Homework 7.

Problem 1 – 10 points.

Consider a flat (no torsion, $\kappa = 0$) trajectory in a bending magnet, with constant radius. Assume that there is no electric field ($\vec{E} = 0$, energy is constant) and there in no x-y coupling (skew quadrupole components and longitudinal magnetic field are zero: $\frac{\partial B_x}{\partial x} = \frac{\partial B_y}{\partial y} = 0$; $B_s = 0$ - this also make mechanical and Canonical momenta equal). Assume that there is a quadrupole component allowed in the magnet

$$\frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} \neq 0$$

The particle's Hamiltonian is then split in two uncoupled parts: (y, p_y) and (x, p_x, τ, δ)

$$\tilde{h} = h_H(x, p_x, \tau, \delta) + h_V(y, p_y)\ h_H = \frac{p_x^2}{2 \cdot p_o} + F \cdot \frac{x^2}{2} + \frac{\delta^2}{2 \cdot p_o} \cdot \left(\frac{mc}{p_o}\right)^2 + g_x \cdot x \cdot \delta;\ h_V = \frac{p_y^2}{2 \cdot p_o} + G \cdot \frac{y^2}{2};$$

with (see Lecture 4 formulae (140 and 141)

$$K_1 = -\frac{G}{p_o} = -\frac{e}{p_o c}\frac{\partial B_x}{\partial y};\ \frac{F}{p_o} = K^2 + K_1;\ K \equiv K_o = -\frac{eB_y}{p_o c};\ \beta_o = \frac{p_o c}{E_o};\ g_x = -\frac{K}{\beta_o};\ \frac{mc}{p_o} = \frac{1}{\gamma_o \beta_o};$$

where I introduced K_o and K_1 notions frequently used in accelerator literature. Because p_o is constant, I would recommend you switch to switch to dimensionless momenta

$$p_{x,y} \to \frac{p_{x,y}}{p_o};\ \delta \to \frac{\delta}{p_o};$$

$$h_H = \frac{p_x^2}{2} + \left(K_o^{\ 2} + K_1\right) \cdot \frac{x^2}{2} + \frac{\delta^2}{2 \cdot (\gamma_o \beta_o)^2} - \frac{K_o}{\beta_o} \cdot x \cdot \delta;\ h_V = \frac{p_y^2}{2} - K_1 \cdot \frac{y^2}{2};$$

Since vertical motion is completely separated from that horizontal plane, there is a little incentive to use 6x6 matrices.

3 point. Unless you want to use 6x6 matrices, derive explicit 2x2 matrices for vertical motion without any assumptions about the sign and value of K_1 (i.e. all 3 cases).

7 point. Similarly, derive 4x4 matrices to calculate motion in horizontal plane without any assumptions of the sign and value of K_1 and value of $\beta_o \neq 0$. Compare it with expression we derived in Lecture 8.

Hint: Motion in horizontal plane has zero *det***H**.