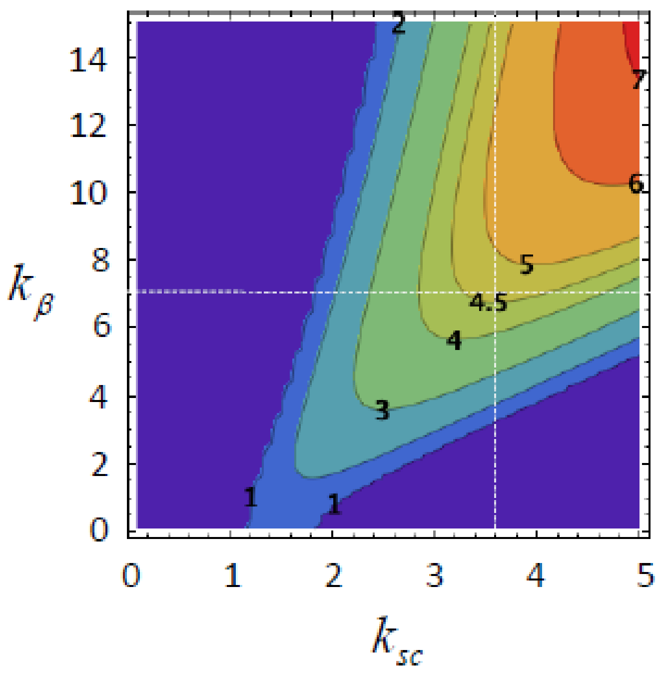
# Updates of 1-D PCA model

G. Wang

# Parameters for PCA-based CeC experiment

| **Energy, γ** | **28.5** |
|---|---|
| Electron beam peak current, A | 100 |
| Bunch length, ns | 0.015 |
| Bunch charge, nC | 1.5 |
| Modulator length, m | 3 |
| Amplifier length, m | 8 (4 sections) |
| Beam width at modulator, mm | 0.94 |
| Amplifier gain (Cold, infinitely wide), $g_{amp}$ | 200 |
| RMS energy spread | 1e-4 |
| KV envelope norm. emittance, µm | 8 |
| Minimal beam width at PCA, mm | 0.2 |

$k_{sc} = 3.6 \qquad k_\beta = 7$




$|\lambda| \approx 4.5 \qquad g_{amp} \equiv \frac{1}{2}|\lambda|^4 \approx 200$

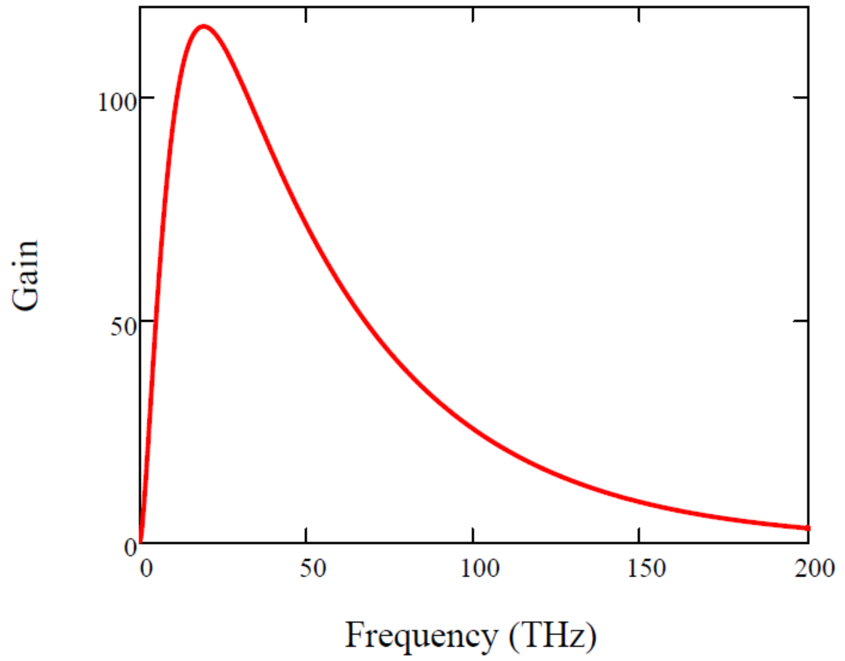
# What we did for the previous estimate…

$$\hat{n}'' + 2k_{sc}^2 \hat{a}(\hat{s})^{-2} \hat{n} = 0$$

$$\hat{a}'' = k_{sc}^2 \hat{a}^{-1} + k_\beta^2 \hat{a}^{-3}$$

$$g_{amp} = \frac{1}{2}|\lambda|^4 = 200$$

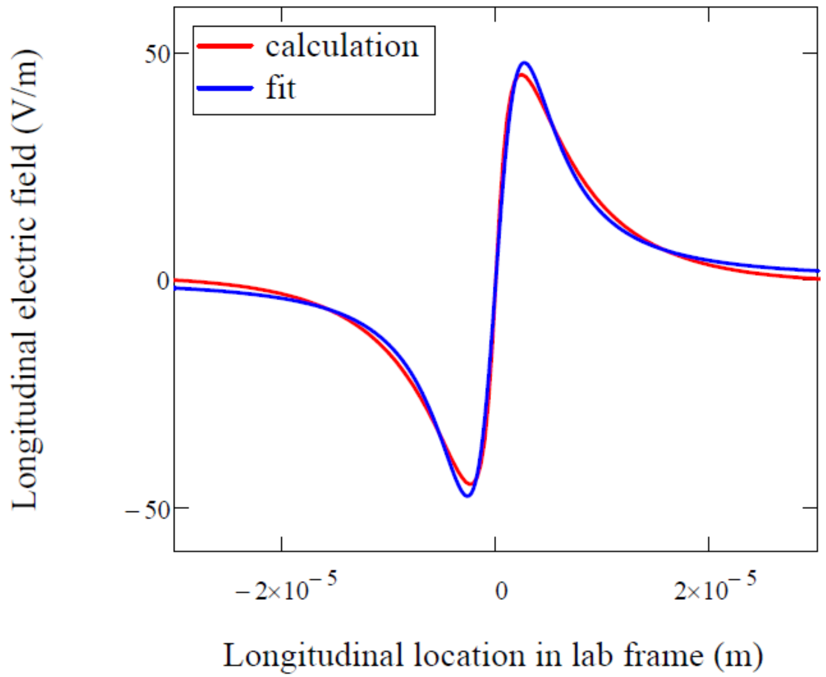
Estimated Gain of PCA




$$\tilde{\rho}_2(k_z) = g_{amp} R_{amp}(k_z) \exp\left(-|k_z|\beta_z \frac{L_{amp}}{\gamma c}\right) \tilde{\rho}_1(k_z)$$

Gain reduction factor due to finite transverse beam size. We used field reduction factor for gain reduction as a rough estimate.

Landau damping factor for Lorentzian energy distribution

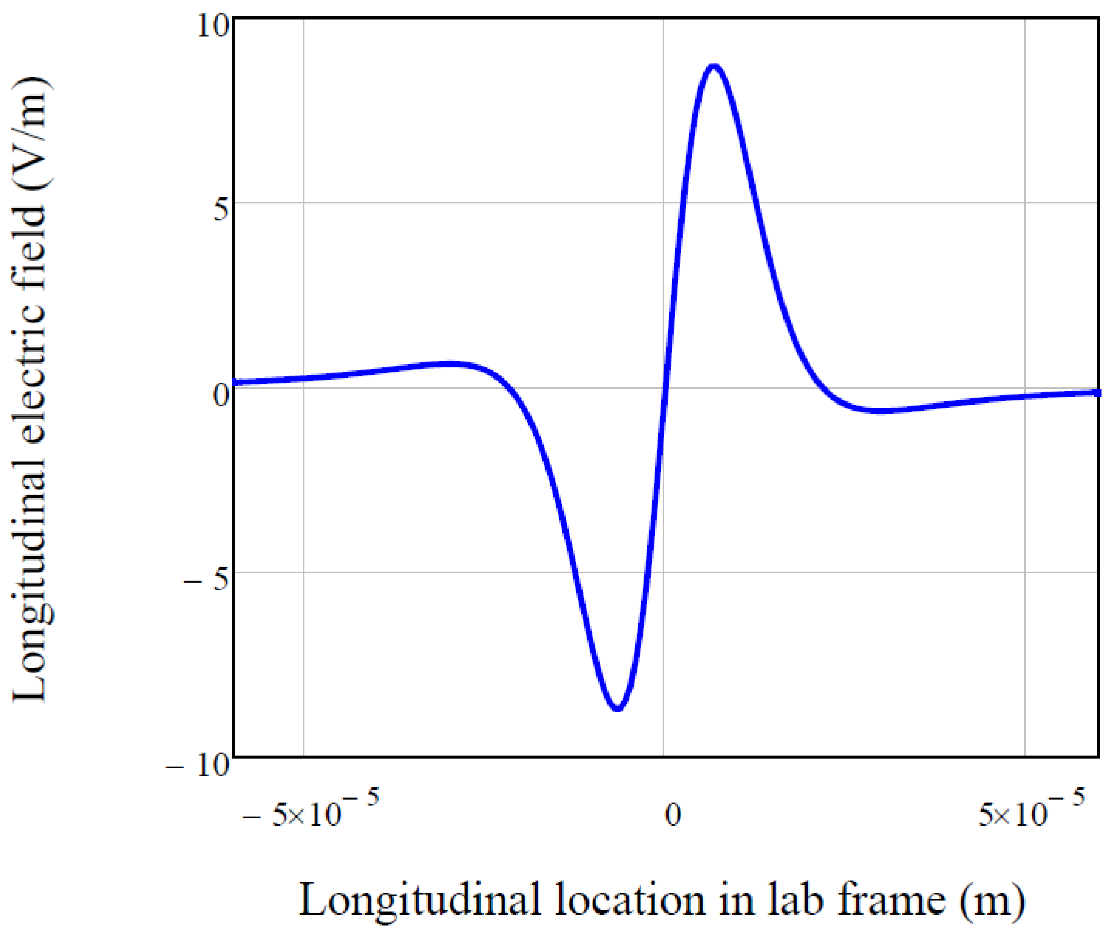


# It's possible to have anti-cooling for wakes calculated with rough low-frequency cut-off.

$a_{\min} = 0.1mm$

$a_{\mathrm{mod}} = 0.094mm$

$\delta_p = 1.5 \times 10^{-3}$



# Updates on Frequency Dependence of PCA Gain

Eq. (1)

$$\frac{d^2}{d\hat{s}^2}\tilde{Q}_1(k,\hat{s})+\frac{2k_{sc}^2 R(k,\hat{s})}{a^2(\hat{s})}\tilde{Q}_1(k,\hat{s})=0$$

$$R(k,\hat{s})\approx R(k,\hat{s}^*)\equiv R^*(k)$$

$$\frac{d^2}{d\hat{s}^2}\tilde{Q}_1(k,\hat{s})+\frac{2k_{sc}^2 R(k,\hat{s})}{a^2(\hat{s})}\tilde{Q}_1(k,\hat{s})=0$$

$$\hat{a}^2\approx k_\beta^2\hat{s}^2+1$$

Eq. (2)

$$\frac{d^2}{d\hat{s}^2}\tilde{Q}_1(k,\hat{s})+\frac{2k_{sc}^2 R^*(k)}{k_\beta^2\hat{s}^2+1}\tilde{Q}_1(k,\hat{s})=0$$

$$\tilde{Q}_1(k,\hat{s})\equiv\tilde{\rho}_1(k,\hat{s})e^{|k|\sigma_\delta\hat{s}l/(\gamma c)}$$

$$\hat{a}''=k_{sc}^2\hat{a}^{-1}+k_\beta^2\hat{a}^{-3}$$

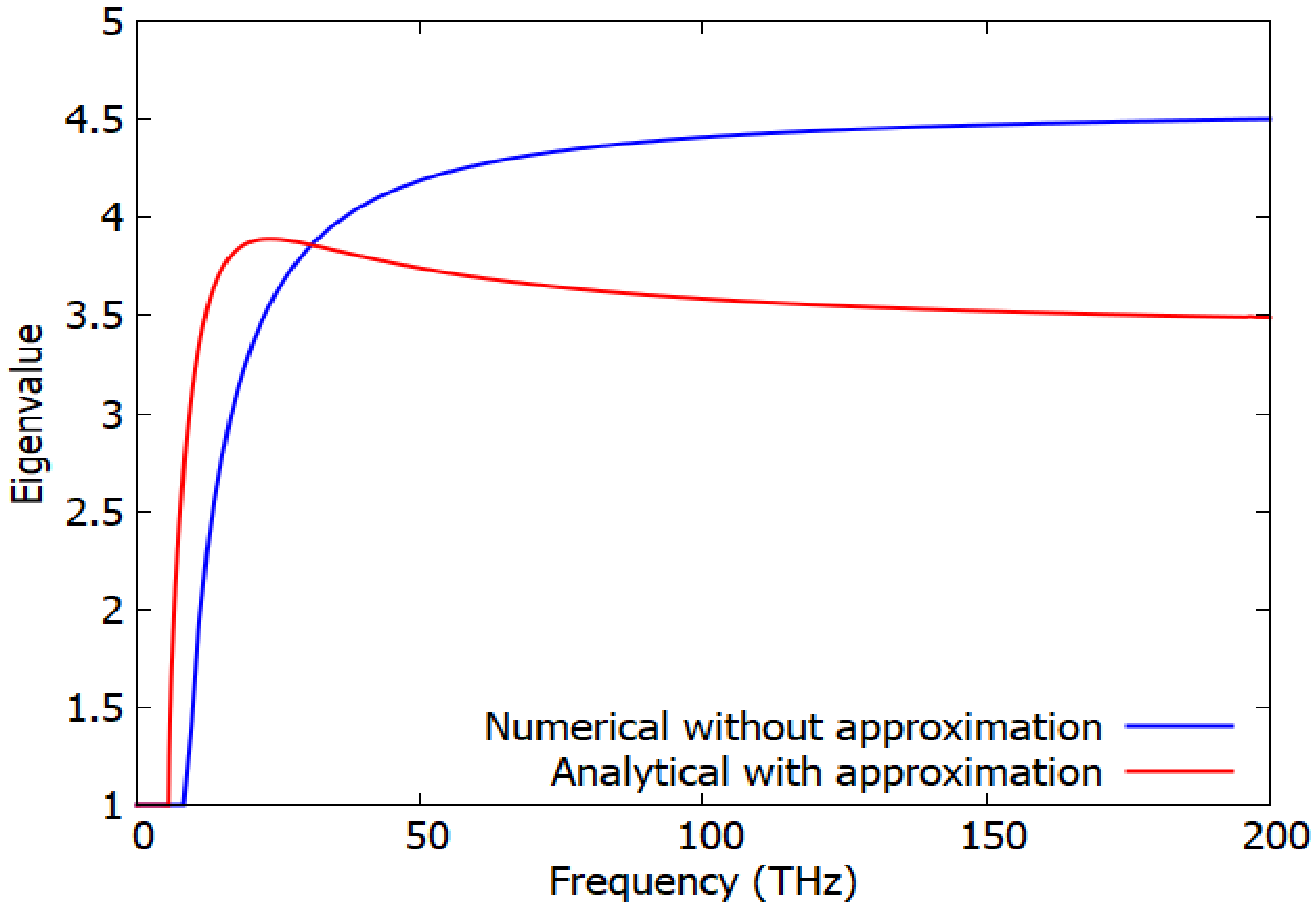
$$R(ka)\equiv\frac{4}{(ka)^2}\int_0^{ka}I_1(\tau)K_0(\tau)\tau^2 d\tau$$

- The gain of PCA can be calculated from numerically solving the coupled Hill's equation and envelope equation;
- If we use the approximation $R(k,s)\approx R(k,s^*)\equiv R^*(k)$ and assuming the electron beam is in the emittance dominated regime, i.e. $k_\beta >> k_{sc}$ , we can get analytical expression of the gain for such system in terms of hypergeometric functions.

$$\lambda(k)=M_{11}\pm\sqrt{M_{11}^{\ 2}-1}\qquad\begin{pmatrix}\hat{n}(\hat{s})\\ \hat{n}'(\hat{s})\end{pmatrix}=\begin{pmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{pmatrix}\begin{pmatrix}\hat{n}(0)\\ \hat{n}'(0)\end{pmatrix}$$

$$M_{11}=2F\left(\alpha_1(k),\beta_1(k);\frac{1}{2};-k_\beta^2\right)\times$$
$$\left\{F\left(\alpha_1(k)+\frac{1}{2},\beta_1(k)+\frac{1}{2};\frac{3}{2};-k_\beta^2\right)-\frac{2}{3}k_{sc}^*(k)^2 F\left(\alpha_1(k)+\frac{3}{2},\beta_1(k)+\frac{3}{2};\frac{5}{2};-k_\beta^2\right)\right\}-1$$

# Eigenvalue of the system




$$a_{\min} = 200\mu m$$

$$k_{sc} = 3.6$$

$$k_{\beta} = 7$$

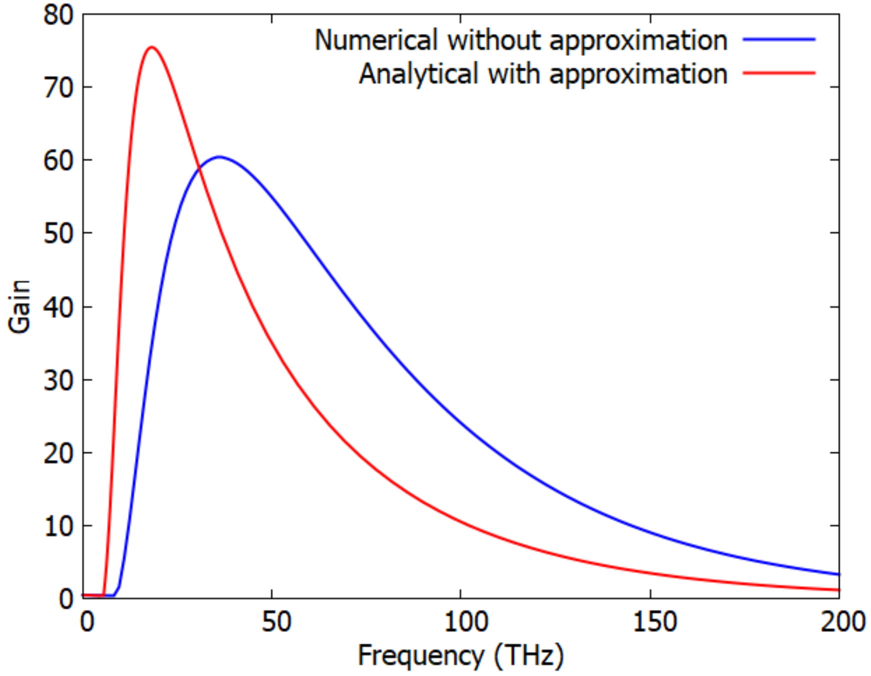
# Gain for 4-cell PCA with Landau damping

$$\tilde{\rho}_2(k) = G(k)\tilde{\rho}_1(k)$$

$$G(k) = \frac{1}{2}|\lambda|^4 \mathrm{e}^{-|k|\sigma_\delta l_{amp}/(\gamma c)}$$

$$E_z(r,z) = -\frac{\partial\varphi}{\partial z} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{E}_z(r,k_z)e^{ik_z z}dk_z$$

$$\tilde{E}_z(k) = -i\frac{\tilde{\rho}_2(k)}{\pi\varepsilon_0 ka^2}R(k) \qquad R(k) = \frac{4}{k^2a^2}\int_0^{ka} I_1(\tau)K_0(\tau)\tau^2 d\tau$$

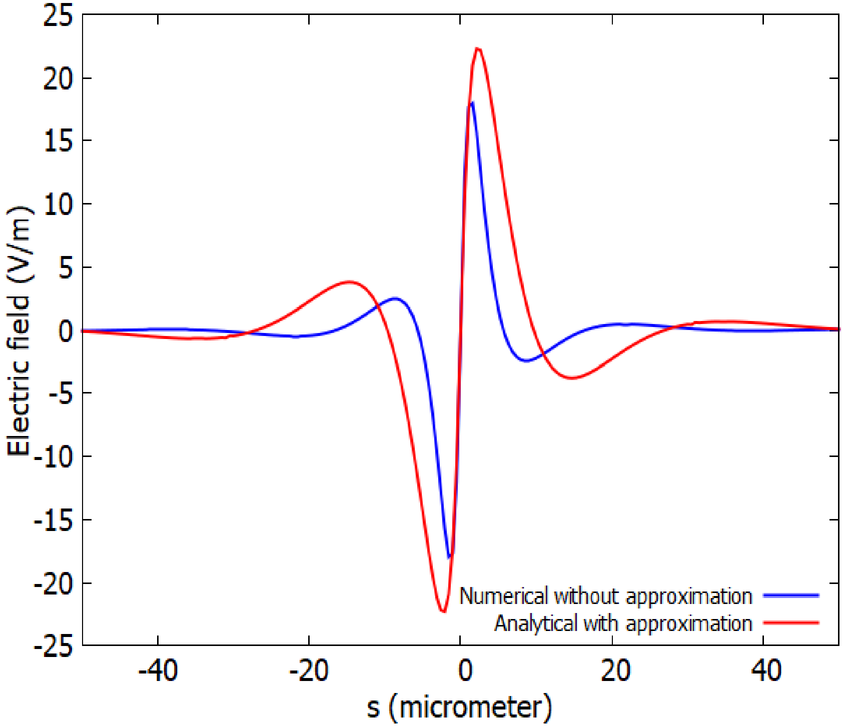



$$a_{\min} = 200\mu m$$

$$k_{sc} = 3.6$$

$$k_\beta = 7$$

$$\delta_p = 1\times10^{-4}$$



# Is the oscillating part come from the low frequency?

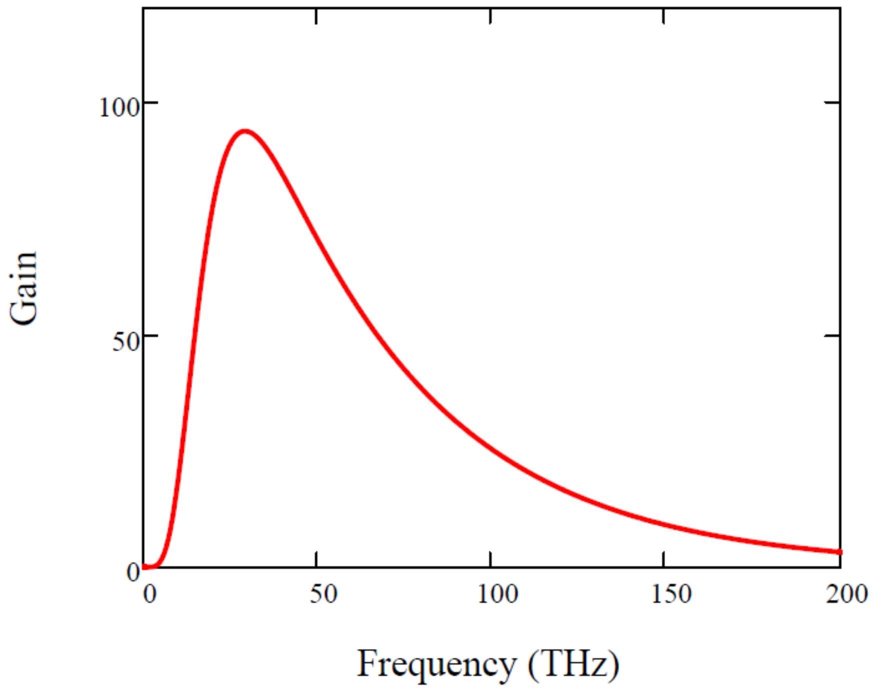
Estimated Gain of PCA

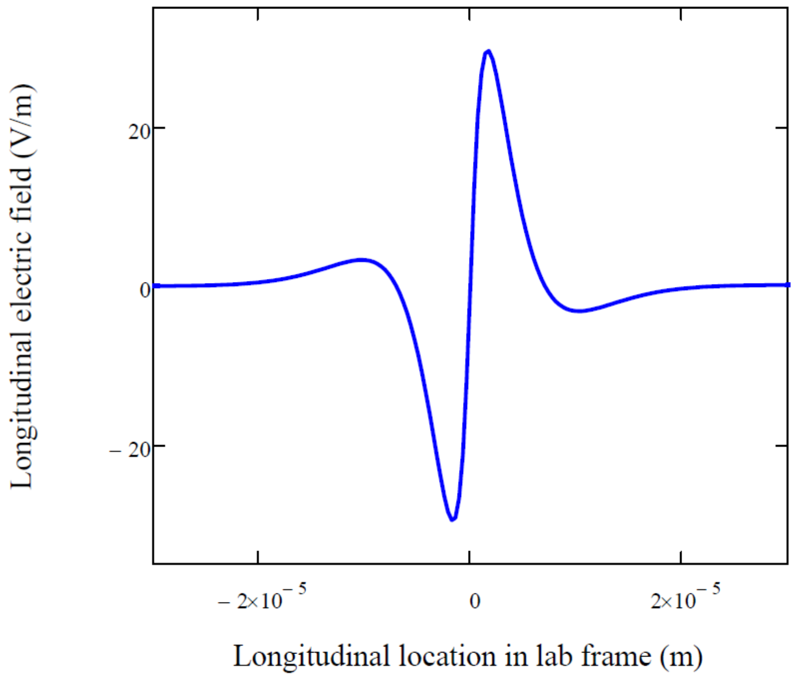
$$\tilde{\rho}_2(k_z) = g_{amp} R_{amp}(k_z)^4 \exp\left(-|k_z|\beta_z \frac{L_{amp}}{\gamma c}\right)\tilde{\rho}_1(k_z)$$

Gain reduction factor due to finite transverse beam size. We used field reduction factor for gain reduction as a rough estimate.

Artificial power to make low frequency 'stop band' in gain curve.

Landau damping factor for Lorentzian energy distribution



# Thank you