PHY 554 Mid-term exam (total 100 pts)

1. **Light source and DBA (30 pts)**
   NSLS-II at BNL is a third generation light source, which adopts DBA lattice (each DBA cell is illustrated in Fig. 1)

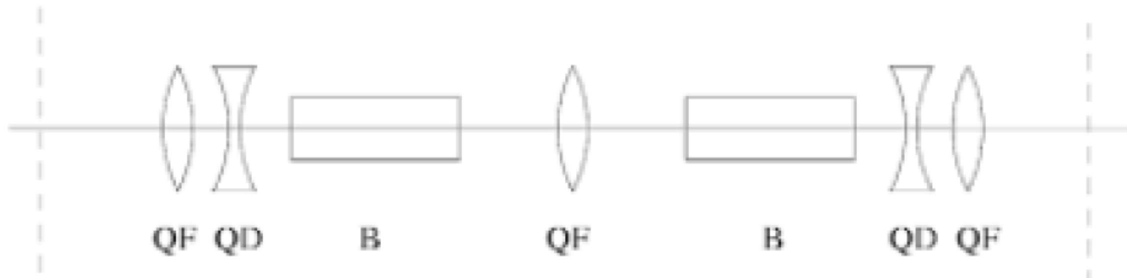

Figure 1: DBA lattice

The parameter of the electron ring is given by the following table.

Table 1: NSLS II parameters

Parameters	Values
Energy [GeV]	3.0
Circumference [m]	780
Number of dipoles	60
Dipole field [T]	0.4
Beam current [A]	0.5
RF frequency [MHz]	499.68
Harmonic number	1320

Using the design parameters in the table, find the answers of the following questions:

a) (10) Find the length of the dipoles assuming they are all equal (Hint: use beam rigidity)

b) (10) In DBA lattice, dispersion D and dispersion slope D′ are zero at both end. Find dispersion function inside the dipole magnet (as the distance s into the dipole).

c) (10) Find the momentum compaction factor $\alpha_c = \frac{1}{C}\oint \frac{D}{\rho} ds$

Solution:

NSLS II has 60 dipoles to form a closed loop, therefore each dipole bends 6 degree, which is $6/180 * \pi = 0.105$ rad. The radius of the dipole can be found as $P = eB\rho$, therefore $\rho = 3GeV/c/0.4m = 25m$. The length of each dipole is $L_D = 2\pi\rho/60 = 2.618m$.

Since at one end has $d = 0$ and $d' = 0$, we can calculate the dispersion function in the dipole from this end using small angle approximation:

$$\begin{pmatrix} d(s) \\ d'(s) \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & l & \rho\theta^2/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d(0) = 0 \\ d'(0) = 0 \\ 1 \end{pmatrix}$$

The dispersion function in dipole is $d(s) = s^2/2/\rho$.

The compaction factor is

$$\begin{aligned} \alpha_c &= \frac{1}{C} \oint \frac{D(s)}{\rho} ds \\ &= \frac{60}{C} \int_0^{L_D} s^2/2/\rho^2 ds \\ &= \frac{10}{C} \frac{L_D^3}{\rho^2} = 3.68 \times 10^{-4} \end{aligned}$$

2. **Synchrotron Radiation (30 pts, each 5 pts)**
   i) The LHC accelerates the proton beam to 7 TeV in a superconducting storage ring with a 26.7 km circumference. The magnetic field in the superconducting bending dipole is 8.3 Tesla. Calculate:
      a) The radiation energy of the 7 TeV proton per turn
      b) The critical energy of the photons
      c) The Radiation power for a beam current of 800 mA
   ii) The LHC tunnel (Hint: thus same bending radius with scaled B field in dipoles) was used for the LEP (large electron project). LEP accelerates the electron beam to 100 GeV, the highest electron energy achieved in a collider. Calculate:
      d) The radiation energy of the 100 GeV electron per turn
      e) The critical energy of the photons
      f) The Radiation power for a beam current of 800 mA

Solution:

The radiation energy per turn is given by:

$$U = C_\gamma E^4/\rho$$

The critical photon energy is:

$$E_c = \frac{3\hbar\gamma^3 c}{2\rho}$$

The answers for LHC and LEP is listed below

Table 2: Radiation of LHC and LEP

	LHC	LEP
Energy	7 TeV	100 GeV
γ	7462	1.957×10^5
Dipole radius	2811 m	
Radiation energy	6.6 KeV	3.1 GeV
Critical photon energy	44 eV	0.79 MeV
Radiatio power @0.8A	5.3 KW	2.5 GW

3. **Dispersion suppressor with FODO cell (20 pts)**
For a FODO cell with dipole and quads (QF/2, B, QD, B, QF/2), we find the optics at the middle plane of the focusing quad are β_F and D_F, from the periodic boundary condition. The phase advance of the cell is ϕ.
a) (5) Find the 3 by 3 matrix M for the cell using known parameters (β_F, D_F, and ϕ). (Hint: what is α_F, γ_F and D_F'? Write 2 by 2 matrix using Courant-Snyder parameterization and then construct 3 by 3 matrix)
b) (5) To match the cell's dispersion function to zero, we need to attach a dispersion suppressor to its end. Show that using the same FODO cells with zero bending angle will not do the job.
c) (10) To design a proper suppressor, we can use another two FODO cells with reduced bending angles. The cell 1 has bending angle θ_1 and cell 2

has bending angle θ_2. Find θ_1/θ and θ_2/θ using known parameters (θ is the main FODO cell's dipole's angle).

Solution:

The 3-by-3 matrix is given by:

$$\mathcal{M} = \begin{pmatrix} \cos\Phi & \beta_F \sin\Phi & M_{13} \\ -\sin\Phi/\beta_F & \cos\Phi & M_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

The unknown part is linked with disperion by:

$$\begin{pmatrix} M_{13} \\ M_{23} \end{pmatrix} = (I - M)\begin{pmatrix} D \\ D' \end{pmatrix}$$
$$= \begin{pmatrix} 1-\cos\Phi & -\beta_F \sin\Phi \\ \sin\Phi/\beta_F & 1-\cos\Phi \end{pmatrix}\begin{pmatrix} D_F \\ 0 \end{pmatrix}$$

Then the matrix give

$$\mathcal{M} = \begin{pmatrix} \cos\Phi & \beta_F \sin\Phi & D_F(1-\cos\Phi) \\ -\sin\Phi/\beta_F & \cos\Phi & D_F \sin\Phi/\beta_F \\ 0 & 0 & 1 \end{pmatrix}$$

Using two cells to match the dispersion to zero, with $a_1 = \theta_1/\theta$ and $a_2 = \theta_2/\theta$ we have:

$$\begin{pmatrix} D_F \\ 0 \\ 1 \end{pmatrix} = M_1 M_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} D_F \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\Phi & \beta_F \sin\Phi & a_1 D_F (1-\cos\Phi) \\ -\sin\Phi/\beta_F & \cos\Phi & a_1 D_F \sin\Phi/\beta_F \\ 0 & 0 & 1 \end{pmatrix}$$

$$\cdot \begin{pmatrix} \cos\Phi & \beta_F \sin\Phi & a_2 D_F (1-\cos\Phi) \\ -\sin\Phi/\beta_F & \cos\Phi & a_2 D_F \sin\Phi/\beta_F \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\Phi & \beta_F \sin\Phi & a_1 D_F (1-\cos\Phi) \\ -\sin\Phi/\beta_F & \cos\Phi & a_1 D_F \sin\Phi/\beta_F \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_2 D_F (1-\cos\Phi) \\ a_2 D_F \sin\Phi/\beta_F \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} a_2 D_F (1-\cos\Phi)\cos\Phi + a_2 D_F \sin^2\Phi + a_1 D_F (1-\cos\Phi) \\ -a_2 D_F (1-2\cos\Phi)\sin\Phi/\beta_F + a_1 D_F \sin\Phi/\beta_F \\ 1 \end{pmatrix}$$

Then it its easy to solve that

$$a_2 = \frac{1}{2(1-\cos\Phi)}$$

and

$$a_1 = \frac{1-2\cos\Phi}{2(1-\cos\Phi)}$$

4. **Heat load in a cavity (20 pts)**
   In RF cavity operating at 500 MHz, amplitude of the magnetic field at the part surface is 500 Gs or 500 Oe. Find power losses per square meter of the surface for:
   (a) (10) Cu cavity*
   (b) (10) SRF cavity with surface resistance, $R_s = 5\ 10^{-9}$ Ohm.

   How much water you can heat from 20 C^o to 40 C^o in one hour (3,600 second) by cooling such Cu cavity?
   Hint: you may use the conductivity of Cu or scale R_s from results shown in Lecture 12. Thermal capacitance of water is 4,179 J/kg/ C^o.

Solution:

We should use formula for surface losses for a good conductor (it is in SI units):

$$\frac{P_{loss}}{A} = \frac{1}{2} R_s \left| \vec{\mathbf{H}}_{//} \right|^2$$

The most confusing is to transfer H from CGS (Gs = Oe) units to SI (A/m) with coefficient 1000/4π : H=3.98 10^4 A/m: With A=1 m^2 power lost is simply

$$P_{loss} = \frac{1}{2} R_s \left| \vec{\mathbf{H}}_{//} \right|^2 A = \frac{1}{2} R_s \left| \vec{\mathbf{H}}_{//} \right|^2$$

and for SRF cavity we would have 3.96 W losses per one square meter of the surface. For Cu surface impedance scales with the frequency

$$R_s = \sqrt{\frac{\omega\mu}{\sigma}} [\Omega]$$

In slide 36, lecture 12 we shown that for Cu R_s=10 mOhm at frequency of 1.5 GHz, which is 3 time higher than in our case. Thus

$$R_s(Cu, 500 Mhz) = \frac{10 m\Omega}{\sqrt{3}} \approx 5.8 m\Omega$$

and power loss density is 4.57 MW per m^2. In one hour the EM field generates 1.65 10^{10} J in 1 m^2 of the Cu surface. Heating one kg (e.g. one liter) of water by 20K requires 83.6 kJ: hence this power will heat from 20 C^o to 40 C^o 197 tons of water! e.g. a cube approximately 6m x 6 m x 6m.