Homework 19. Due November 18

Problem 1. 20 points. 1D emittance

For an ensemble or a distribution function of particles 1D geometrical emittance is defined as

$$\varepsilon_y^{\ 2} = \langle y^2 \rangle \langle y'^2 \rangle - \langle yy' \rangle^2;$$

$$\langle g(y,y') \rangle = \frac{\sum_{n=1}^{N_p} g(y_n, y'_n)}{N_p} = \int f(y,y') g(y,y') \, dy \, dy';$$

1. Show that the emittance is invariant to a Canonical linear (symplectic matrix) transformation of

$$\begin{bmatrix} x \\ x' \end{bmatrix} = M \begin{bmatrix} x_1 \\ x'_1 \end{bmatrix}$$

Note: use the fact that $\varepsilon_y^{\ 2} = \det \Sigma; \Sigma = \begin{bmatrix} \langle y^2 \rangle & \langle yy' \rangle \\ \langle yy' \rangle & \langle y'^2 \rangle \end{bmatrix}$; and find transformation rule for the Σ matrix.

2. For one-dimensional betatron (y) distribution find components of eigen vector $\mathbf{w}_y$ and $\mathbf{w'}_y$ generating a given (positively defined)

$$\Sigma = \begin{bmatrix} \langle y^2 \rangle & \langle yy' \rangle \\ \langle yy' \rangle & \langle y'^2 \rangle \end{bmatrix};$$

This operation is called matching the beam into the beam-line optics.