Homework 14. Due November 2

Problem 1. 2 x 12 points. Sextupoles and octupoles, or un-harmonic oscillators

For an simple harmonic oscillator

$$H = \frac{p^2}{2} + \omega^2 \frac{x^2}{2}$$

consider adding and additional term:

(a) 4th order (octupole):

$$\delta H = \varepsilon \frac{x^4}{4}$$

(b) 3rd order (sextupole)

$$H = \frac{p^2}{2} + \omega^2 \frac{x^2}{2} + \varepsilon \frac{x^3}{3}$$

1. For both case write reduces equations of motion using action (or amplitude – whatever you prefer) and angle variables. Use perturbation method and find first order variation of the "slow" variables. Show that there is a first order frequency dependence (e.g. a constant time-averaged phase advance) on the action of the oscillator (amplitude of oscillation squared) for 4th order perturbation but there is no such term. Show that simply averaging the δH over time gives the same result.
2. For sextupole case, go to the second order of the perturbation theory. Calculate $\varepsilon\left\langle F(\xi, s) + \varepsilon\tilde{F}\right\rangle$ from Bogolyubov and Metropolsky method. Show that there is a frequency dependence on the amplitude of oscillation $\sim \varepsilon^2 a^2$.