Influence of energy jitter to PCA-based CeC (updated)

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Estimate Cooling Force for PCA-based CeC: Longitudinal Electric Field in the Kicker Section

The electric potential induced by the line density perturbation is determined by the following equations

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\varphi(r,z)\right)\right]+\frac{\partial^2}{\partial z^2}\varphi(r,z)=\frac{1}{\varepsilon_0}\rho_2(z)f_\perp(r)$$

If we take the transverse distribution of the electrons as

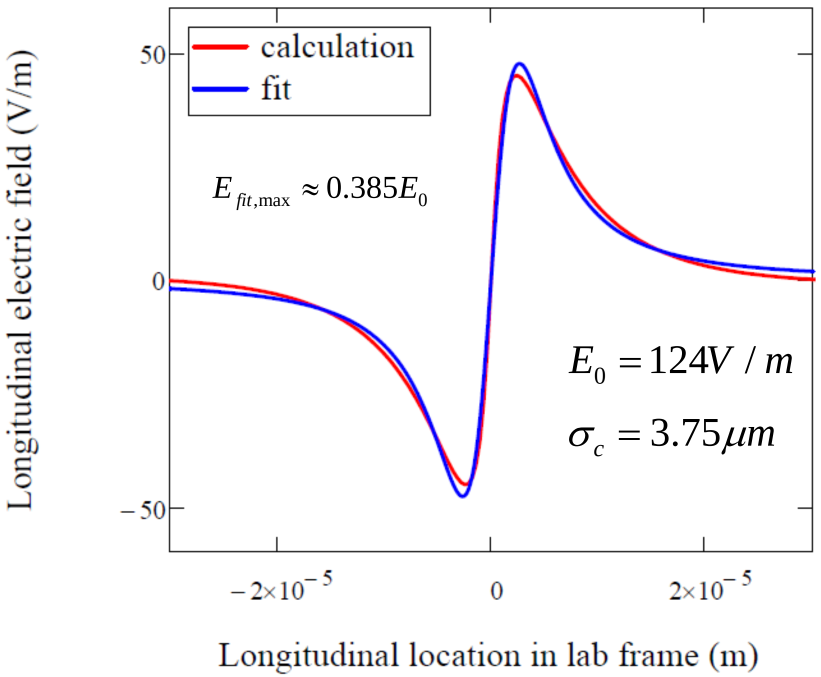
$$f_\perp(r)=\frac{1}{\pi a^2}H(a-r)$$

The electric field can be solved as

$$E_z(r,z)=-\frac{\partial\varphi}{\partial z}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{E}_z(r,k_z)e^{ik_z z}dk_z$$

$$\tilde{E}_z(r)=-ik_z\frac{\tilde{\rho}_2(k_z)}{\pi\varepsilon_0}$$

$$\times\left[I_0(k_z r)\int_{r/a}^{1}\eta K_0(k_z a\cdot\eta)d\eta+K_0(k_z r)\int_{0}^{r/a}\eta I_0(k_z a\cdot\eta)d\eta\right]$$



For easy implementation into ion tracking code, we use the fitting formula:

$$E_{fit}(z)=E_0\cdot\frac{z}{\sigma_c}\left[1+\frac{z^2}{\sigma_c^2}\right]^{-3/2}$$

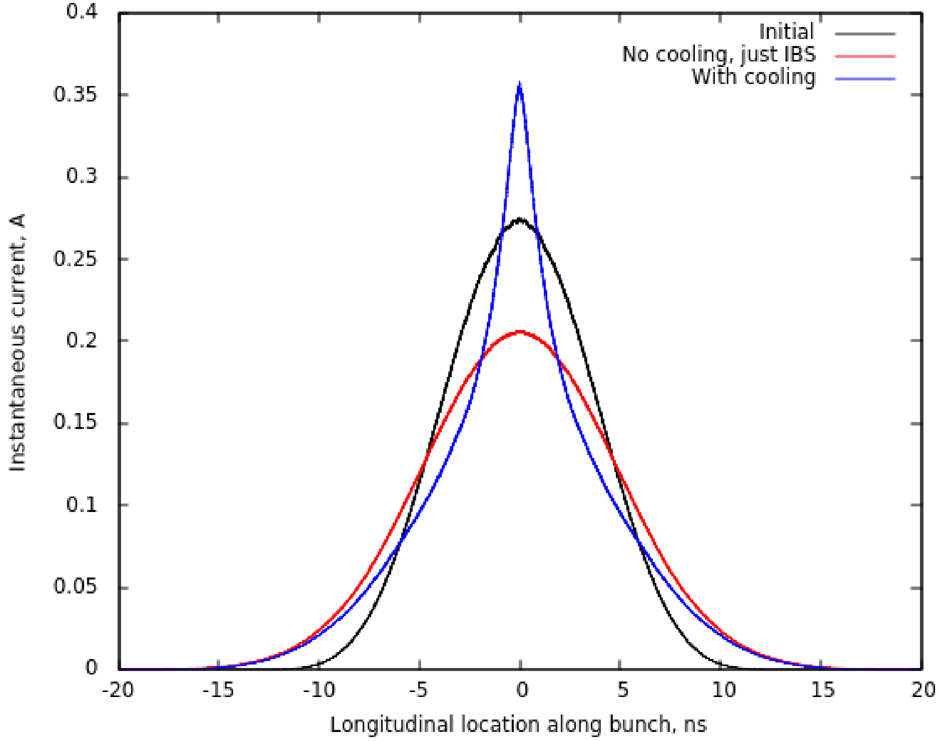
Single-pass Kick and Tracking Results for PCA-based CeC

$$\Delta\gamma_{j,N} = -g_\gamma \frac{\left(D\cdot\delta_{j,N}\right)}{\sigma_c}\left[1+\frac{\left(D\cdot\delta_{j,N}\right)^2}{\sigma_c^2}\right]^{-3/2} + g_\gamma\sqrt{\frac{3\pi}{8}\rho_{ion}\left(z_{j,N}\right)\sigma_c}\cdot X_{j,N} + \frac{g_\gamma}{Z_i}\sqrt{\frac{3\pi}{8}\rho_e\left(z_{j,N}\right)\sigma_c}\cdot Y_{j,N}$$

$$g_\gamma = Z_i e E_0 L_k \big/ \left(A_i m_u c^2\right)$$

Parameters used in ion tracking	
E_0*	62 V/m
σ_c	3.75 µm
Z_{ion}	79
Ion bunch intensity	2E8
D (R_{56})	1.2 cm

*E_0 is reduced by a factor of 2 to account for reduced cooling for ions with large betatron amplitude

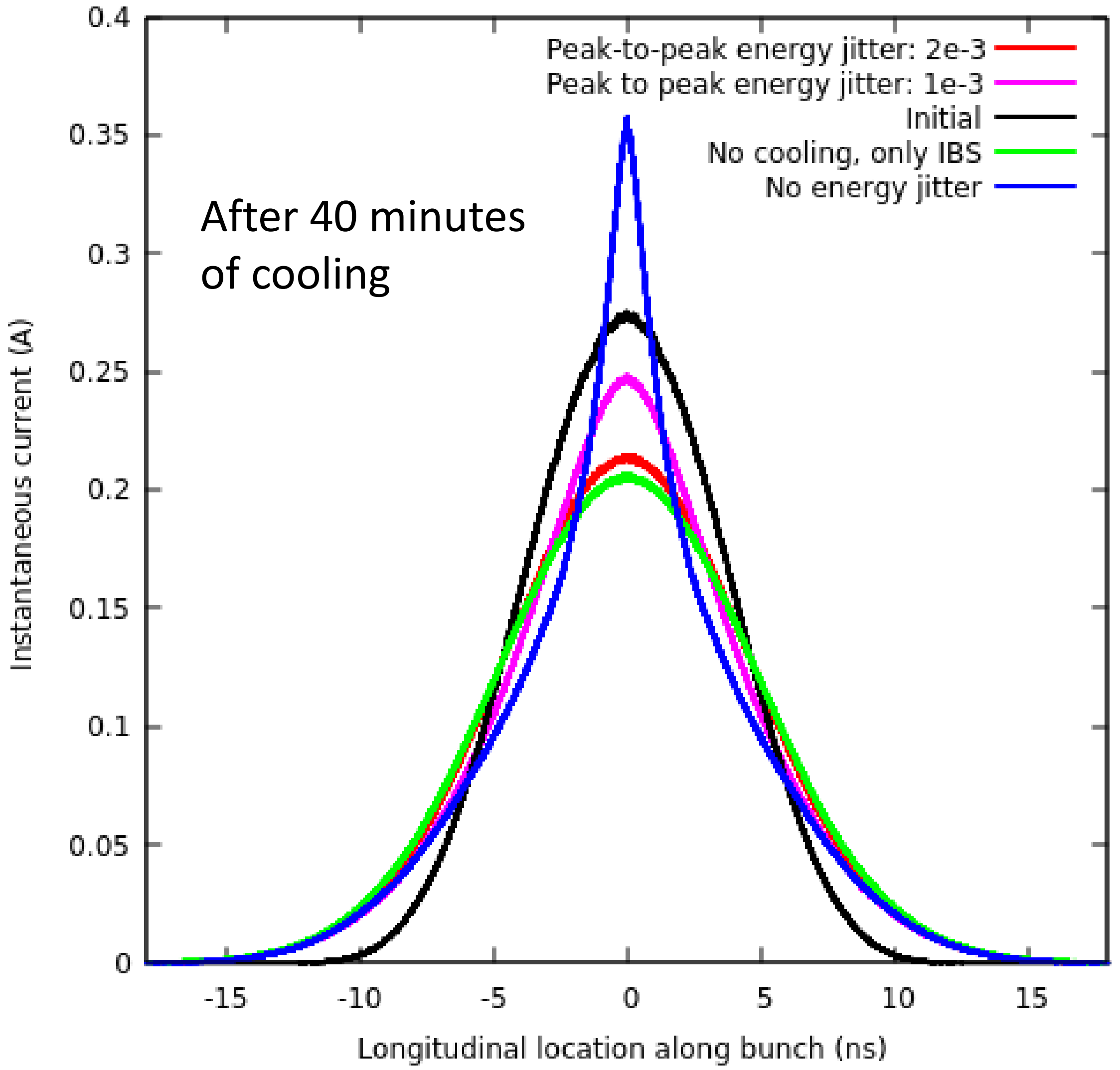


Influence of energy jitter

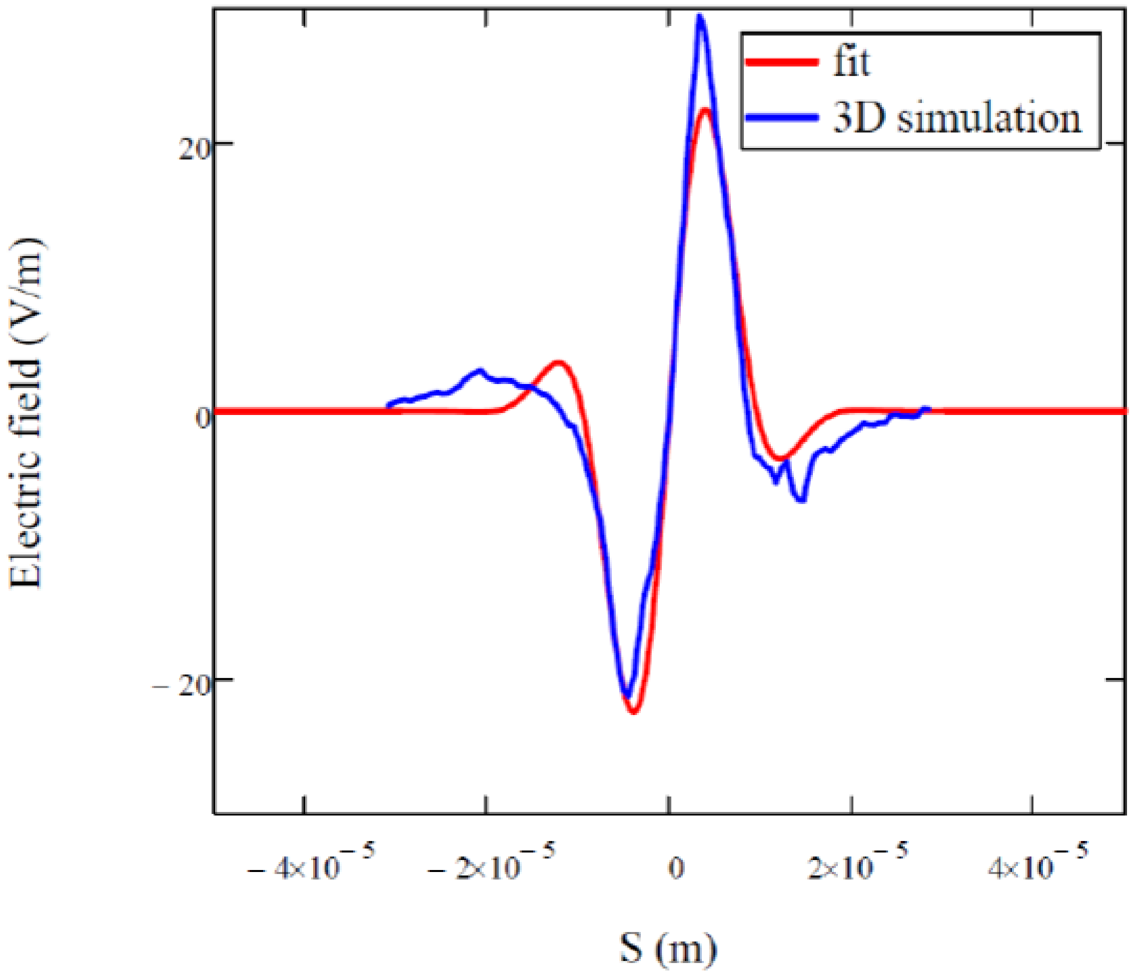
$$\Delta\gamma_{j,N} = -g_\gamma \frac{D\cdot\left(\delta_{j,N}+\Delta\delta_N\right)}{\sigma_c}\left[1+\frac{\left(\left[D\cdot\left(\delta_{j,N}+\Delta\delta_N\right)\right]\right)^2}{\sigma_c^2}\right]^{-3/2}$$

$$+g_\gamma\sqrt{\frac{3\pi}{8}\rho_{ion}\left(z_{j,N}\right)\sigma_c}\cdot X_{j,N}+\frac{g_\gamma}{Z_i}\sqrt{\frac{3\pi}{8}\rho_e\left(z_{j,N}\right)\sigma_c}\cdot Y_{j,N}$$

$\Delta\delta_N$ is the energy jitter for turn N, which is a random number uniformly distributed in the interval $\left[-\frac{\Delta\delta_{\max}}{2}:\frac{\Delta\delta_{\max}}{2}\right]$



Updates with 3D simulated field for cooling



$$E_{fit}(z) = E_0 \exp\left(-\frac{z^2}{2\sigma_c^2}\right)\sin(k_0 z),$$

$$E_0 = 28.1 V/m$$

$$\sigma_c = 6.475 \mu m$$

$$k_0 = 3.276\times 10^5 m^{-1} (\lambda_0 = 19.18 \mu m)$$

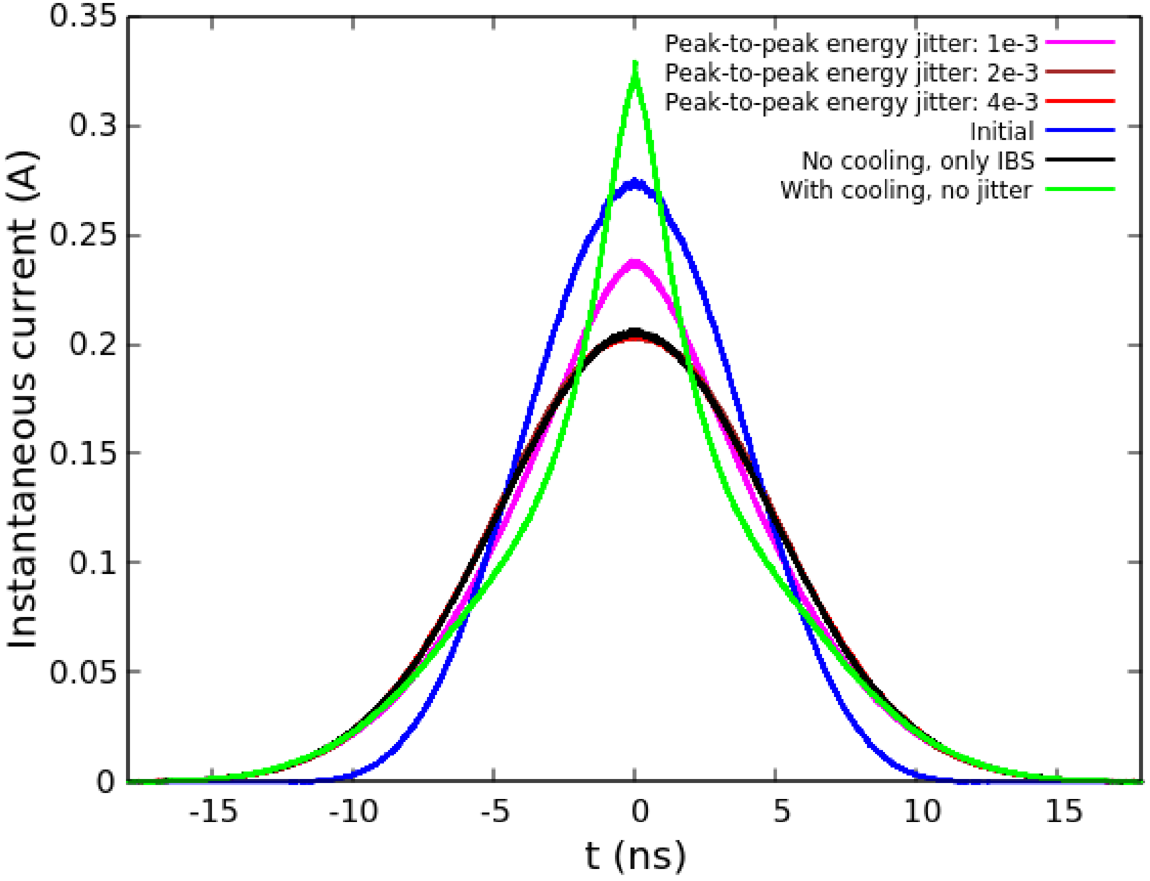
Simulation results

$$\Delta\gamma_{j,N} = -g_{\gamma}\sin\left[k_0 D\cdot\left(\delta_{j,N}+\Delta\delta_N\right)\right]\exp\left[-\frac{D^2\cdot\left(\delta_{j,N}+\Delta\delta_N\right)^2}{2\sigma_c^2}\right]$$

$$+g_{\gamma}\sqrt{\frac{3\sqrt{\pi}}{2}\rho_{ion}\left(z_{j,N}\right)\sigma_c\left(1-e^{-k_0^2\sigma_c^2}\right)}\cdot X_{j,N}$$

$$+\frac{g_{\gamma}}{Z_i}\sqrt{\frac{3\sqrt{\pi}}{2}\rho_e\left(z_{j,N}\right)\sigma_c\left(1-e^{-k_0^2\sigma_c^2}\right)}\cdot Y_{j,N}$$

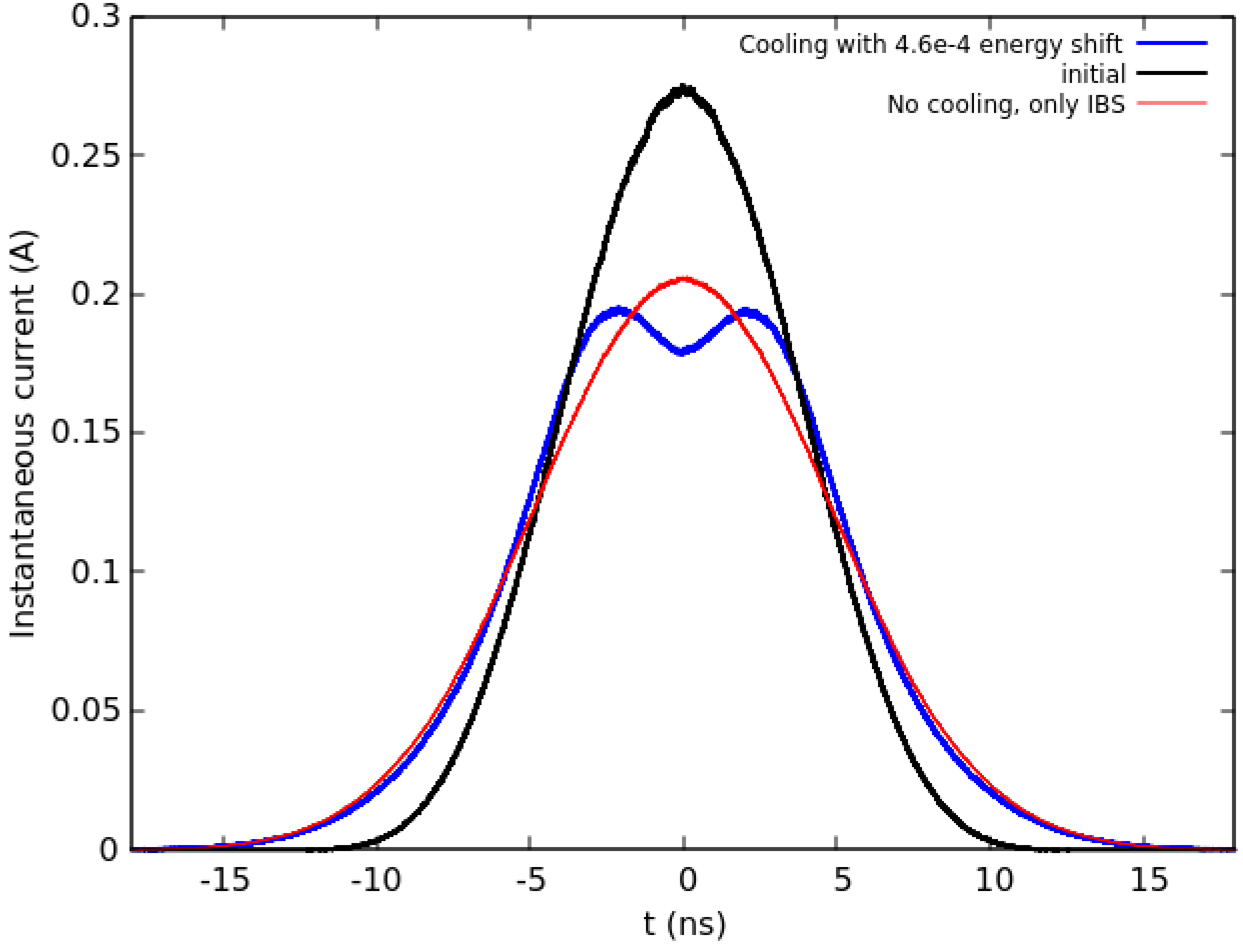
$\Delta\delta_N$ is the energy jitter for turn N, which is a random number uniformly distributed in the interval $\left[-\frac{\Delta\delta_{\max}}{2}:\frac{\Delta\delta_{\max}}{2}\right]$.

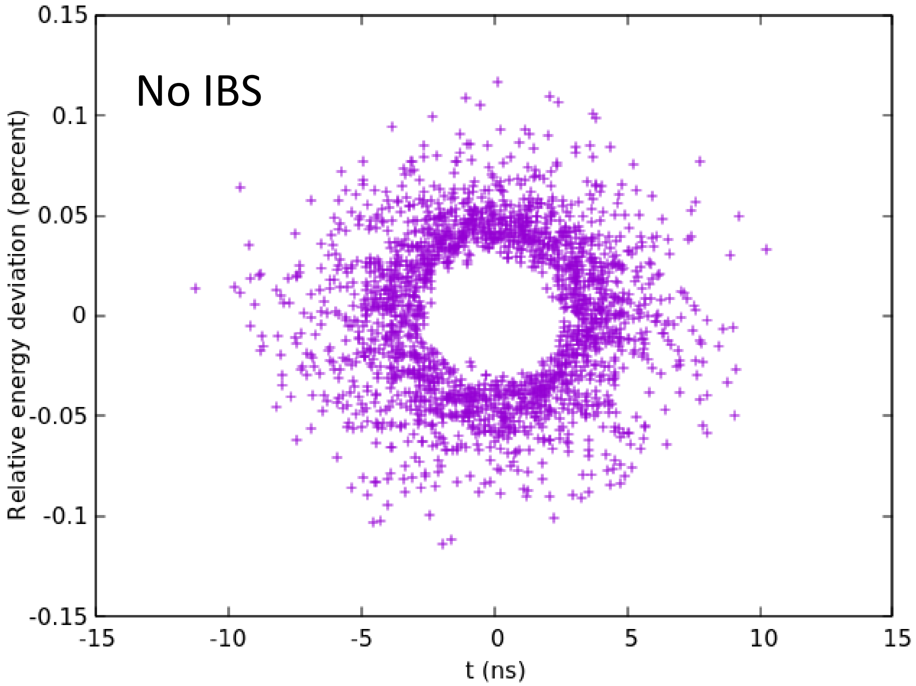


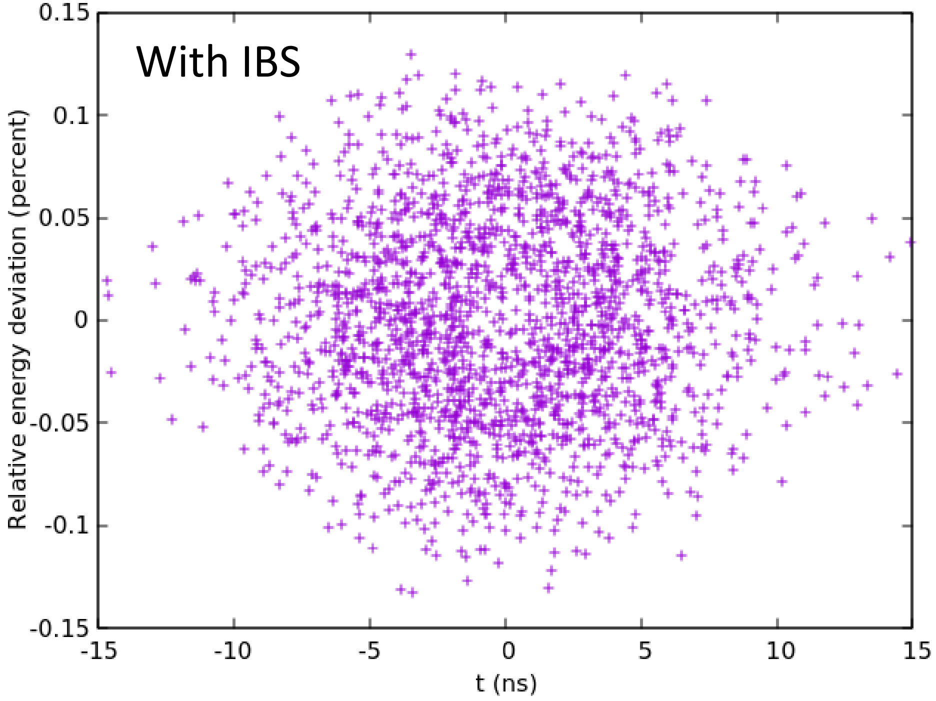
For the details of the code, see Phys. Rev. Accel. Beams **22**, 111002 (2019)

Influence of unmatched energy between electrons and ions

$$\frac{\gamma_e - \gamma_{ion}}{\gamma_{ion}} = 4.64\times10^{-4} \qquad \Delta z = 6.5\mu m$$







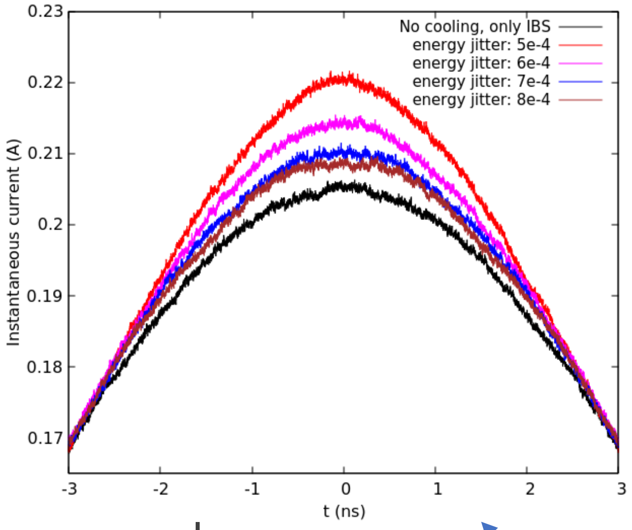
Summary

- For uniform energy jitter, peak-to-peak relative amplitude of 0.1% should still allow us to observe some cooling, assuming no additional noise in the electron beam other than Poisson noise;
- If the peak-to-peak amplitude of the jitter increases above 0.2%, the cooled bunch behaves similarly as the reference bunch, i.e. no cooling can be observed.
- Further increase of energy jitter amplitude to 0.4% will not noticeably heat electron bunch.
- In the absence of excessive noise in electrons and energy jitter, energy mismatch on the level of 5e-4 can noticeably change the longitudinal profile of the ion bunch, even with IBS.

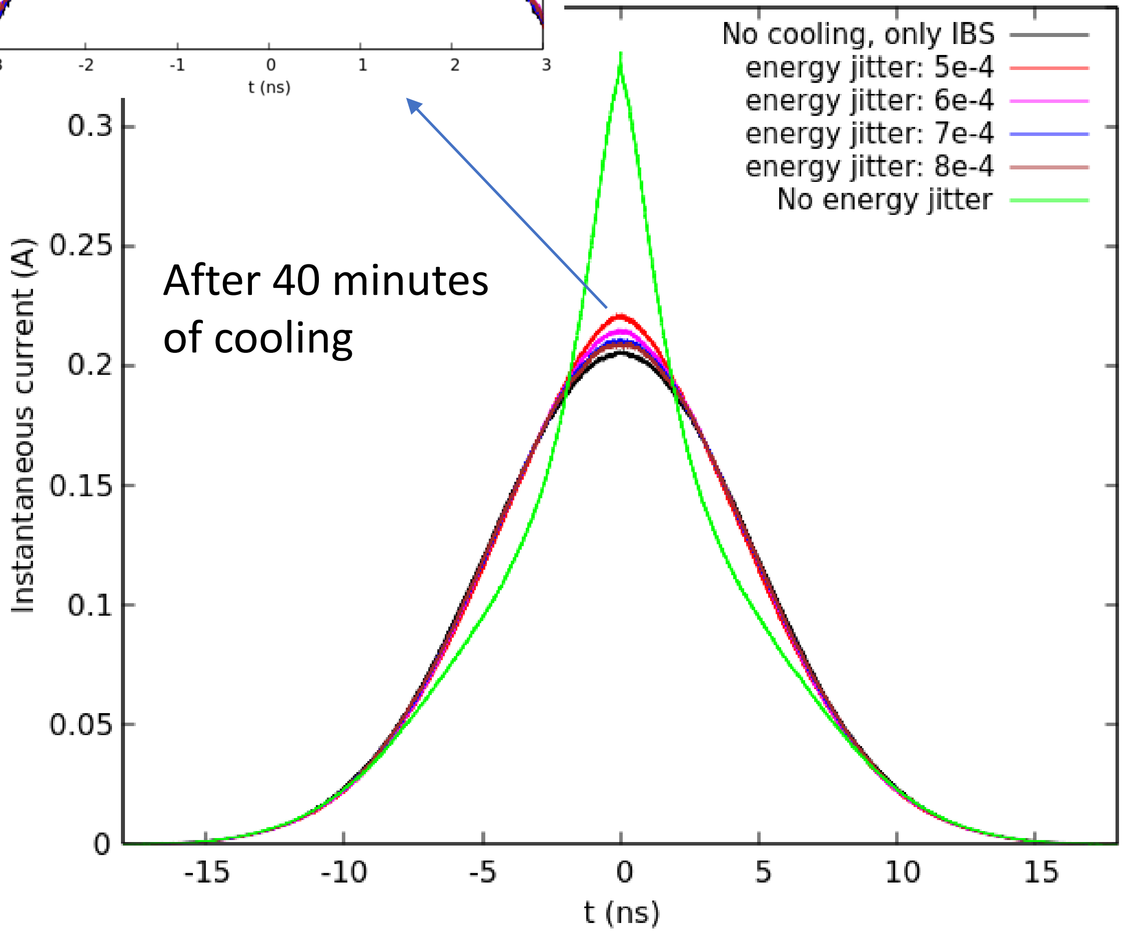
Energy jitter with Gaussian distribution

$$\Delta\gamma_{j,N} = -g_\gamma \sin\left[k_0 D\cdot\left(\delta_{j,N}+\Delta\delta_N\right)\right]\exp\left[-\frac{D^2\cdot\left(\delta_{j,N}+\Delta\delta_N\right)^2}{2\sigma_c^2}\right]$$
$$+g_\gamma\sqrt{\frac{3\sqrt{\pi}}{2}\rho_{ion}\left(z_{j,N}\right)\sigma_c\left(1-e^{-k_0^2\sigma_c^2}\right)}\cdot X_{j,N}$$
$$+\frac{g_\gamma}{Z_i}\sqrt{\frac{3\sqrt{\pi}}{2}\rho_e\left(z_{j,N}\right)\sigma_c\left(1-e^{-k_0^2\sigma_c^2}\right)}\cdot Y_{j,N}$$

$\Delta\delta_N$ is the energy jitter for turn N, which is a Gaussian random number with RMS variation of σ_{jitter} .



Could this slow CeC cooling due to energy jitter is what we observed at the experiment?

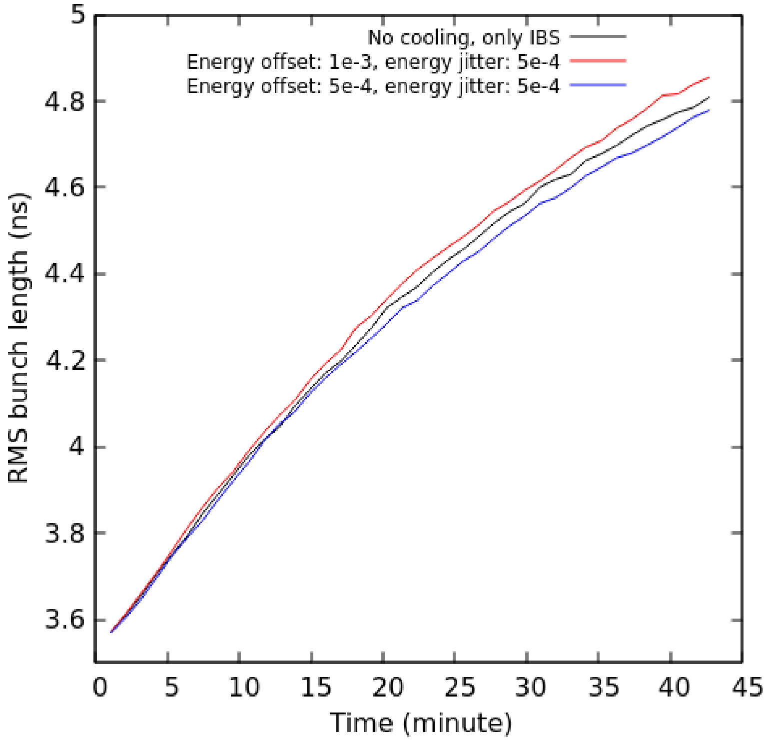


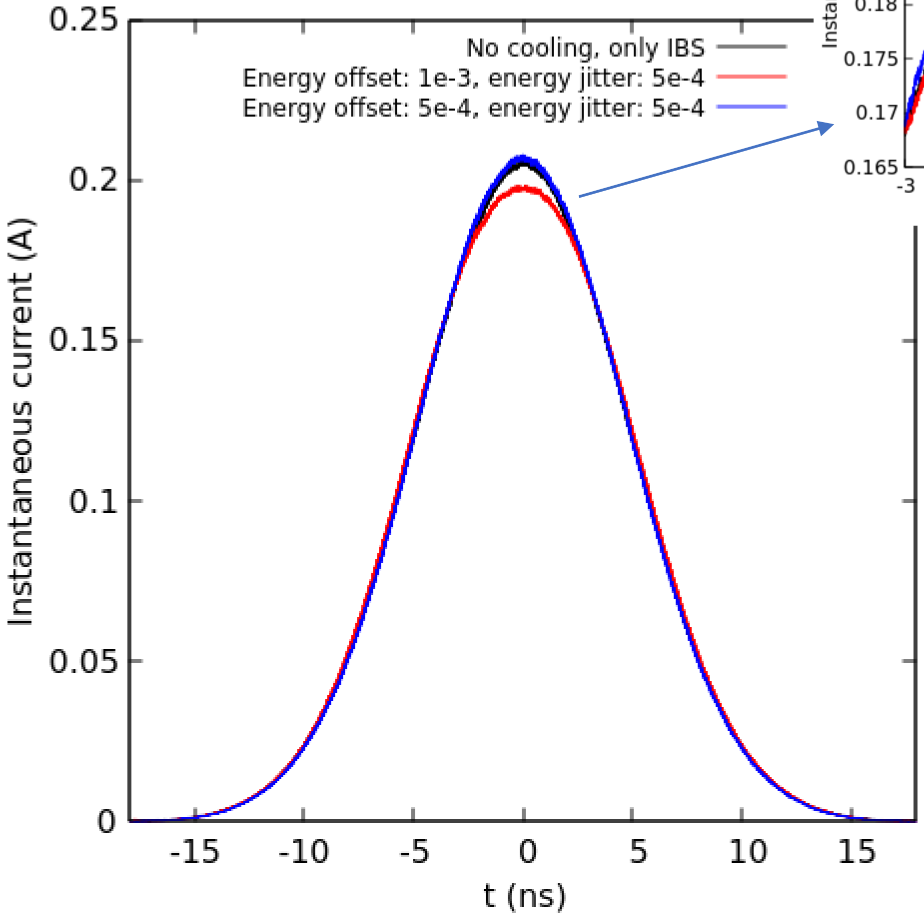
Cooling with energy jitter and energy offset

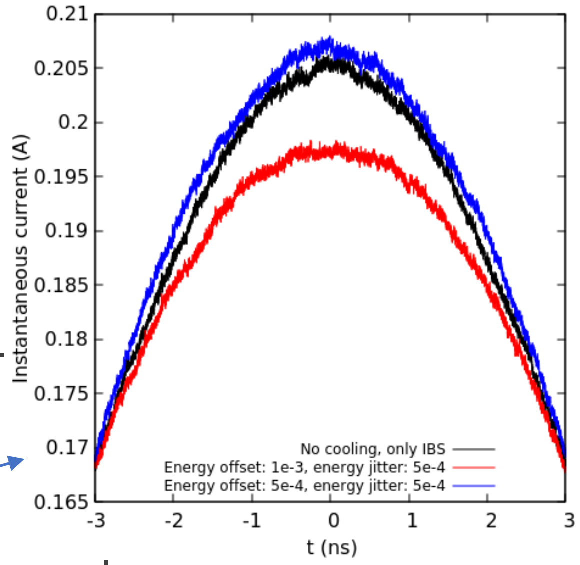
$$\Delta\gamma_{j,N} = -g_\gamma \sin\left[k_0 D\cdot\left(\delta_{j,N} + \Delta\delta_N + \delta_{offset}\right)\right]\exp\left[-\frac{D^2\cdot\left(\delta_{j,N}+\Delta\delta_N+\delta_{offset}\right)^2}{2\sigma_c^2}\right]$$

$$+g_\gamma\sqrt{\frac{3\sqrt{\pi}}{2}\rho_{ion}\left(z_{j,N}\right)\sigma_c\left(1-e^{-k_0^2\sigma_c^2}\right)}\cdot X_{j,N}$$

$$+\frac{g_\gamma}{Z_i}\sqrt{\frac{3\sqrt{\pi}}{2}\rho_e\left(z_{j,N}\right)\sigma_c\left(1-e^{-k_0^2\sigma_c^2}\right)}\cdot Y_{j,N}$$







Unfortunately, it is not likely related to our observation in the CeC experiment since introducing energy difference between electron beam and ion beam at even 4e-3 level will not change slow cooling to heating….

Backup slides

Energy kick in cooling section

$$\Delta E_j = -Z_i e E_0 l_1 \sin\left[k_0 D\cdot\delta_j\right]\exp\left[-\frac{\left(D\cdot\delta_j\right)^2}{2\sigma_c^2}\right]$$

$$-Z_i e E_0 l_1 \sum_{i\neq j}\sin\left[k_0\left(D\cdot\delta_j+\varsigma_j-\varsigma_i\right)\right]\exp\left[-\frac{\left(D\cdot\delta_j+\varsigma_j-\varsigma_i\right)^2}{2\sigma_c^2}\right],$$

$$= \Delta E_{coh,j} + \Delta E_{inc,j}$$

Diffusive kick at cooling section

$$\left\langle \Delta E_{inc,j}^2 \right\rangle \equiv \left(Z_i e E_0 l_1\right)^2 \sum_{i,k\neq j} \left\langle \begin{array}{l} \sin\left[k_0\left(D\cdot\delta_j+\varsigma_j-\varsigma_i\right)\right]\sin\left[k_0\left(D\cdot\delta_j+\varsigma_j-\varsigma_k\right)\right] \\ \times\exp\left[-\frac{\left(D\cdot\delta_j+\varsigma_j-\varsigma_i\right)^2+\left(D\cdot\delta_j+\varsigma_j-\varsigma_k\right)^2}{2\sigma_c^2}\right] \end{array} \right\rangle$$

$$= \left(Z_i e E_0 l_1\right)^2 \sum_{i\neq j} \left\langle \sin^2\left[k_0\left(z_j-\varsigma_i\right)\right]\exp\left[-\frac{\left(z_j-\varsigma_i\right)^2}{\sigma_c^2}\right] \right\rangle$$

$$+\left(Z_i e E_0 l_1\right)^2 \sum_{i\neq k\neq j} \left\langle \sin\left[k_0\left(z_j-\varsigma_i\right)\right]\sin\left[k_0\left(z_j-\varsigma_k\right)\right]\exp\left[-\frac{\left(z_j-\varsigma_i\right)^2+\left(z_j-\varsigma_k\right)^2}{2\sigma_c^2}\right] \right\rangle ,$$

Diffusive kick at cooling section

$$\left\langle \Delta E_{inc,j}^{2} \right\rangle = \left(Z_i e E_0 l_1\right)^2 \sum_{i \neq j} \left\langle \sin^2\left[k_0\left(z_j - \varsigma_i\right)\right] \exp\left[-\frac{\left(z_j - \varsigma_i\right)^2}{\sigma_c^2}\right] \right\rangle$$

$$= \left(Z_i e E_0 l_1\right)^2 \int_{-\infty}^{\infty} \rho\left(\varsigma_i\right) \sin^2\left[k_0\left(z_j - \varsigma_i\right)\right] \exp\left[-\frac{\left(z_j - \varsigma_i\right)^2}{\sigma_c^2}\right] d\varsigma_i$$

$$\approx \left(Z_i e E_0 l_1\right)^2 \rho\left(\varsigma_j\right) \int_{-\infty}^{\infty} \sin^2\left[k_0\left(z_j - \varsigma_i\right)\right] \exp\left[-\frac{\left(z_j - \varsigma_i\right)^2}{\sigma_c^2}\right] d\varsigma_i$$

$$= \frac{\sqrt{\pi}\rho\left(\varsigma_j\right)\sigma_c}{2} \left(Z_i e E_0 l_1\right)^2 \left[1 - \exp\left(-k_0^{\,2} \sigma_c^{\,2}\right)\right]$$