PHY 554

Fundamentals of Accelerator Physics

Lecture 13: Beam Dynamics in an Electron Storage Ring

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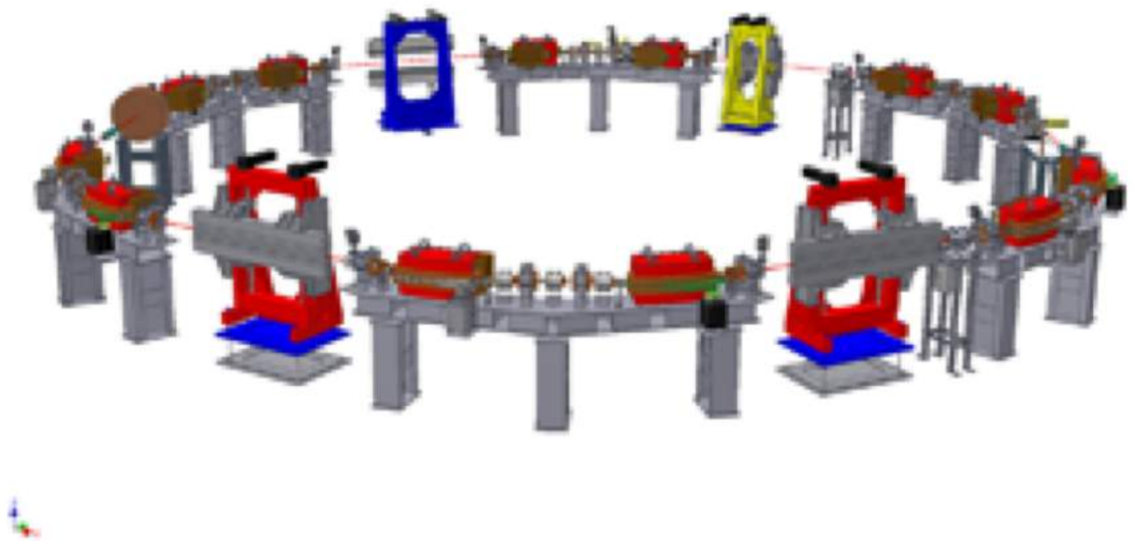
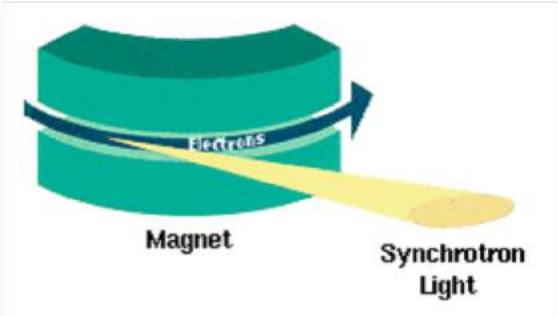


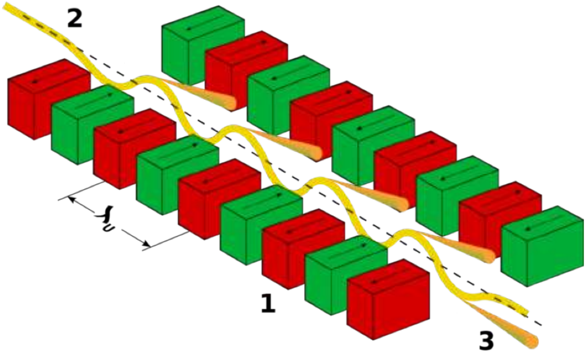
http://case.physics.stonybrook.edu/index.php/PHY554_fall_2018

Why electron/positron storage rings are different?

SYNCHROTRON RADIATION

- Origin: Energy emitted to infinity or other boundary condition.
  - Form: Electromagnetic wave
  - Source: the charged particles
  - Direction: Along the tangent of the beam trajectory curve.





SR Power

The power and its distribution can be calculated from the ‘retarded potential’ - *there will be a dedicated lecture on SR*

$$P = \frac{2e^2}{3c^3}\gamma^6\left[\left|\dot{\vec{\beta}}\right|^2 - \left[\vec{\beta}\times\dot{\vec{\beta}}\right]^2\right]$$

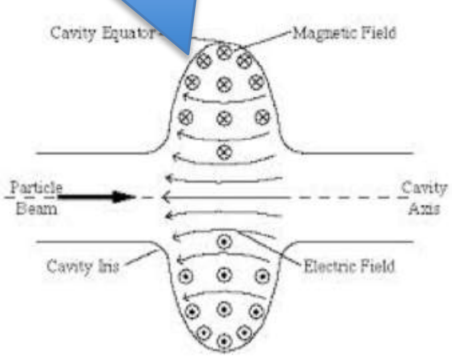
$$\frac{dE_{rad}}{dt} = \frac{2}{3}\frac{e^4\gamma^2}{m^2c^3}\left\{\left(\vec{E}-[\vec{\beta}\times\vec{B}]\right)^2 - \left(\vec{\beta}\cdot\vec{E}\right)^2\right\};\ \vec{\beta}=\vec{\mathrm{v}}/c$$

SGS

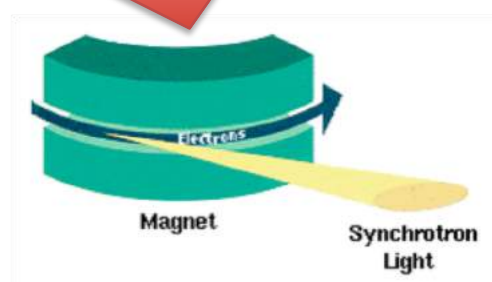
$$P = \frac{e^2}{6\pi\varepsilon_0 c}\gamma^6\left[\left|\dot{\vec{\beta}}\right|^2 - \left|\vec{\beta}\times\dot{\vec{\beta}}\right|^2\right]$$

SI

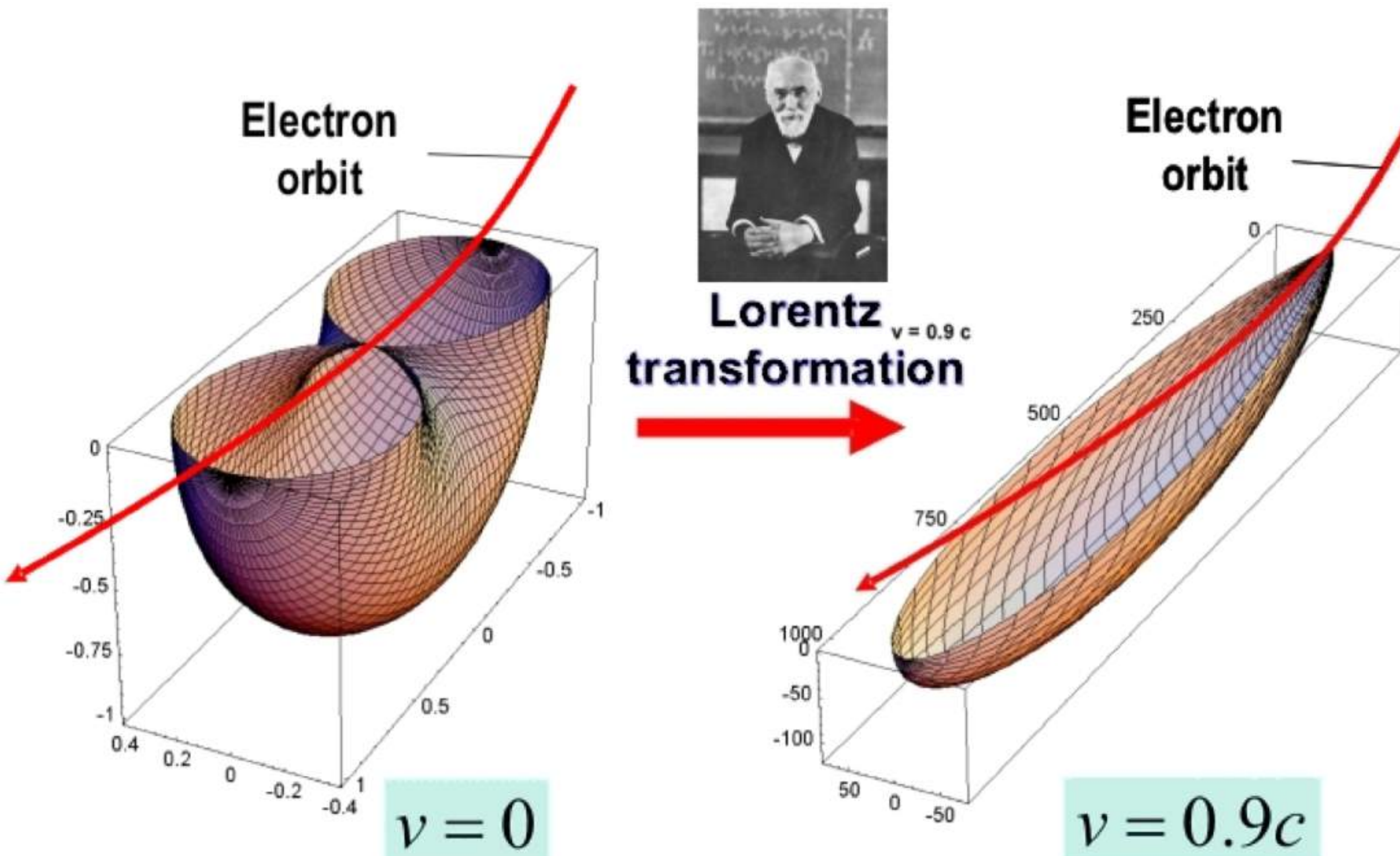
Radiation due to Acceleration (Negligible)



Radiation due to Bending (Dominating)



Radiation Angular Distribution



Opening angle in lab frame: $\sim 1/\gamma$

SR in storage ring

- The power of SR radiation in a dipole magnet

$$P = \frac{2e^2}{3c^3}\gamma^6\left[\left|\dot{\vec{\beta}}\right|^2 - \left[\vec{\beta}\times\dot{\vec{\beta}}\right]^2\right]$$

$$\frac{dE_{rad}}{dt} = \frac{2}{3}\frac{e^4\gamma^2}{m^2c^3}\left\{\left(\vec{E} - [\vec{\beta}\times\vec{B}]\right)^2 - \left(\vec{\beta}\cdot\vec{E}\right)^2\right\};\ eB = \gamma\frac{mc^2}{\rho}$$

$$P = \frac{2e^2c}{3\rho^2}\gamma^4$$

SGS

$$P = \frac{e^2c}{6\pi\varepsilon_0}\frac{\gamma^4}{\rho^2}$$

SI

Energy Loss in e-ring

The 2nd radiation integral I_2.

- In one turn, the energy loss is

SGS

$$\Delta U = \oint_C P \frac{ds}{c} = \frac{2e^2}{3} \gamma^4 \oint_C \frac{ds}{\rho^2} \equiv \frac{2e^2}{3_\rho} \gamma^4 \cdot I_2$$

$$I_2 = \oint_C \frac{ds}{\rho^2} \equiv \oint_C K_o^{\,2}(s)\, ds$$

SI

$$\Delta U = -\oint P_{SR}(t) ds/c$$

$$= -\frac{e^2\gamma^4}{6\pi\epsilon_0} \oint \frac{1}{\rho^2} ds$$

$$= -\frac{e^2\gamma^4}{6\pi\epsilon_0} I_2$$

- In a iso-magnetic ring:

$$I_2 = \oint_C K_o^{\,2}(s)\, ds = 2\pi K_o$$

$$\Delta U = \frac{4\pi e^2}{3\rho} \gamma^4$$

$$\Delta U = \frac{e^2}{3\epsilon_0} \frac{\gamma^4}{\rho}$$

Energy losses, practical units

- For electrons and positrons:

$$\Delta U_e(keV) = 88.46 \frac{E(GeV)^4}{\rho(m)}$$

- For Protons:

$$\Delta U_p(keV) = 6.03 \frac{E(TeV)^4}{\rho(m)}$$

- Typically the energy loss per turn is much less than the beam energy, and should be restored by RF cavity.

What we get so far

- The SR energy loss per turn and power have stong energy dependence.

$$U_{SR} = \frac{C_\gamma E^4}{2\pi} \oint \frac{ds}{\rho^2}$$

The 2nd radiation integral I_2.

$$I_2 = \oint_C \frac{ds}{\rho^2} \equiv \oint_C K_o^2(s)\,ds$$

$$c_\gamma = \frac{4\pi}{3} \frac{r_0}{m^3c^6} = 8.85 \times 10^{-5} m/(GeV)^3$$

$$P = \frac{cC_\gamma}{2\pi} \frac{E^4}{\rho^2} = \frac{e^2c^3}{2\pi} C_\gamma E^2 B^2$$

SGS

$$r_o = \frac{e^2}{mc^2}$$

SI

$$r_0 = \frac{e^2}{4\pi\epsilon_0 mc^2}$$

The longitudinal motion, revisit

$$\dot{\delta} = \frac{eV\omega_0}{2\pi\beta^2 E_0}\left(\sin\phi - \sin\phi_s\right)$$

$$\frac{d\Delta E}{dt} = \frac{eV\omega_0 \cos\phi_s}{2\pi}\omega_{rf}\tau - \frac{\omega_0}{2\pi}\frac{dU}{dE}\Delta E$$

$$\dot{\phi} = h\omega_0\eta\delta$$

$$\frac{d\tau}{dt} = \eta\frac{\Delta E}{E_0}$$

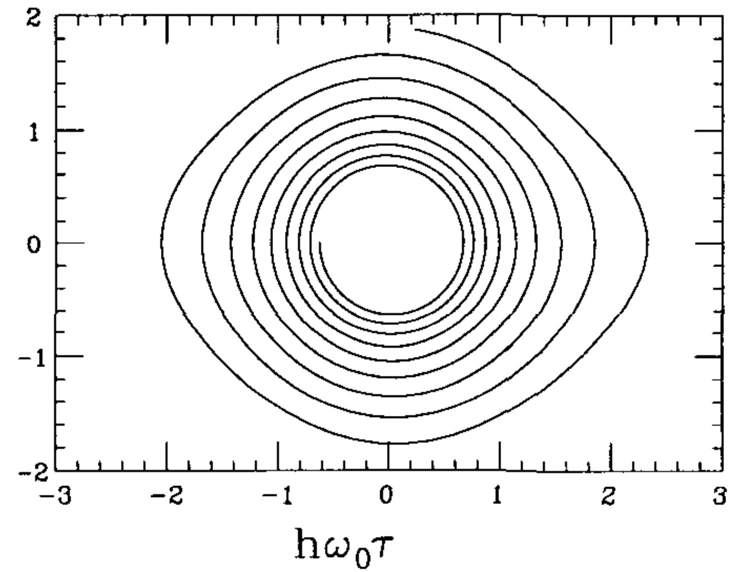
$$\frac{d^2\Delta E}{dt^2} = \frac{eV\omega_0^2 h\cos\phi_s\eta}{2\pi E_0}\Delta E - \frac{\omega_0}{2\pi}\frac{dU}{dE}\frac{\Delta E}{dt}$$

Damped Motion

The second order differential equation becomes

$$\frac{d^2\Delta E}{dt^2} + 2\alpha_E \frac{\Delta E}{dt} + \omega_s^2 \Delta E = 0$$

$$\alpha_E = \frac{1}{2T_0}\frac{dU}{dE}$$



If the damping effect is very small, the oscillation and damping are separated

$$\Delta E = A\exp(-\alpha_E t)\cos(\omega_s t + \phi_0)$$

Energy Loss per Turn

In this lecture, we will consider ultra-relativistic particles with $\gamma >> 1$
In longitudinal dynamics, we want to know the SR energy loss per turn for non-synchronous particle.

- Different energy has different radiation power
- Different energy has different travelling time

$$U = \oint P_{SR} dt = \oint P_{SR}(1 + x/\rho) ds/c$$

$$= \oint P_{SR}(1 + \frac{D}{\rho}\frac{\Delta E}{E_0}) ds/c$$

Radiation Power

$$P \sim E^4/\rho^2 \sim E^2 B^2$$

Both energy and radius are function of energy deviation.

Dipole field is constant for any energy deviation. How about quadrupole?

$$\frac{dP}{dE} = \frac{2P}{E_0} + \frac{2P}{B_0}\frac{dB}{dE} = \frac{2P}{E_0} + \frac{2PD}{B_0 E_0}\frac{dB}{dx}$$

Energy loss dependence

Here, we ignore second order terms:

$$\frac{dU}{dE} = \oint \left[\frac{dP}{dE} + \frac{DP}{\rho E}\right]\Bigg|_{E=E_0} ds/c$$

$$\frac{dU}{dE} = \oint \left[\frac{2P}{E_0} + \frac{2PD}{B_0 E_0}\frac{dB}{dx} + \frac{PD}{E\rho}\right]\Bigg|_{E=E_0} ds/c$$

$$= \frac{2U_0}{E_0} + \frac{1}{cE_0}\oint PD\left[\frac{2}{B_0}\frac{dB}{dx} + \frac{1}{\rho}\right]\Bigg|_{E=E_0} ds/c$$

$$\alpha_E = \frac{1}{2T_0}\frac{dU}{dE} = \frac{U_0}{2T_0 E_0}\left[2 + \oint \frac{D}{\rho}\left(1/\rho^2 + K(s)\right) ds / I_2\right]$$

Damping Partition Number

$$\bar{D} = \oint \frac{D}{\rho} \left(1/\rho^2 + K(s)\right) ds / I_2 \equiv I_4 / I_2$$

$$I_2 = \oint 1/\rho^2 ds$$

And the damping factor becomes

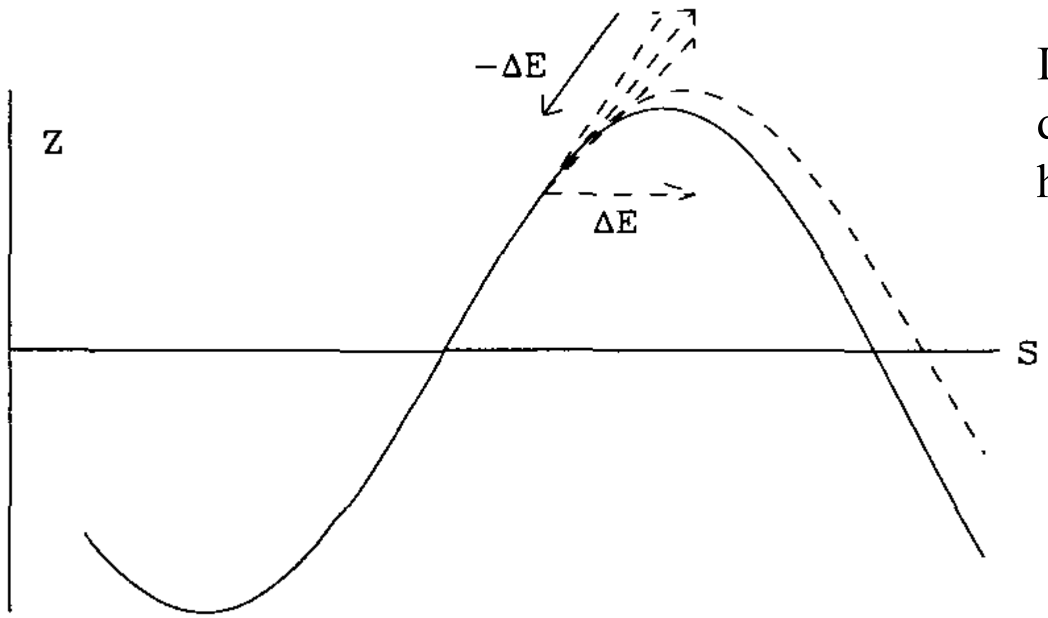
$$\alpha_E = \frac{U_0}{2T_0 E}(2 + \bar{D})$$

For separate function, iso-magnet ring:

$$\bar{D} = \frac{\alpha_c C}{2\pi\rho} \ll 1 \qquad \alpha_E \sim \frac{U_0}{T_0 E_0}$$

Transverse Damping (Vertical)

- The particle loose it's momentum in the very narrow cone ~1/γ along the direction of motion, and regain its momentum in RF in z-direction.

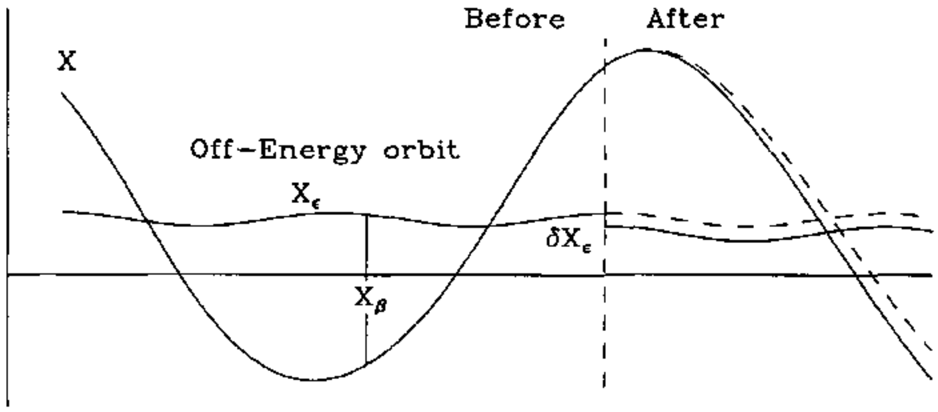


If we jump to the result: the damping rate in vertical plain is half of that in longitudinal plane.

$$\alpha_y = \frac{U_0}{2T_0E_0}$$

Transverse Damping (Horizontal)

- This case is more complicated because of the coupling with longitudinal motion via dispersion function.



Dispersion will 'heat up' the horizontal motion. Luckily we have similar damping scheme as in vertical plain.

$$\alpha_x = \frac{U_0}{2T_0E_0}(1-\bar{D})$$

$$\alpha_y = \frac{U_0}{2T_0E_0} \qquad \alpha_E = \frac{U_0}{2T_0E_0}(2+\bar{D})$$

Quantum Fluctuations of Synchrotron Radiation

- Synchrotron radiation is not a continuous emission
- Instead, energy is radiated at random moments and energy loss has quantum nature
- The emission obey Poisson distribution.
- It serves as a source of the noise that excites (heat) motion of particles in the electron beam.
- If u is the emitted photon energy, the average amplitude of energy deviation is

$$\frac{d\left\langle A^2\right\rangle}{dt} = \int n(u)u^2du = N\left\langle u^2\right\rangle$$

Equilibrium energy spread

- The damping and excitation will reach the a balance point.

$$\frac{d\left\langle A^2\right\rangle}{dt} = N\left\langle u^2\right\rangle - \frac{\left\langle A^2\right\rangle}{\tau_E/2}$$

- The rms energy spread is:

$$\left\langle A^2\right\rangle = N\left\langle u^2\right\rangle \tau_E/2 \sim 2\sigma_E^2$$

$$N \sim P_{SR}/\hbar\omega_c \qquad u \sim \hbar\omega_c \qquad \tau_E \sim E/P_{SR}$$

Equilibrium energy spread II

- The equilibrium of the energy spread:

$$\left(\frac{\delta_E}{E}\right)^2 = \frac{C_q \gamma^2}{(2+\bar{D})} \frac{\langle 1/\rho^3 \rangle}{\langle 1/\rho^2 \rangle}$$

Radiation Integral I_3

Radiation Integral I_2

$$C_q = \frac{55\hbar}{32\sqrt{3}mc} = 3.83 \times 10^{-13} m$$

- Amazingly it does not depend on the RF voltage. However the bunch length does.

Transverse equilibrium

- Horizontal equilibrium

$$\sigma_x^2(s) = C_q \frac{\gamma^2}{\rho} \left[\frac{\beta_x(s) \langle H \rangle}{1 - \bar{D}} + \frac{D^2(s)}{2 + \bar{D}} \right]$$

$$H = \beta_x D'^2 + 2\alpha_x D D' + \gamma_x D^2$$

- Vertical equilibrium

$$\sigma_y^2 \sim C_q \left\langle \beta_z^2 \right\rangle / \rho$$

Almost zero size in vertical direction.

Transverse Coupling

- In reality vertical emittance/beam size cannot be zero
- There are other effects that dominate over the equilibrium→ Coupling

$$\epsilon_x = \frac{1}{1+\kappa}\epsilon_{nat}; \epsilon_y = \frac{\kappa}{1+\kappa}\epsilon_{nat}$$

Summary: Radiation Integral

Index	Integrals	Properties
1	$I_1 = \oint D/\rho ds$	$\alpha_c = I_1 \times C$
2	$I_2 = \oint 1/\rho^2 ds$	$U_{SR} = \frac{C_\gamma E^4}{2\pi} I_2$
3	$I_3 = \oint 1/\left\|\rho\right\|^3 ds$	$\left(\frac{\sigma_E}{E}\right)^2 = C_q \gamma^2 \frac{I_3}{2I_2 + I_4}$
4	$I_4 = \oint (D/\rho)\left(1/\rho^2 + 2K(s)\right) ds$	$\bar{D} = I_4/I_2$
5	$I_5 = \oint H(s)/\left\|\rho\right\|^3 ds$	$\epsilon_x = C_q \gamma^2 \frac{I_5}{I_2 - I_4}$

Beam life-time

- Quantum lifetime
  - Although the equilibrium emittance is small, there is chance that, for one single electron, continuous random emission drive the electron out of aperture
  - Longitudinal or Transverse.
- Touschek lifetime
  - Coulomb scattering in the bunch may transfer transverse momentum to longitudinal plane and cause beam loss.

Typical “good’ numbers

- Revolution time: ~ micro second
- Longitudinal oscillation: sub millisecond
- Damping time: few thousand turns
  - Several millisecond
- Energy spread $\sim 10^{-3}$
- Rms transverse emittance sub nm-rad
- Rms vertical emittance several pm-rad