Home work 3

Problem 1 – 5 points

We proved already that for an arbitrary set of m (does not matter odd or even!) ordinary first order linear differential equation

$$\frac{d}{dt}X = \mathbf{D}(t) \times X; \tag{1}$$

solution can be written in form of transport matrix

$$X(t) = \mathbf{M}(t) \times X; \ \frac{d}{dt}\mathbf{M} = \mathbf{D}(t) \times \mathbf{M} \tag{2}$$

show that

$$\frac{d}{dt}(det\mathbf{M}) = Trace(\mathbf{D}(t) \times det\mathbf{M} \tag{3}$$

Hint: consider a infinitesimal step in t:

$$\mathbf{M}(t + dt) = (\mathbf{I} + \mathbf{D}(t)dt) \times \mathbf{M}(t) + O(dt^2)$$

and show that

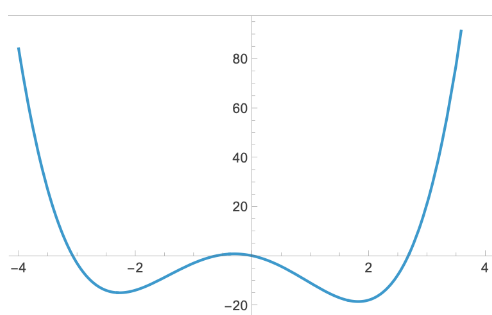
$$det(\mathbf{I} + \mathbf{D}(t)ds) = \mathrm{Trace}\mathbf{D}(t) \times dt + O(dt^2)$$

Problem 2 – 5 points

For time (or s – does not matter) independent Hamiltonian of

$$H = \frac{p^2}{2} + U(x); \ U(x) = -5x - 8x^2 + x^3 + x^4 \tag{1}$$

The graph of U(x) looks like and has extrema at approximately at x=-1.82; x=-0.321 and x=2.14



Write equations for stationary point and draw (either by hand or using computer) trajectories in x-p plane in x range from -3 to +3 and p from -8 to +8. The most important is designating stable and unstable points at the phase plot, drawing a separatrix which goes through a local maximum at x=-0.321 and approximating trajectories for *H > U(-0.321)* and *H < U(-0.321)*.