Homework 14. Due October 28

Problem 1. 15 points. Oscillating external force.

For a linear one-dimensional motion of particle in an storage ring with circumference C consider an oscillating force applied to the particle:

$$\frac{d}{ds}\begin{bmatrix} x \\ p \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K_1(s) & 0 \end{bmatrix}\begin{bmatrix} x \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ f(s)\cos\omega t \end{bmatrix};$$

$$K_1(s+C) = K_1(s); f(s+C) = f(s); t = \frac{s}{v_o}; \mu_e = C\frac{\omega}{v_o} = 2\pi Q_e.$$

Betatron tune and the eigen vectors are known function

$$\mu_x = 2\pi Q_x;\ \ Y(s) = Y(s+C) = \begin{bmatrix} w(s) \\ w'(s) + \dfrac{i}{w(s)} \end{bmatrix}$$

are considered to be known.

(a) Find solution in a form

$$x = x_e(s) + x_o(s)$$

where is $x_o(s)$ well know free oscillations and forced oscillations

$$x_e(s) = b(s)\cos(\omega t + \varphi); b(s+C) = b(s)$$

Find expression for $b(s)$ in a form of integral over the ring circumference.

Hint: use class notes for a general case and apply it to 1D

(b) Find and write down resonant conditions, when amplitude of oscillation is unlimited.