Homework 11

Problem 1. 20 points. A weak transverse coupling.
***** STAR part - 50 points**

Consider a fully uncoupled x and y betatron motion in a storage ring with circumference C

$$\tilde{h}_o = \frac{\pi_1^2 + \pi_3^2}{2} + f(s)\frac{x^2}{2} + g(s)\frac{y^2}{2}$$

described by eigen vectors:

$$\mu_{x,y} = 2\pi Q_{x,y};\ \ Y_x(s) = \begin{bmatrix} \mathrm{w}_x \\ \mathrm{w}'_x + \dfrac{i}{\mathrm{w}_x} \\ 0 \\ 0 \end{bmatrix}; Y_y(s) = \begin{bmatrix} 0 \\ 0 \\ \mathrm{w}_y \\ \mathrm{w}'_y + \dfrac{i}{\mathrm{w}_y} \end{bmatrix}$$

The eigen vectors and tunes are considered to be known. Introduce a week coupling by SQ-quadrupole and solenoidal fields (for torsion equal zero):

$$\delta\tilde{h} = \delta f \frac{x^2}{2} + \delta n \cdot xy + \delta g \frac{y^2}{2} + \delta L(x\pi_3 - y\pi_1)$$

with

$$\delta n(s) = \frac{e}{2p_o c}\left[\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}\right]; \delta L(s) = \frac{e\delta B_s}{2p_o c}; \delta f(s) = \delta g(s) = \delta L^2(s);$$

(a) Write explicitly expressions for new betatron tunes using our developed perturbation method. Show that there is linear term on $\delta n, \delta L$ only in case of coupling resonance when $\mu_x = \pm\mu_y + 2\pi m$.

(b) For the case $\mu_x \neq \mu_y$ write expressions for new eigen vectors. Use perturbation method developed in class. Normalize eigen vectors symplectically.

STAR Part:

(*) Using new eigen vectors and substituting them into the Hamiltonian, find tune change expression on next order of $\delta n, \delta L$ for the case $\mu_x \neq \mu_y$. It is fine if it is just an integral.

(**)What should we do with eigen vectors at coupling resonance, when $\mu_x = \pm\mu_y + 2\pi m$.

Hint – look what is done with two energy degenerated levels in quantum mechanics.