Space charge effect

Dec 7th, PHY554
Kentaro Mihara
Outline

- Space charge overview (Direct space charge)
- Space charge in KV envelope equation
- Transverse tune shift
- Beam Break up effect (Indirect space charge)
- Longitudinal tune shift
- Compensation technique for S.C
Space charge

- “Collective effect” space charge is the coulomb force the particle experiences by its surrounding
- Proportional to its intensity
- Adversely affect the beam stability
Cancelation at ultrarelativity

- Repulsive force by radial E field is canceled by azimuthal B field at relativistic limit.
Nonlinearity of S.C

- Although S.C has linear form for uniform distribution, it gives rise to non-linear form for more realistic dist.
Phase space SRF112Mhz

- 2000 particles with 2nC and 1.5MeV with and without S.C
Charge as variance

- Fig. S.C effect for 112 SRF Gun
- **Spotsizes** and **emittance** with charge 0.25 ~ 2 nC
- Solid line has 1.5mm and dashed line has 0.75mm laser spot size
S.C in envelope equation

- Envelope equation
- Simplest consistent model incorporating applied focusing, space-charge defocusing, and thermal defocusing forces
- Starting point of almost all practical machine design!
Hill’s equation

- Transverse particle dynamics
- Deviation from reference particle in Frenet-Serret coordinate

\[
\frac{d^2}{ds^2} x(s) + \kappa_x(s)x(s) = 0
\]

\[
\begin{pmatrix}
  x(s) \\
  x'(s)
\end{pmatrix} = M_x(s | s_i) \cdot \begin{pmatrix}
  x(s_i) \\
  x'(s_i)
\end{pmatrix}
\]

\[x(s) = A_i w(s) \cos \psi(s)\]

\[w(s + L_p) = w(s) \quad w(s) > 0\]
Courant Snyder Invariant

- Hill’s eqn (linear beam dynamics) gives rise to invariant.

\[
x(s) = A_i w(s) \cos \psi(s)
\]

\[
x'(s) = A_i w'(s) \cos \psi(s) - \frac{A_i}{w(s)} \sin \psi(s)
\]

\[
\left(\frac{x}{w}\right)^2 + (wx' - w'x)^2 = A_i^2 = \text{const}
\]

This Ai is called Courant Snyder Invariant
Energy of the beam, amplitude of oscillation
KV distribution

- 4D KV. D gives elliptical uniform dist in any 2d projection
- Not realistic but highly investigated and used for accelerator designing.

So the Poisson eqn would be calculated as
Space charge in KV.D

- Poison eqn is written by

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \phi = \begin{cases} 
- \frac{\lambda}{\pi \varepsilon_0 r_x r_y}, & \text{if } \frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} < 1 \\
0, & \text{if } \frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} > 1
\end{cases}
\]

- Analytical solution of space charge potential inside of the beam is linear form

\[
\phi = -\frac{\lambda}{2\pi \varepsilon_0} \left\{ \frac{x^2}{r_x (r_x + r_y)} + \frac{y^2}{r_y (r_x + r_y)} \right\} + \text{const}
\]
Envelop equation

- Define emittance as maximum C.S invariant
  \[ \varepsilon_x \equiv \text{Max}(A^2_{xi}) \]

- Means the edge of the distribution
  \[ r_x(s) = \sqrt{\varepsilon_x w_x(s)} \]

- Hill’s eqn can be written by envelope \( r_x \) as follows
  \[ r''_x(s) + \kappa_x(s)r_x(s) - \frac{2Q}{r_x(s) + r_y(s)} - \frac{\varepsilon_x^2}{r_x^3(s)} = 0 \]
Envelope equation

- Projection of 4D invariant KV distribution
- If not KV dist, E.E is given equivalently by moments (statistical information)
Transverse phase shift

- Space charge acts as defocusing

\[ \kappa_{x}^{\text{eff}}(s) = \kappa_{x}(s) - \frac{2Q}{[r_{x}(s) + r_{y}(s)]r_{x}(s)} \]

- Generate tune shift from designated P.A is called depressed P.A

\[ \Delta Q_{y} \approx -\frac{r_{0}N}{2\pi E_{y}\beta^{2}\gamma^{3}} \frac{2}{(1 + \sqrt{\beta_{x}E_{x}/\beta_{y}E_{y}})} \frac{2}{\sigma_{0}} \leq \frac{\sigma}{\sigma_{0}} \leq 1 \]

\[ \varepsilon \rightarrow 0 \quad Q \rightarrow 0 \]

- Each limit is called S.C or Emittance dominated beam
Incoherent tune shift by S.C

- With constant focusing strength

\[ \frac{\sigma}{\sigma_0} = 0.2 \]

Particle Orbits in Beam

- Depressed
- Undepressed

Static beam

Test particle trajectory

Kicker
Vacuum chamber
Pick-up

Coherent oscillation

Pulse < 1 turn
Frequency
(n-Q) \( \omega \)
Incoherent T.S

- It is cast into tune spread => more likely to cross the resonance tune => undesirable beam loss
- CERN Proton Synchrotron Booster
Longitudinal tune shift

- Longitudinal eqn of motion follows

\[ z'' + \left( \frac{\nu_{s0}}{R} \right)^2 \nu = - \frac{3N r_0 \eta}{\beta^2 \gamma^3 \tilde{z}^3} \left( \ln \frac{b}{a} + \frac{1}{2} \right) \nu \]

\[ \Delta \nu_s = \frac{3N r_0 \eta R^2}{2 \beta^2 \gamma^3 \tilde{z}^3 \nu_{s0}} \left( \ln \frac{b}{a} + \frac{1}{2} \right) \]

Unlikely the transverse one, longitudinally tune shift could be focusing or defocusing depending on whether it is below or above transition
Indirect space charge effect

- Beam interacts with the surrounding (pipe)
- Beam induces surface charges or currents into this environment that act back on the beam, possibly resulting in an ‘indirect’ space-charge
- Called Wake field
- Example) Beam breakup instability
- Bunch entering with offset from the center of the pipe in linac creates transverse wake field and it will acts back on the tail
Indirect space charge effect

- BBU instability

- In long linac with high current beam, this effects amplifying the distortion of the beam shape into a “banana” like shape.
Indirect space charge effect

- BBU instability
So far..

- Non linearity of S.C causes instability of the beam such as emittance growth, incoherent tune shift => tune spread!!
- How can we compensate those effect?
- A lot of work is going on for this subject.
Compensation of S.C

- Octupole

**Octupole** fields could cancel the next-to-leading term in the s.-c. force. For a round beam, the 4th order of term of the direct s.-c. potential varies as $(x^4 + 2x^2y^2 + y^4)$, while the potential of an octupole is proportional to $(x^4 - 6x^2y^2 + y^4)$. Therefore, at least two families of octupoles are needed to reduce the s.-c. tune spread, which are placed...
Compensation of S.C

- Potential of family of Octupole can suppress the 2nd leading order of S.C

**Space charge** +  **Octupole**
Compensation of S.C

- Compensate the S.C force by static force with electrons

- **Electron lenses**, in which a negatively charged electron beam collides with the proton beam inside a strong solenoid field, could also compensate the S.C.

Assuming: free space, coasting beam, $\rho(r)$

- **Proton to Proton**
  
  \[
  F_p(r) = e(E_{r_p}(r) - v_p B_{\phi p}(r)) = \frac{e^2(1 - \beta_p^2)}{\varepsilon_0 r} \int r' \rho_e(r') dr'
  \]

- **Electron to Proton**
  
  \[
  F_e(r) = -\frac{e^2(1 - \beta_e \beta_p)}{\varepsilon_0 r} \int_0^r r' \rho_e(r') dr'
  \]

- **Compensation**
  
  \[
  F_p(r) + F_e(r) = 0
  \]
Compensation of S.C

- Simulation for PS booster at CERN
- 4 electron lens with 10 keV, 1.2A

- Incoherent tune spread is suppressed by seeing 12 particles located at sigma to 2 sigma
Summary

- Nonlinearity of S.C causes instability such as emittance growth
- Introduced the S.C effect in EV envelope eqn
- Transverse and longitudinal tune shift and its incoherence could cause undesirable beam loss
- S.C compensation technique is going on and challenging topic
References

- https://acceleratorinstitute.web.cern.ch/acceleratorinstitute/ACINST89/Schindl_Space_Charge.pdf

- SPACE-CHARGE COMPENSATION OPTIONS FOR THE LHC INJECTOR COMPLEX* M. Aiba, M. Chanel, U. Dorda, R. Garoby

- https://acceleratorinstitute.web.cern.ch/acceleratorinstitute/ACINST89/Schindl_Space_Charge.pdf


- Transverse Equilibrium Distributions* Prof. Steven M. Lund

- USPAS2016 Collective Instability by Alex Chao
Thank you for your attention!!
Longitudinal S.C

- Unbunched uniform longitudinal dist gives only transverse field by symmetry
- Non uniform dist gives rise to S.C in s direction.
- Elamda(s-betact) with lamda being line charge density
- S.C proportional to lamda prime and 1/gamma can be gessed.
Faraday’s law gives $E_s$
Longitudinal S.C

- Opening angle is small
- So $E_r$ and $B_{\text{beta}}$ would be

$$E_r = \frac{B_\theta}{\beta} = 2e\lambda(s - \beta ct) \begin{cases} 0 \quad \text{if } r < a \\ \frac{1}{r} \quad \text{if } a < r < b \end{cases}$$

- Line integral for Faraday’s law gives $E_s$ as follows

$$\oint d\vec{l} \cdot \vec{E} = -\frac{1}{c} \frac{\partial}{\partial t} \int d\vec{A} \cdot \vec{B}$$