BEAM DIAGNOSTIC IN 112 MHz SUPER- CONDUCTING RF ELECTRON GUN WITH CsK2Sb PHOTO-CATHODE: EMITTANCE MEASUREMENT AND PHASE SPACE RECONSTRUCTION

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06/06

OUTLINE

- Vlasov Equation and Emittance
- Magnet Scan
- Slit Method
- Phase Space Reconstruction

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In classical mechanics, Hamiltonian formalism leads to the conservation of phase space density => Liouville's theorem $\frac{d\rho}{dt} = 0$.

It leads to Vlasov equation even under the self-consistent collective field created by the charged plasma, equation

$$\underbrace{\frac{d}{dt}\int_{V_6} f(\mathbf{x},\mathbf{v},t)d^3xd^3v}_{N(t)} + \underbrace{\int_{V_6} \nabla_{\mathbf{x},\mathbf{v}} \cdot [f(\mathbf{x},\mathbf{v},t)(\dot{\mathbf{x}},\dot{\mathbf{v}})]d^3xd^3v}_{\oint_{S_6} f(\mathbf{x},\mathbf{v},t)(\dot{\mathbf{x}},\dot{\mathbf{v}})\cdot d\mathbf{s}_6} = 0$$

$$\frac{d}{dt}N(t) + \oint_{S_6} f(\mathbf{x},\mathbf{v},t)(\dot{\mathbf{x}},\dot{\mathbf{v}})\cdot d\mathbf{s}_6 = 0,$$

$$P(p_x) = C_p \exp\left(-\frac{p_x^2}{2\gamma_0 m_0 k_B T_x}\right) \equiv C_p \exp\left(-\frac{p_x^2}{2\sigma_{p_x}^2}\right)$$

states that the rate of increase (decrease) of the total number of particles in equals the partcle flux into (out of) volume.

Equilibrium solution indicates the Gaussian momentum distribution
called Maxwellian distribution in classical statistical mechanics.
Denoted in terms of transverse temperature

Moments of distribuiton function

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^n p_x^m f(x, p_x)\, dx\, dp_x.$$

$$\sigma_{11} \equiv \sigma_x^2 = \langle x^2 \rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 f_x(x, x')\, dx\, dx'$$

$$\sigma_{22} \equiv \sigma_{x'}^2 = \langle x'^2 \rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x'^2 f_x(x, x')\, dx\, dx'$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_x(x, x')\, dx\, dx' = 1$$

$$\sigma_{12} = \sigma_{21} \equiv \sigma_{xx'} = \langle xx' \rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xx' f(x, x')\, dx\, dx'.$$

$$\langle x' \rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x' f_x(x, x')\, dx\, dx'.$$

The full distribution function contains all of the information needed to describe the state of non-interacting ensemble of beam particles. But instead of whole information moments of the distribution are used to arrive at a simpler description of the distribution’s evolution

Linear transformations of the first and second moments

$$\bar{X}_i = \langle \bar{x}_i \rangle = \langle \sum_{j=1}^{n} R_{ij} x_j \rangle = \sum_{j=1}^{n} R_{ij} \langle x_j \rangle = \sum_{j=1}^{n} R_{ij} X_j$$

$$\bar{\sigma}_{ij} = \langle \bar{x}_i \bar{x}_j \rangle = \langle \sum_{k=1}^{n} R_{ik} x_k \sum_{l=1}^{n} R_{jl} x_l \rangle = \sum_{k=1}^{n} R_{ik} \sum_{l=1}^{n} R_{jl} \langle x_k x_l \rangle = \sum_{k=1}^{n} \sum_{l=1}^{n} R_{ik} R_{jl} \sigma_{kl}$$

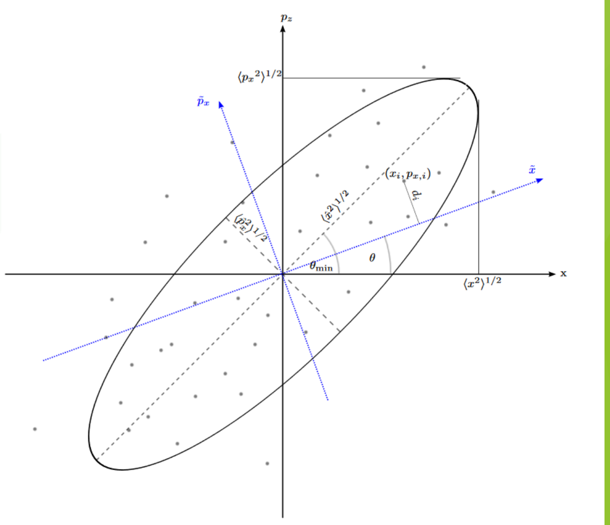
or in matrix form $\bar{\sigma} = R \sigma R^T$

If the initial trace space is transformed by transportation matrix representing the portion of beam line,
Using the fact that transportation matrix has unity determinant

$$\sigma \equiv \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

$$\det \bar{\sigma} = \det(R \sigma R^T) = \det(R) \det(\sigma) \det(R^T) = \det \sigma$$

$$\epsilon_{tr,rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x\, x' \rangle^2}$$



This constant of motion for linear transformation is related to the trace space invariants which corresponds to the phase space area we formalized in single particle dynamics

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Application of second moment transformation to emittance measurement

$$
\begin{aligned}
{x_{rms}}^2 = \langle x^2 \rangle &= \langle (C_{11}\, x_0)^2 \rangle + \langle 2\, C_{11}\, C_{12}\, x_0\, x_0' \rangle + \langle (C_{12}\, x_0')^2 \rangle \\
&= {C_{11}}^2\, \langle {x_0}^2 \rangle + 2\, C_{11}\, C_{12}\, \langle x_0\, x_0' \rangle + {C_{12}}^2\, \langle {x_0'}^2 \rangle .
\end{aligned}
$$

$$
\langle x'^2 \rangle = \frac{{\epsilon_{rms}}^2}{\langle x^2 \rangle} + \frac{\langle x\, x' \rangle^2}{\langle x^2 \rangle} = \frac{{\epsilon_{rms}}^2}{{x_{rms}}^2} + (x_{rms})'^2
$$

$$
{x_{rms}}^2 = {C_{11}}^2\, {x_{0,rms}}^2 + 2\, C_{11}\, C_{12}\, x_{0,rms}\, (x_{0,rms})' + {C_{12}}^2 \left(\frac{{\epsilon_{rms}}^2}{{x_{0,rms}}^2} + (x_{0,rms})'^2 \right) .
$$

Simple system which we used for magnetic scan is solenoid and drift space, under the thin lens approximation,

$$
\begin{aligned}
\Sigma_{11} (= \langle x^2 \rangle) = &({S_{11}}^2 \Sigma_{11_0} + 2 S_{11} S_{12} \Sigma_{12_0} + {S_{12}}^2 \Sigma_{22_0}) \\
&+ (2 S_{11} S_{12} \Sigma_{11_0} + 2 {S_{12}}^2 \Sigma_{12_0}) K + {S_{12}}^2 \Sigma_{11_0} K^2
\end{aligned}
$$

Particle dynamics inside of solenoid

The rotationally symmetric fields can be expanded in a polynomial series

$$B_z(z,r) = B_{z,0} - \frac{r^2}{4}\frac{\mathrm{d}^2}{\mathrm{d}z^2}B_z(z) + \frac{r^4}{64}\frac{\mathrm{d}^4}{\mathrm{d}z^4}B_z(z)\ldots$$

$$B_r(z,r) = -\frac{r}{2}\frac{\mathrm{d}}{\mathrm{d}z}B_z(z) + \frac{r^3}{16}\frac{\mathrm{d}^3}{\mathrm{d}z^3}B_z(z) - \frac{r^5}{384}\frac{\mathrm{d}^5}{\mathrm{d}z^5}B_z(z)\ldots$$

The equations of motion of a charged particle inside a solenoid field are coupled differential equations. In cylindrical coordinate

$$\frac{1}{f} = \int \left(\frac{q\,{B_z}^2}{2\,p_z}\right)^2 \mathrm{d}z.$$

Focusing: $$\gamma\, m_0 \left(\ddot{r} - r\,\dot{\theta}^2\right) = q\, r\, \dot{\theta}\, B_z$$

Rotation: $$\gamma\, m_0 \left(2\,\dot{r}\,\dot{\theta} + r\,\ddot{\theta}\right) = -q\left(\frac{r\,\dot{z}}{2}\,{B_z}' + \dot{r}\,B_z\right)$$

$$\theta = -\int \frac{q\,B_z}{2\,p_z}\,\mathrm{d}z.$$

Demonstration of principal measurement by Solenoid Scan method

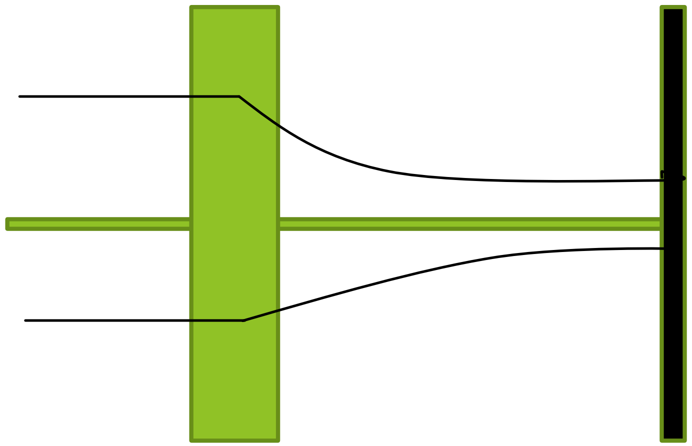


Fig: Simple system we use:
solenoid+drift

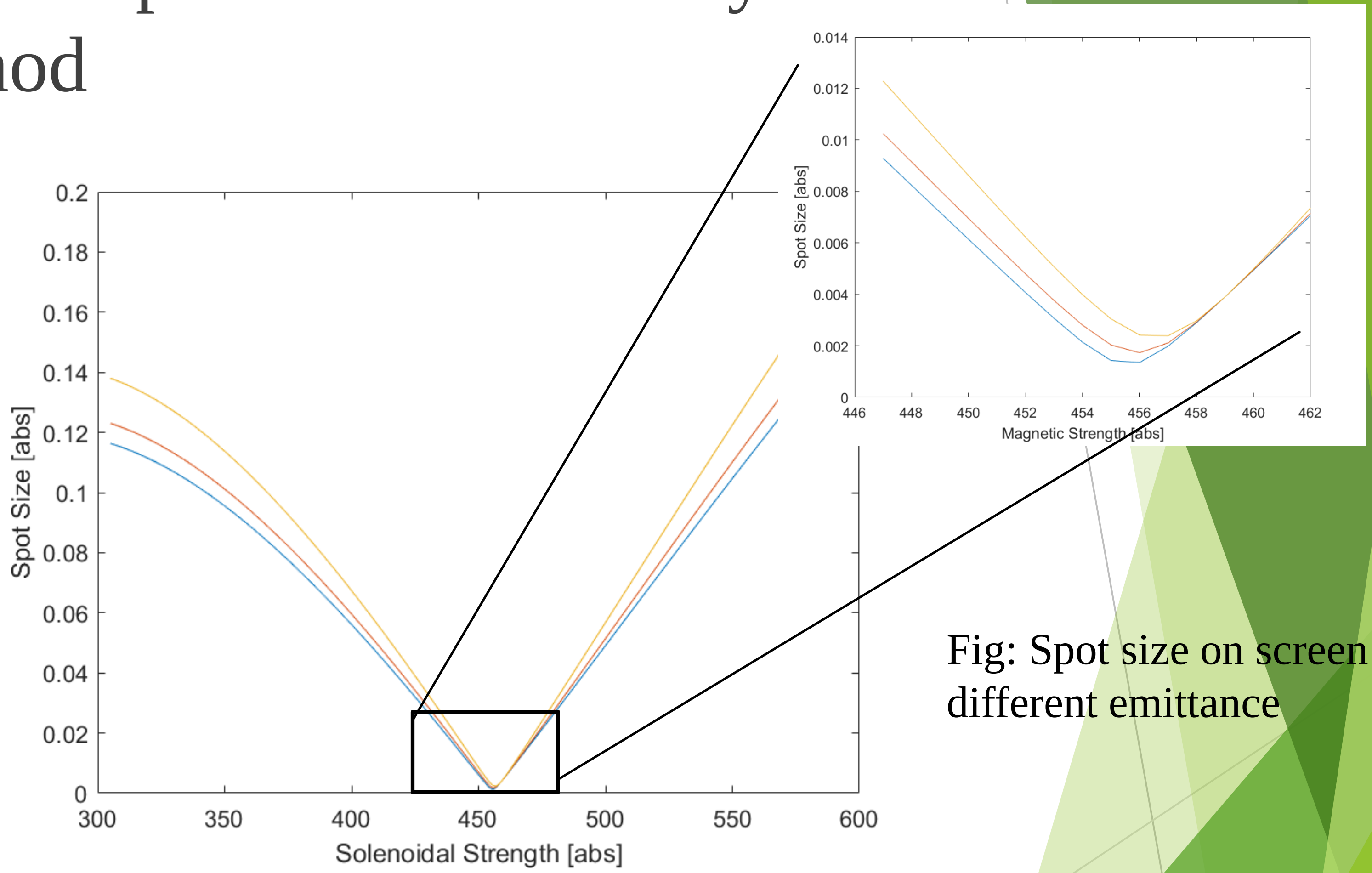
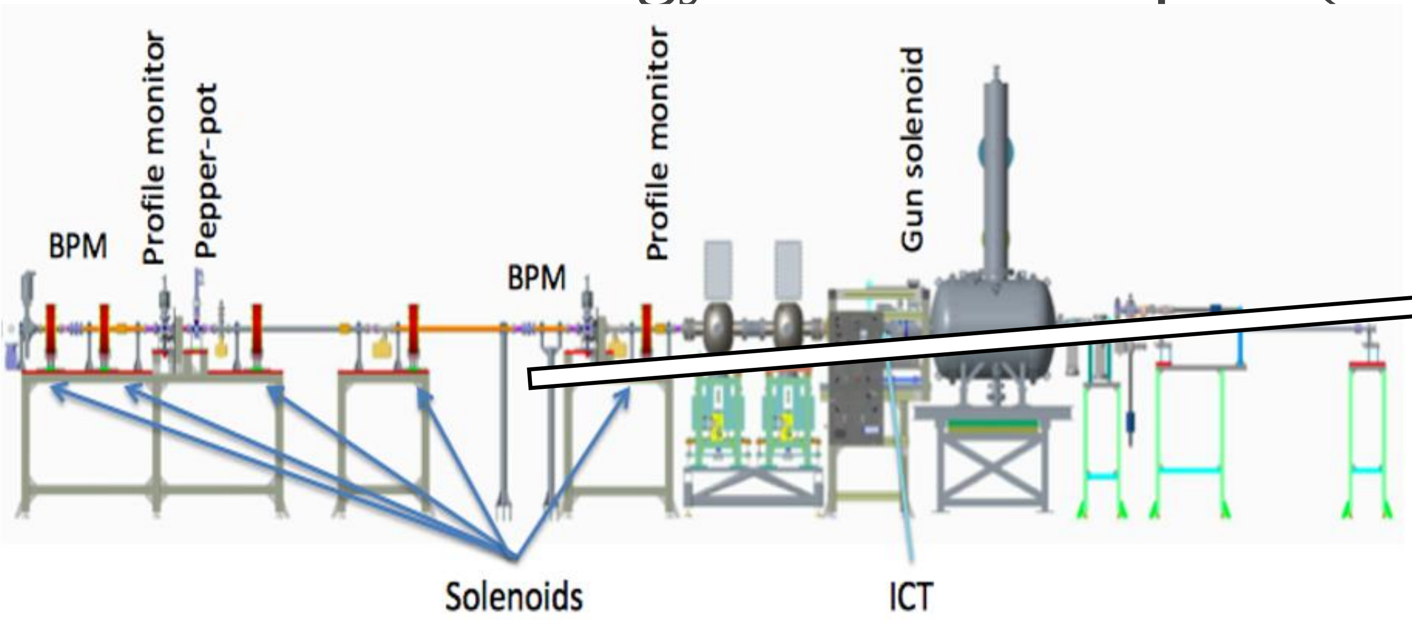
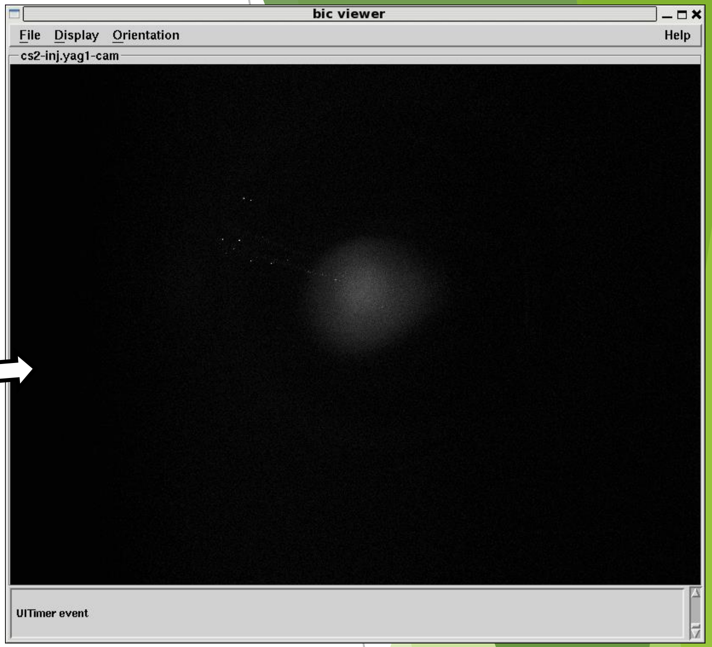


Fig: Spot size on screen with different emittance

Slight change of depth of minima indicates this method is most sensitive to beam size right at the minimum so major aim of this method to determine the minima of spot size.

CeC Low Energy Beam Transport (LEBT)





At 4.28 m, 8.14m

Gun Solenoid

Parameter	Value
Maximal current	13.4 A
Peak axial field	0.693 kGs
Integral B_z dz	12.63 kGs cm
Linear magnetic length	18.2 cm
Integral B_z^2dz	6.14 kGs^2 cm
Quadratic magnetic length	12.8 cm
Resistance	1.12 Ohm

At 0.65 m

Beam Line Solenoids

Parameter	Value
Maximal current	8.4 A
Peak axial field	1 kGs
Integral B_z dz	12.24 kGs cm
Linear magnetic length	12.2 cm
Integral B_z^2dz	8.47 kGs^2 cm
Quadratic magnetic length	16.7 cm
Resistance	1.6 Ohm

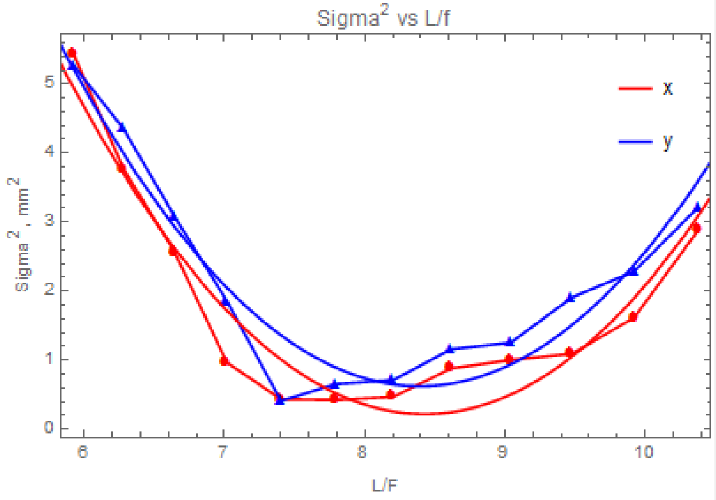
At 3.65 m, 7.17m

Fig; LEBT Schematics and Solenoid Property (Left), Picture obtained in YAG Profile Monitor 1 (Top Right)

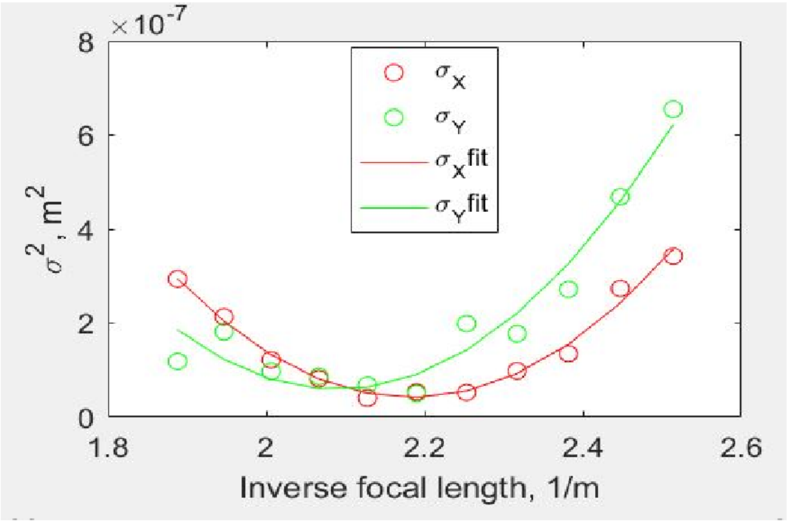
There are one Gun Compensation Solenoid and 5 Solenoids to keep focusing the beam.

Sol-Scan Result

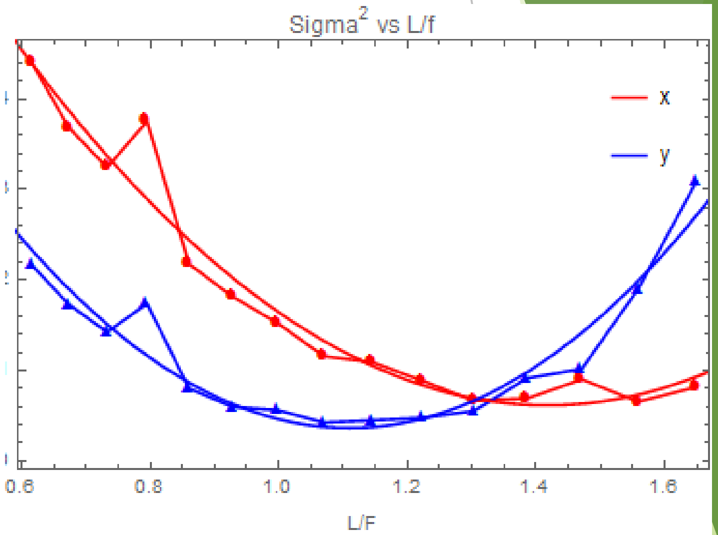
Beam Kinetic Energy	Charge per Bunch
1.05 MeV	0.5 nC



(a)



(b)



(c)

Fig: Results of Three Emittance Measurements Performed Using Three Different Solenoid's Scans: (a) the Gun Solenoid and YAG1 Profile Monitor; (b) the LEBT1 Solenoid and YAG1 Profile Monitor; (c) LEBT3 Solenoid and YAG2 Profile Monitor

(a), (b), (c) gave 0.32, 0.94, 0.54 mm mrad normalized emittance

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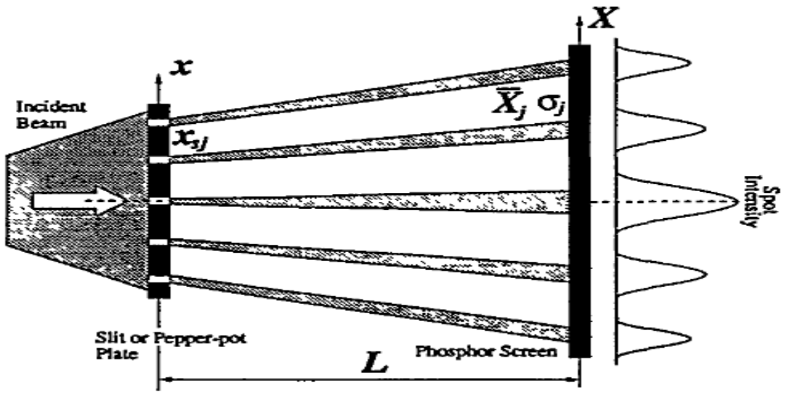
Slit measurement

Using the two coordinates x on the slit, X on the phosphor screen. Phase space information x and x' have simple relation as

$$x_i' \equiv \frac{X_i - x_i}{L}$$



$$\epsilon_x \equiv \sqrt{< x^2 >< x'^2 > - < xx' >^2}$$

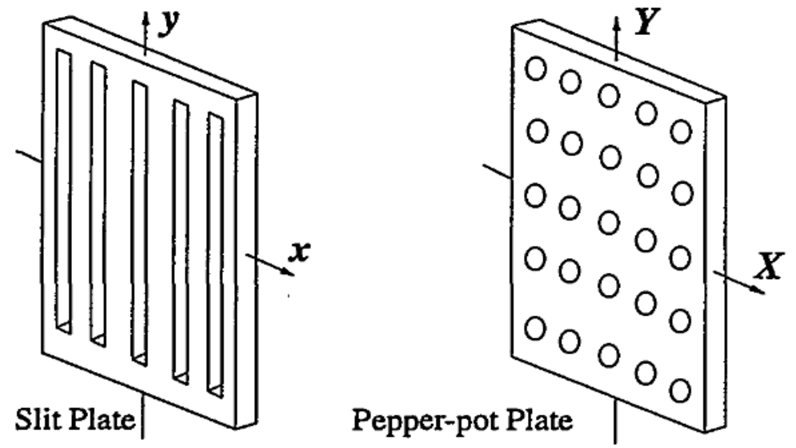


Fig: beam mask in emittance measurement: slit plate (left) and pepper pot plate (right)

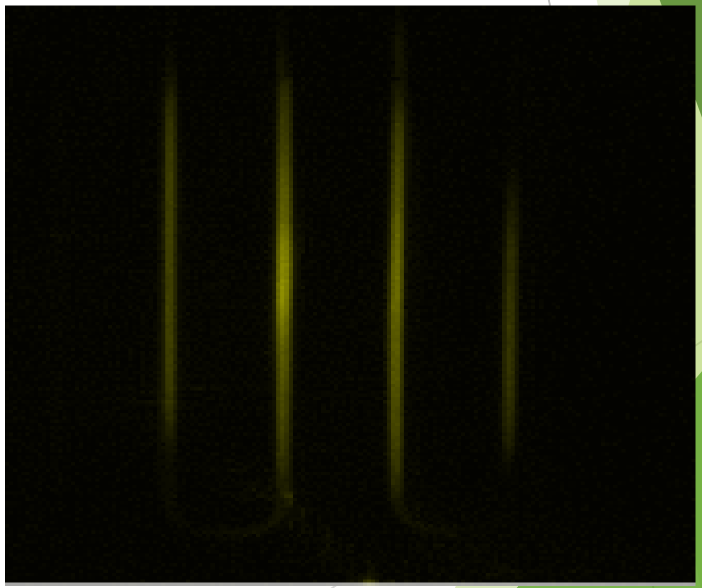
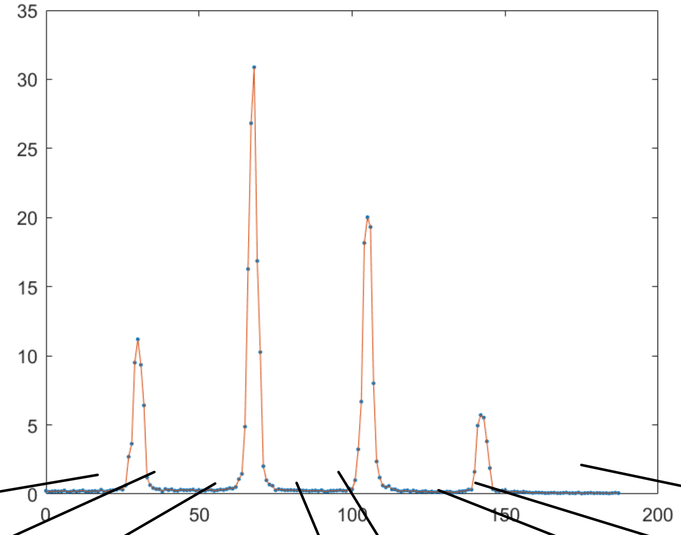


Fig: Picture taken at YAG2 Profile (at 8.14m) Monitor with Slit Inserted (at 7.79m) with Slit Width 0.2mm Separated by 2 mm

Slit Measurement

Each spike corresponds to X,
X’ and its spread is determined by Gaussian fitting



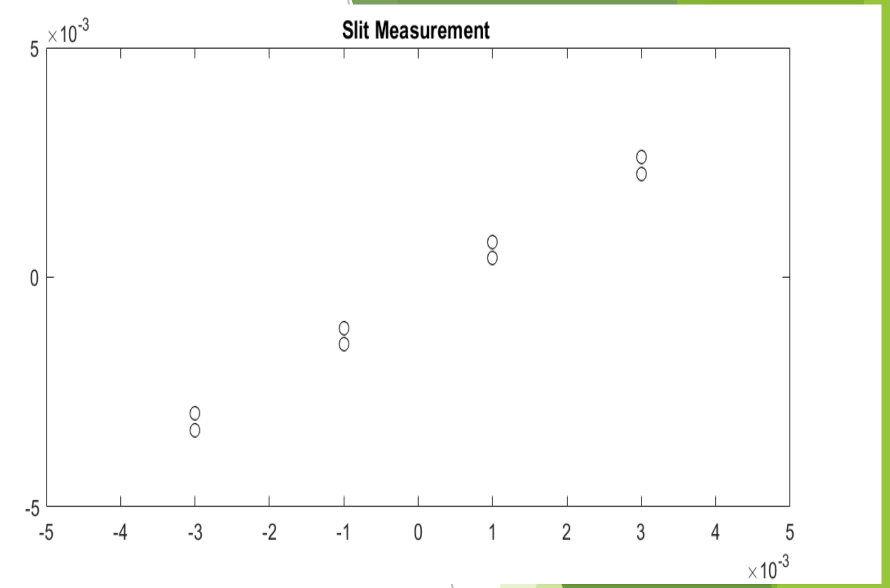
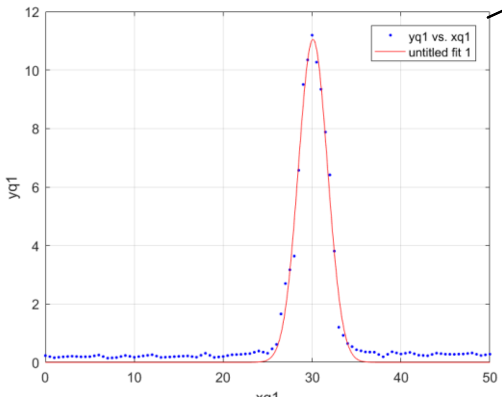
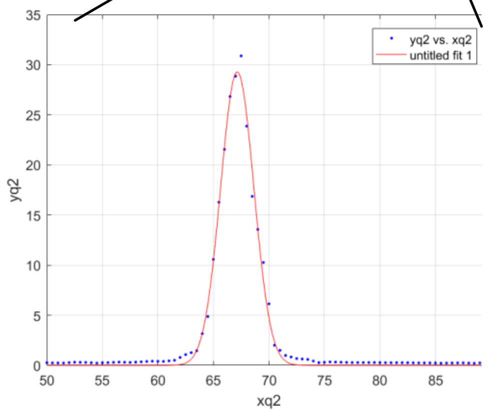


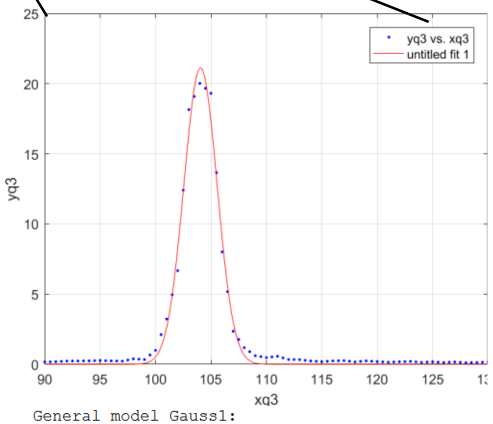
Fig: Slit Measurement Result



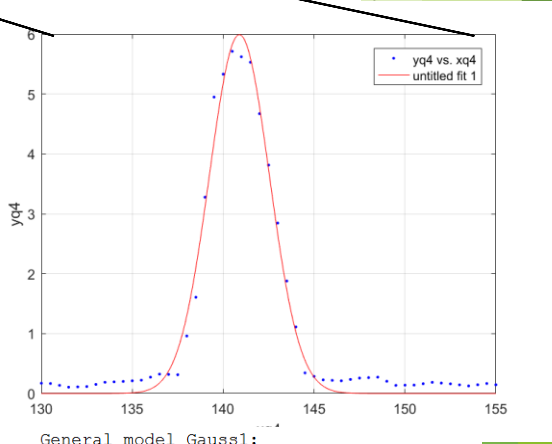

```
General model Gauss1:
a1(x) =  a1*exp(-((x-b1)/c1)^2)
Coefficients (with 95% confidence bounds):
  a1 =       11.05  (10.71, 11.39)
  b1 =        30.1  (30.04, 30.16)
  c1 =       2.313  (2.232, 2.395)
```




```
General model Gauss1:
a2(x) =  a1*exp(-((x-b1)/c1)^2)
Coefficients (with 95% confidence bounds):
  a1 =       29.28  (28.63, 29.93)
  b1 =       67.15  (67.11, 67.19)
  c1 =       2.119  (2.064, 2.173)
```




```
General model Gauss1:
a3(x) =  a1*exp(-((x-b1)/c1)^2)
Coefficients (with 95% confidence bounds):
  a1 =       21.13  (20.59, 21.66)
  b1 =       104.1  (104, 104.1)
  c1 =       2.117  (2.055, 2.179)
```




```
General model Gauss1:
a4(x) =  a1*exp(-((x-b1)/c1)^2)
Coefficients (with 95% confidence bounds):
  a1 =        5.99  (5.759, 6.221)
  b1 =       140.9  (140.8, 141)
  c1 =       2.339  (2.235, 2.443)
```

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Tomography for Reconstruction and Radon Transformation

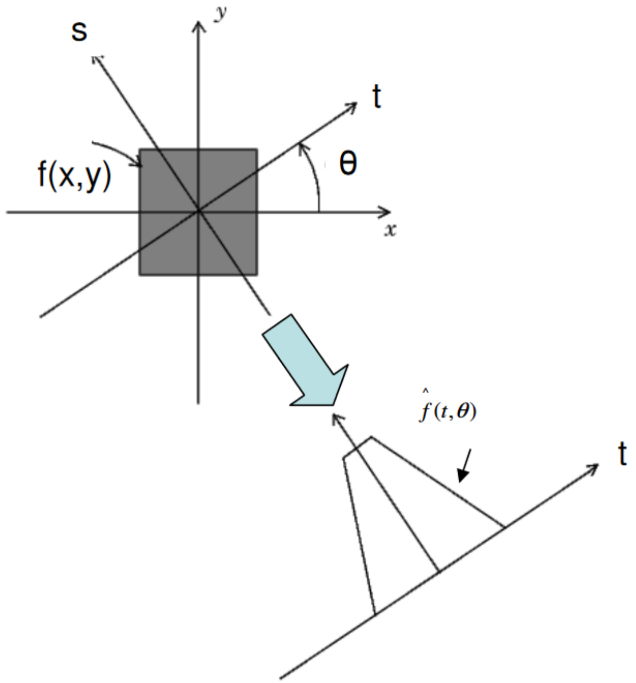


Fig: Illustration of the Radon Transform

$$\hat{f}(t,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dxdy f(x,y)\delta(x\cos\theta + y\sin\theta - t)$$

$$S(w,\theta) = \int_{-\infty}^{\infty} \hat{f}(t,\theta) e^{-j2\pi wt} dt$$

$$f(x,y) = \int_{0}^{\pi}\int_{-\infty}^{\infty} S(w,\theta)\,|w|\,e^{j2\pi wt} dw d\theta,$$

if a number of projections between 0 and π are known, the distribution can be reconstructed by back-projecting the filtered version of the projections.

Extension to the beam physics

Profile on X plane calculated from configuration space

$$c(x) = \int g(x, y)dy = \iiint \mu(x, x', y, y')dx'dy'dy$$

Profile on X plane calculated from Phase space

$$h(x) = \int \mu(x, x')_z dx' = \iiint \mu(x, x', y, y')dydy'dx'$$

Hence, radon transformation of phase space is related to that of configuration.

- Extension to the beam physics

$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = M_1 \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$

$$c(x) = \iint \mu(x_1, x_1')_{z_1} \delta(x_1 - x) dx_1 dx_1' \qquad \mu(x_0, x_0')_{z_0} = \mu(x_1, x_1')_{z_1}$$

$$c(x) = \iint \mu(x_0, x_0')_{z_0} \delta(M_{11} x_0 + M_{12} x_0' - x) dx_0 dx_0'$$

$$s = \sqrt{M_{11}^2 + M_{12}^2} \qquad \tan\theta = \frac{M_{12}}{M_{11}}$$

$$c(x) = \frac{1}{s} \iint \mu(x_0, x_0')_{z_0} \delta(x_0 \cos\theta + x_0' \sin\theta - \rho) dx_0 dx_0'$$

Introduce the delta function in integral.
Using the fact that distribution density is not changed along the transportation, Expression becomes like on the last line.

- Extension to the beam physics

$$c(x) = \frac{1}{s}\int\int \mu(x_0, x_0^{'})_{z_0}\,\delta(x_0\cos\theta + x_0^{'}\sin\theta - \rho)\,dx_0 dx_0^{'}$$

$$\rho = x/s$$

$$\hat{f}(t,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dxdy f(x,y)\delta(x\cos\theta + y\sin\theta - t)$$

$$\hat{\mu}(\rho,\theta)_{z_0} = \hat{\mu}(x/s,\theta)_{z_0} = sc(x/s)$$

we can deduce that a simple scaling equation derived previous slide relates the spatial beam projections to the Radon transform of the transverse phase space.

- Procedure and current setting
- 1.Calculate the transportation matrix + scaling factor and angle
- 2.Get the image and profile of beam at Phosphor screen
- 3.Relate the profile and radon transformation
- 4.Collect the enough profile and have them back-filtered
- 5.Take integration over the angle to reconstruct the phase space

Using Two Solenoid LEBT2 and LEBT3 solenoid, rotation angle and scaling factor need to be determined.
I used fixed LEBT2 solenoid and varying LEBT3 solenoid to cover all the angle from -90 to 90 degree.

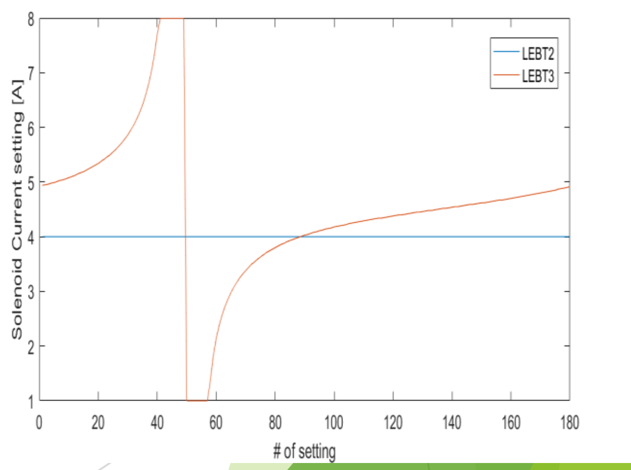
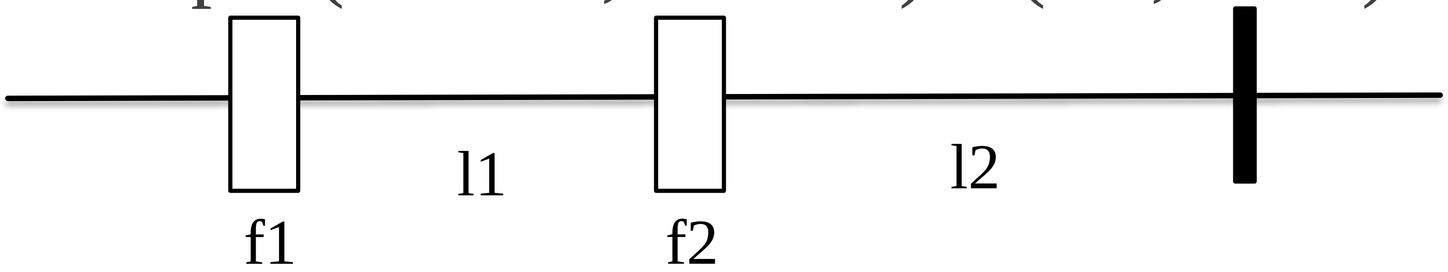


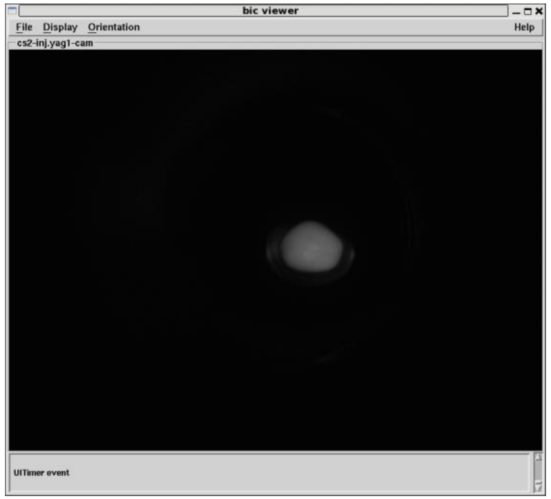
Fig: Solenoid Setting Used for Tomography

Example (LEBT2, LEBT3) = (4A, 5.6A) at 6.17, 7.17 m



$$\begin{pmatrix} 1 & 0.9638 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -3.88786 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1.01 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1.9836 & 1 \end{pmatrix} = \begin{pmatrix} 0.84477 & -1.81079 \\ 1.91763 & -2.92674 \end{pmatrix}$$

Θ = ArcTan(0.845/(-1.81)) = -64.99, s = Sqrt(0.845^2 + (-1.81^2)) = 1.998



At 8.14 m

Projection

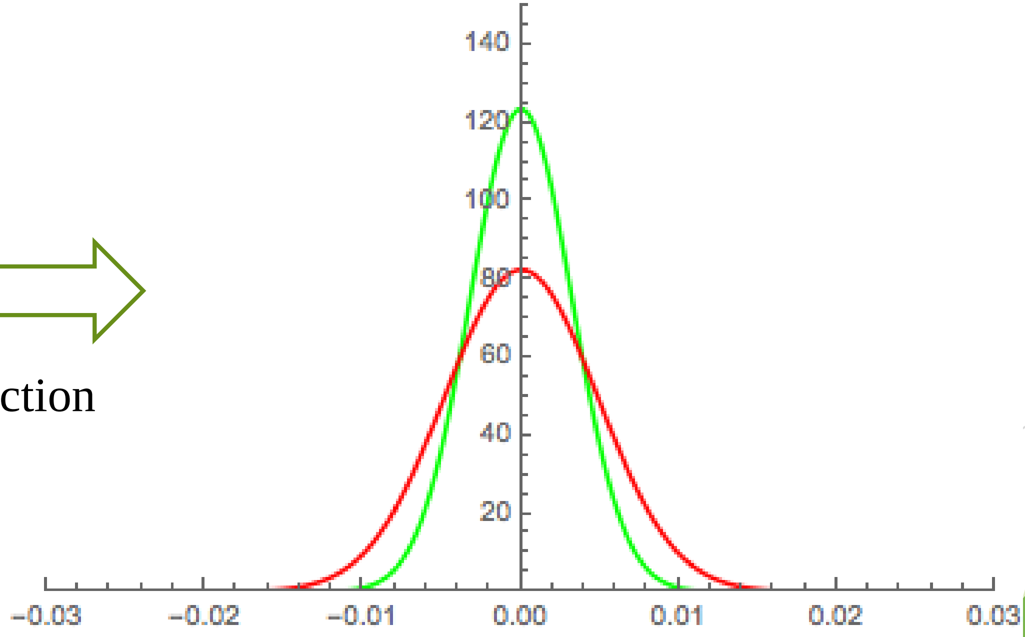
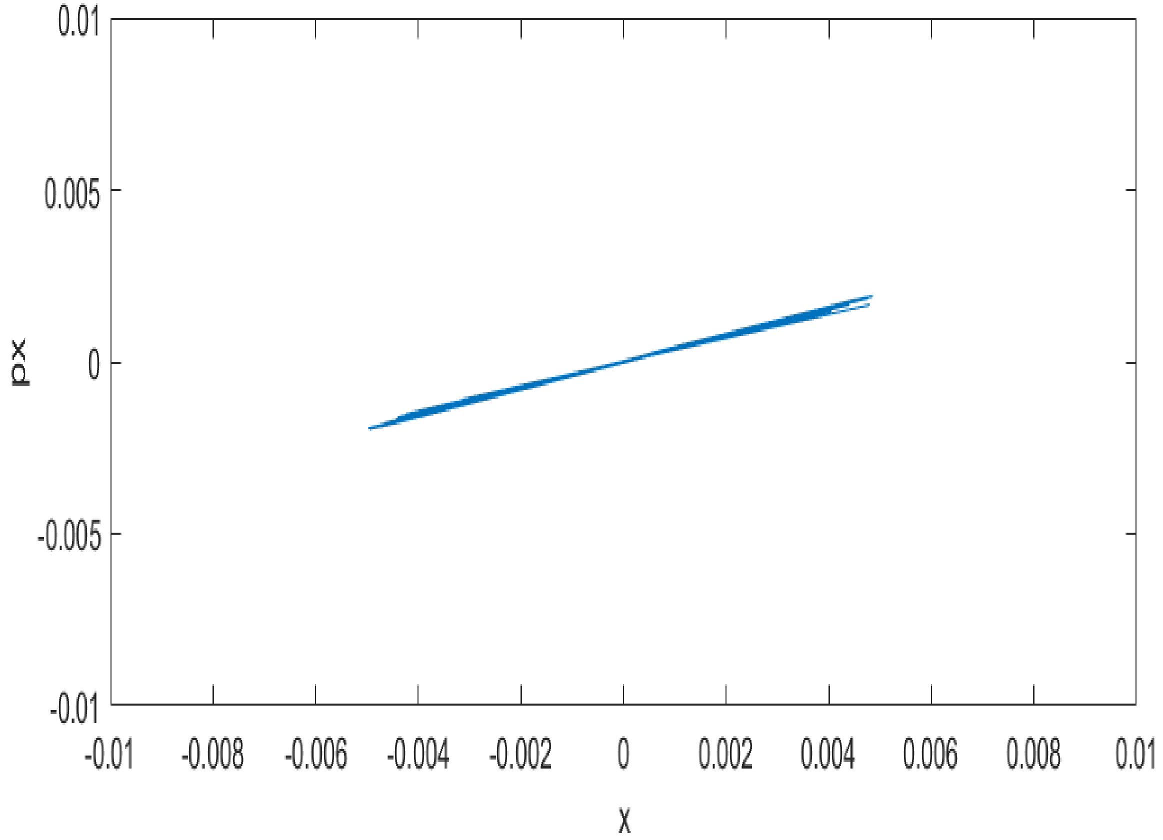


Fig: Configuration Space Distribution(Left), Transformation from projection (Red) to the radon transformation (Green) by scaling (Right)

Comparison of the Result



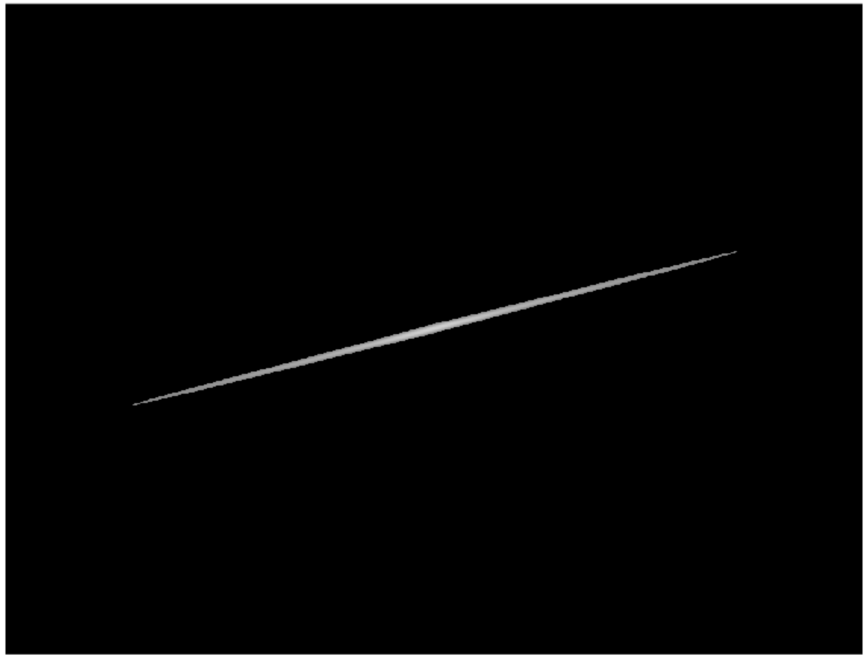


Fig: Phase Space at the entrance of LEBT2 Solenoid Simulated by PARMELA (Left) and Same by Reconstructed by Filtered Back Projection Method (Right)

- Conclusion

1. Started from the vlasov equation to generalize the description of charged particle by second moments.

2. We saw the lowest order information called envelope follows the linear transformation and led to the constant of motion.

3. Emittance measurement by solenoidal scan and slit method has been presented.

4. 112 Mhz Gun developed for CeC has extraordinary emittance performance for high bunch charge.

5. Transverse phase space reconstruction via Tomography method is presented.

Thank you for your attention !