 15.13, Sect. 15.2.6):

Table 10.2 Parameters of SATURNE II separated function FODO lattice. ρ_0 denotes the reference bending radius in the main dipole; the reference orbit, wave numbers, etc., are taken along that radius

Orbit length, C	m	105.5556
Average radius, $R = C/2\pi$	m	16.8
Length of long straight section	m	
Wave numbers, ν_x; ν_y		3.64; 3.60
Chromaticities, ξ_x; ξ_y		negative, a few units
Momentum compaction α		0.015
Injection energy (proton)	MeV	20
Top energy	GeV	3
$\dot{B}$	T/s	4.2
Synchronous energy gain	keV/turn	1.160
RF harmonic		2
Dipole:		
- bend angle, α	deg	$\pi/8$
- magnetic length, $\rho\alpha$	m	2.489
- magnetic radius, ρ	m	6.3381
- wedge angle, ε	deg	2.45
Quadrupole:		
- gradient	T/m	0.5 - 10.56
- magnetic length F/D	m	0.46723 / 0.486273

$$C_0 = 0.2401,\ C_1 = 1.8639,\ C_2 = -0.5572,\ C_3 = 0.3904,\ C_4 = C_5 = 0$$

Produce a graph of the field across the dipole along the reference orbit, in the median pnae and at 5 cm vertical distance. Produce the transport matrix, check against theory. Compare with the matrix of the hard edge model.

Simulate the F and D quadrupoles, using respectively QUADRUPOLE and MULTIPOL. Compare matrices with theory.

Construct the cell. Produce machine parameters (tunes, chromacities), check against data, Tab. 10.2.

Construct the 4-cell ring. Produce a graph of the optical functions.

(b) Accelerate a bunch with Gaussian densities comprised of a few tens of particles (it can be defined using MCOBJET), from injection to top energy; use harmonic 3 RF frequency, and (unrealistic, for a reduced number of turns) peak RF voltage $\hat{V} = 1$ MV.

Produce a graph of the three phase spaces. Check the transverse betatron damping.

(c) Simulate multiturn injection in the ring. Take the injection point at the center of a long straight section.

(d) Simulate resonant extraction from the ring, on $\nu_x = 11/3$. Take the extraction point at the center of a long straight section.

10.2 Depolarizing Resonances In SATURNE II

The input data file to simulate the ring is given in Tab. 17.73, an outcome of exercise 10.1.

(a) Calculate the strength of the intrinsic depolarizing resonances (systematic and non-systematic) over 0.5-3ǴeV, using Eq. **??**.

(b) Ggamma=7-ν_y was found to be a potentially harmful depolarizing resonance - unexpectedly as this is not a systematic resonance. Produce a crossing of that resonance, for a 100-particle bunch. Get its strength from this simulation, compare with (a).

(c) Multiple resonance xing - ref to Phys. Rev. article ***

10.3 Cornell electron RCS. Radiative Energy Loss

Short intro .... energy loss by synchrotron radiation [24]

Tab.: RCS parameter list

(a) Cornell RCS parameters are given in Tab. **??**. Construct the ring, produce its optical parameters. Poduce a graph of the optical functions.

(b) Raytrace a few tens of particles over 3000 turns in Cornell RCS, from *** to *** GeV. Assume emittances epsilx=, epsily=, Gaussian densities, initial *rms* $\delta p/p = 10^{-4}$. Produce a graph of the three phase spaces. produce graphs of horizontal and vertical transverse excursions versus turn number.

(c) Re-do (b) with synchrotron radiation energy loss.

(d) Produce the average beam polarization obtained in (c).

(c) Multiple resonance crossing.

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