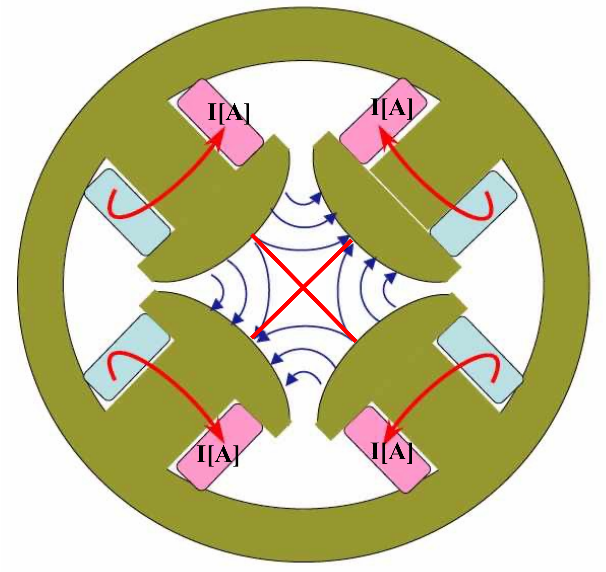
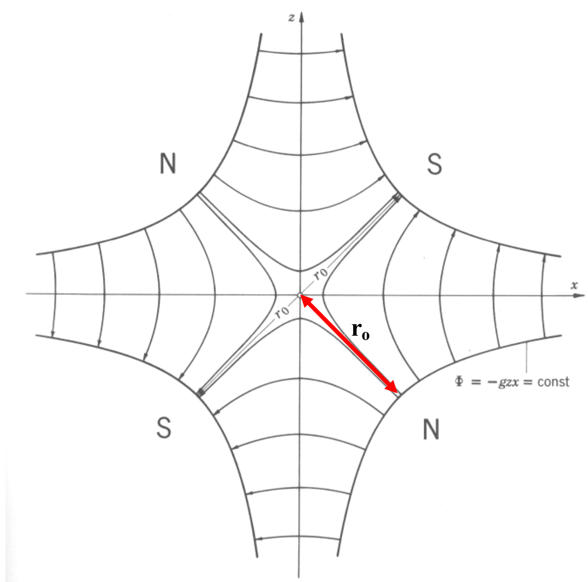
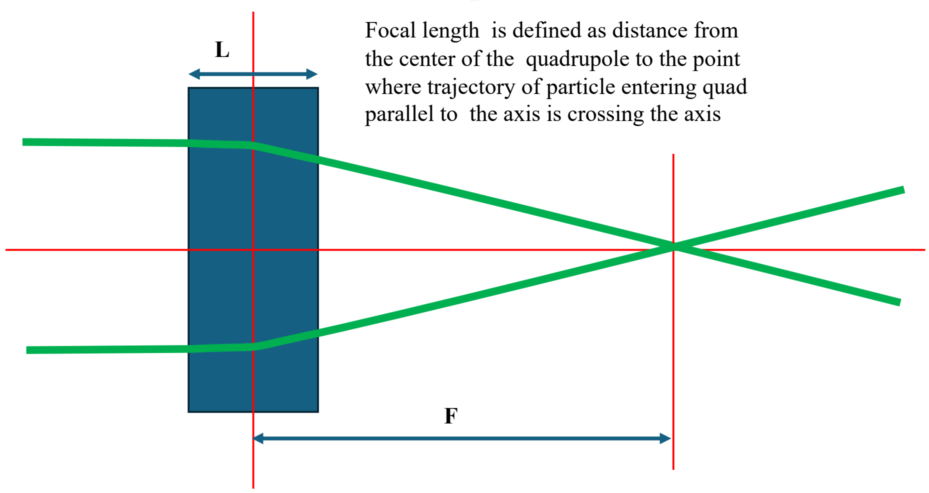
PHY 564 Midterm open-book, take home exam

Sent – evening Monday, March 23, 2026, Due -mid-night, Wednesday 25, 2026

Problem 1. Design a quadrupole for 3 GeV electron beam with pole radius of r_o=*2 cm* (0.*02 m),* field gradient of *5 kGs/cm (50 T/m*) and focal length of *F=1* meter. See the figures and specific questions below. **Total points: 40 points**

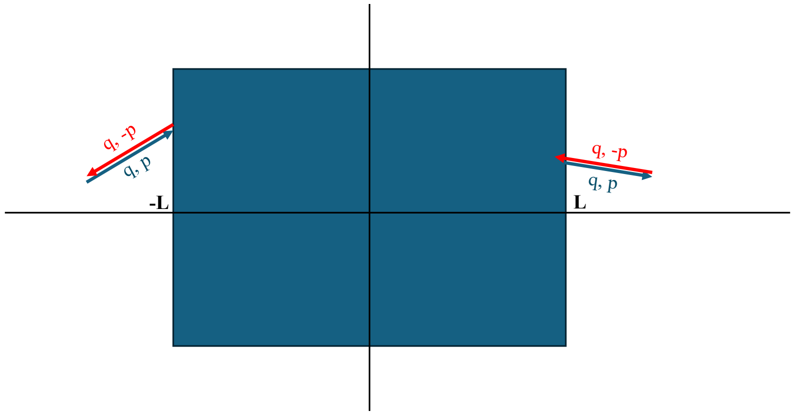






a) Fund value of field on the tip of the quadrupole pole (r=2 cm) – **3 points**
b) What is the total current in each of 4 quadrupole coils – **4 points**
c) In so-called "short quadrupole" assumption, find necessary length of quadrupole needed for focal length of *F=1* meter – **5 points**
d) For a sick (real) quadrupole white equation for length L needed for *F=1* meter – **10 points**
e) Check what is actual focal length of sick quadrupole with length found in a "short-quad" approximation – **8 points**
f) Attempt to solve (even approximately, or using Mathematica and other numerical tools) what is actual length needed for *F=1* meter. – **10 points**

Problem 2. Mirror or bilaterally symmetric lattices. Total 45 points
Consider a linear transport system extending from *-L* to *+L* is *s* (value of L is not important!) and that it has a mirror (bilateral) symmetry *H(-s)=H(s)*. Derive condition on the transport matrix coefficient from *-L* to *+L*.



a) Observe (discuss) the fact that reversed trajectory has the same matrix as that in the fromward direction and that $q_{rev}(+L)=q_{fov}(+L)$ and $P_{rev}(+L)=-P_{fov}(+L)$ will result in $q_{rev}(-L)=q_{fov}(-L)$ amd $P_{rev}(-L)=-P_{lof}(-L)$ – **5 points**
b) Consider the fact that reverse motion is described by the same matrix, but we also can use inverse matrix to connect which can be found using symplecticity conditions – **5 points**
c) Derive conditions for 1D case and 2x2 matrix **– 15 points**
d) Derive conditions for N dimensions and 2Nx2N matrices. - **20 points**

Hint: While it does not matter for 1D case, I suggest that you use overscored (Alex Dragt's) convention for 2Nx2N matrices.

Problem 3. Muon collider – **15 points**
Muons are unstable particles with energy of 105.66 MeV and 2.197 microseconds e-fold lifetime in rest frame. Physics community is considering a multi-TeV circular muon collider where muons circulate and collide for a number of turns. Consider a storage ring with average bending magnetic field of 10 T and derive number of turns (collisions) muon can conclude during it e-fold lifetime. Does this number depend on the muon beam energy?

Suggestions/Hints: consider ultra-relativistic muons moving with speed practically indistinguishable from the speed of light (indeed only such colliders are of interest). Try to understand

Additional explanation to Problem 2.

It is important to implicitly specify the meaning of the bilateral summitry. We know the Hamiltonian equations (*Q* and *P* have *N* components):

$$\frac{dQ}{ds} = \frac{\partial H(Q,P,s)}{\partial P}; \ \frac{dP}{ds} = -\frac{\partial H(Q,P,s)}{\partial Q}; \qquad (1)$$

Let's now use reverse coordinate $z=-s$:

$$\frac{dQ}{dz} = -\frac{dQ}{ds} = -\frac{\partial H(Q,P,-s)}{\partial P}; \ \frac{dP}{dz} = -\frac{dP}{ds} = \frac{\partial H(Q,P,-s)}{\partial Q};$$

Let's also introduce new variable

$$\hat{P} = -P;$$

$$\frac{dQ}{dz} = \frac{\partial H(Q,-\hat{P},-z)}{\partial \hat{P}}; \ \frac{d\hat{P}}{dz} = -\frac{\partial H(Q,-\hat{P},-z)}{\partial Q}.$$

This set of questions will be identical to eq (1) if

$$H(Q,-P,-s) = H(Q,P,s) \qquad (2)$$

or that in addition to bilateral symmetry the Hamiltonian is an even function of momenta. In this case map from {Q(-L),P(-L)} to {Q(+L),P(+L)} is identical to reverse map from Q(+L),-P(+L)} to {Q(-L),-P(-L)}.

In linear system eq. (2) means that there should be no $h_{qipj}(s)q_iP_j$ terms in the Hamiltonian or there sign is flipping with the sign of s: $h_{qipj}(s)=- h_{qipj}(-s)$. For example, a bilaterally symmetric system that has torsion or solenoidal field should have $\kappa(-s) = -\kappa(s)$; $B_s(-s) = -B_s(s)$

I hoped that this would be condition you get in part a) of the problem, but since I already have questions, I decided to add this explanation to the problem.